Exercise 2.1

Q.1 Identity which of the following are rational and irrational numbers?

(i)	$\sqrt{3}$	Irrational number
(ii)	$\frac{1}{6}$	Rational number
(iii)	π	Irrational number
(iv)	$\frac{15}{2}$	Rational number
(v)	7.25	Rational number
(vi)	$\sqrt{29}$	Irrational number

Q.2 Convert the following fractions into decimal fractions.

(i) $\frac{17}{25}$

Solution: $\frac{17}{25}$

$$\begin{array}{r} 0.68 \\ 25\overline{)\ 170} \\ -150 \\ \hline 200 \\ -200 \\ \hline 0 \end{array}$$

$\frac{17}{25} = 0.68$ **Ans**

(ii) $\frac{19}{4}$

Solution: $\frac{19}{4}$

$$\begin{array}{r} 4.75 \\ 4\overline{)19.000} \\ 16 \\ \hline 30 \\ 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

$= \frac{19}{4}$

$= 4.75$ **Ans**

(iii) $\frac{57}{8}$

Solution: $\frac{57}{8}$

$$\begin{array}{r} 7.125 \\ 8\overline{)\ 57} \\ -56 \\ \hline 10 \\ 8 \\ \hline 20 \\ -16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

$= \frac{57}{8}$

$= 7.125$ **Ans**

(iv) $\frac{205}{18}$

Solution: $\frac{205}{18}$

$$\begin{array}{r} 11.388 \\ 18\overline{)205.000} \\ \hline 25 \end{array}$$

18

70
-54

160
-144

160
-144

16

$\frac{208}{18}$

$= 11.3888$

$= 11.3889$ **Ans**

(v) $\frac{5}{8}$

Solution: $\frac{5}{8}$

$$8\overline{)5.000}\quad .625$$

48

20
-16

40
-40

0

$\frac{5}{8}$

$= 0.625$ **Ans**

(vi) $\frac{25}{38}$

Solution: $\frac{25}{38}$

$$38\overline{)250}\quad 0.65789...$$

-228

220
-190

300
-266

340
-304

360
-342

18

$\frac{25}{38}$

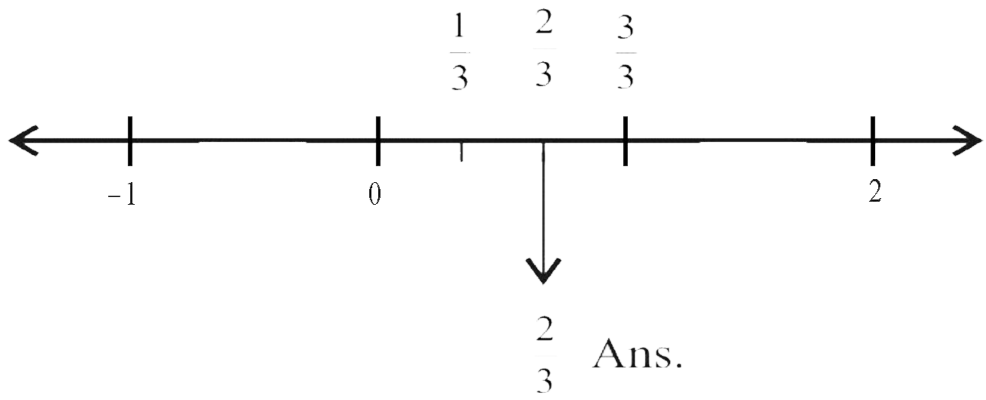
$= 0.65789$ **Ans**

Q.3 Which of the following statements are true and which are false?

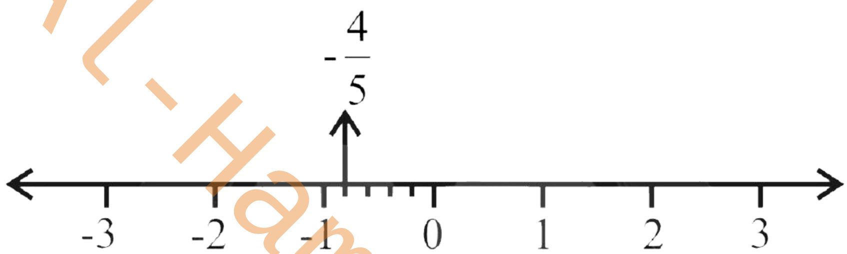
(i)	$\frac{2}{3}$ is an irrational number.	False
(ii)	π is an irrational number.	True
(iii)	$\frac{1}{9}$ is a terminating fraction.	False
(iv)	$\frac{3}{4}$ is a terminating fraction.	True
(v)	$\frac{4}{5}$ is a recurring fraction.	False

Q.4 Represent the following numbers on the number line.

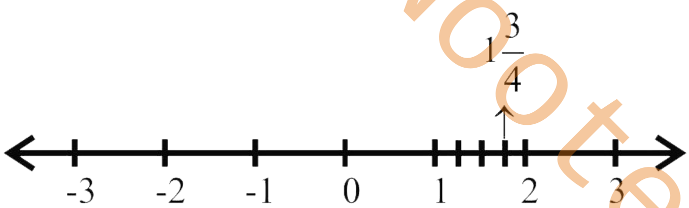
(i) $\frac{2}{3}$



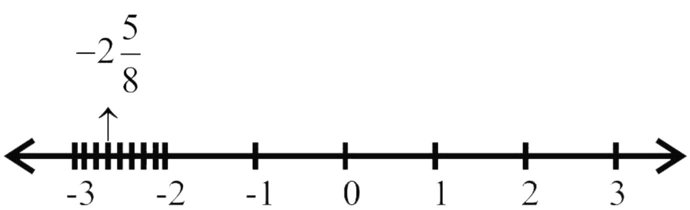
(ii) $-\frac{4}{5}$



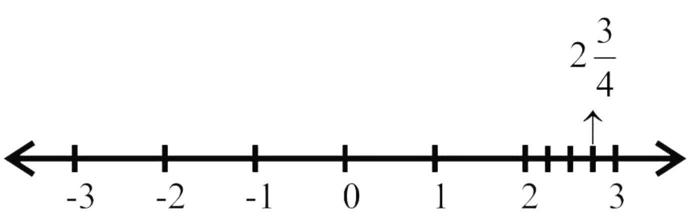
(iii) $1\frac{3}{4}$



(iv) $-2\frac{5}{8}$



(v) $2\frac{3}{4}$



(vi) $\sqrt{5}$

By Pythagoras theorem

$(\text{Hypoteneus})^2 = (\text{Base})^2 + (\text{Perpencicular})^2$

$(\overline{OB})^2 = (2)^2 + (1)^2$

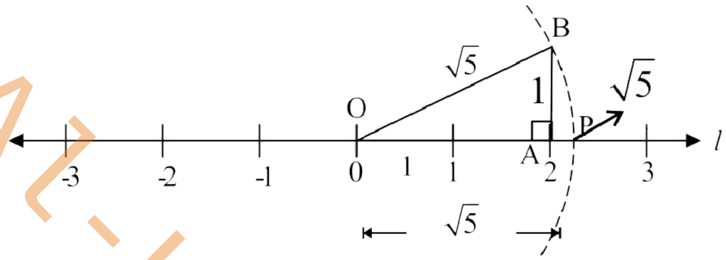
$(\overline{OB})^2 = 4 + 1$

$(\overline{OB})^2 = 5$

Taking square root on both sides

$\sqrt{(\overline{OB})^2} = \sqrt{5}$

$\overline{OB} = \sqrt{5}$



Q.5 Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$

Solution:

Required No between

$\frac{3}{4}$ and $\frac{5}{9}$

$= \left[\frac{3}{4} + \frac{5}{9}\right] \div 2$

$= \left[\frac{27+20}{36}\right] \div 2$

$= \left[\frac{47}{36}\right] \div 2$

$= \frac{47}{36} \times \frac{1}{2}$

$= \frac{47}{72}$ **Ans**

Q.6 Express the following recurring decimals as the rational number $\frac{p}{q}$ where p, q are integer and $q \neq 0$.

(i) $0.\overline{5}$

Solution:

$x = 0.\overline{5}$

$x = 0.555...$

$10 \times x = 10 \times 0.555...$

$10x = 5.555...$

$10x = 5 + 0.555...$

$10x = 5 + x$

$10x - x = 5$

$9x = 5$

$x = \frac{5}{9}$

$\therefore 0.\overline{5} = \frac{5}{9}$ **Ans**

(ii) $0.\overline{13}$

Solutions:

Suppose

$x = 0.\overline{13}$

$x = 0.131313\ldots$

$100^{x}x = 100\times 1.131313\ldots$

$100x = 13.1313\ldots$

$100x = 13 + 0.1313\ldots$

$100x = 13 + x$

$100x - x = 13$

$99x = 13$

$x = \frac{13}{99}$

$\therefore 0.\overline{13} = \frac{13}{99}$ **Ans**

(iii) $0.\overline{67}$

Solutions:

Suppose

$x = 0.\overline{67}$

$x = 0.676767\ldots$

$100\times x = 100\times 0.676767\ldots$

$100x = 67.6767\ldots$

$100x = 67 + 0.6767\ldots$

$100x = 67 + x$

$100x - x = 67$

$99x = 67$

$x = \frac{67}{99}$

$\therefore 0.\overline{67} = \frac{67}{99}$ **Ans**

Exercise 2.2

Q.1 Identify the property used in the following.

(i)	$a+b=b+a$	Commutative Property *w.r.t* addition
(ii)	$(ab)c=a(bc)$	Associative Property *w.r.t* multiplication
(iii)	$7\times 1=7$	Multiplicative Identity
(iv)	$x>y$ or $x=y$ or $x<y$	Trichotomy
(v)	$ab=ba$	Commutative *w.r.t* multiplication
(vi)	$a+c=b+c=a+b$	Cancellation Property of addition
(vii)	$5+(-5)=0$	Additive Inverse
(viii)	$7\times\frac{1}{7}=1$	Multiplicative inverse
(ix)	$a>b\Rightarrow ac>bc(c>0)$	Multiplicative property

Q.2 Fill in the following blanks by stating the properties of real numbers used.

$3x+3(y-x)$

$=3x+3y-3x,\ldots$ Distributive property

$=3x-3x+3y,\ldots$ Commutative

$=0+3y,\ldots$ Additive Inverse

$=3y,\ldots$ Additive identity

Q.3 Give the name of property used in the following.

(i)	$\sqrt{24}+0=\sqrt{24}$	Additive Identity
(ii)	$-\frac{2}{3}\left[5+\frac{7}{2}\right]=\left[-\frac{2}{3}\right](5)+\left[-\frac{2}{3}\right]\left[\frac{7}{2}\right]$	Distributive Property
(iii)	$\pi+(-\pi)=0$	Additive Inverse
(iv)	$\sqrt{3}.\sqrt{3}$ is a real number.	Closure property w.r.t x.
(v)	$\left[-\frac{5}{8}\right]\left[-\frac{8}{5}\right]=1$	Multiplicative Inverse.

Exercise 2.3

Q.1 Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

(i) $\sqrt[3]{-64}$

$=(-64)^{\frac{1}{3}}$

(ii) $2^{\frac{3}{5}}$

$=\sqrt[5]{2^3}$

(iii) $-7^{\frac{1}{3}}$

$-\sqrt[3]{7}$

(iv) $y^{-\frac{2}{3}}$

$=\sqrt[3]{y^{-2}}$

Q.2 Tell whether the following statements are true or false?

(i)	$5^{\frac{1}{5}}=\sqrt{5}$	**False**
(ii)	$2^{\frac{2}{3}}=\sqrt[3]{4}$	**True**
(iii)	$\sqrt{49}=\sqrt{7}$	**False**
(iv)	$\sqrt[3]{x^{27}}=x^3$	**False**

Q.3 Simplify the following radical expression.

(i) $\sqrt[3]{-125}$

Solution:

$=\sqrt[3]{-125}$

$=\sqrt[3]{-5\times-5\times-5}$

$=\sqrt[3]{(-5)^3}$

$= -5$ **Ans**

(ii) $\sqrt[4]{32}$

Solutions:

$$= \sqrt[4]{32}$$
$$= \sqrt[4]{2\times 2\times 2\times 2\times 2}$$
$$= \sqrt[4]{2^4 \times 2}$$
$$= \sqrt[4]{2^4} \times \sqrt[4]{2}$$
$$= 2\sqrt[4]{2} \text{ } \textbf{Ans}$$

(iii) $\sqrt[5]{\frac{3}{32}}$

Solution:

$$= \sqrt[5]{\frac{3}{32}}$$
$$= \frac{\sqrt[5]{3}}{\sqrt[5]{32}}$$
$$= \frac{\sqrt[5]{3}}{\sqrt[5]{2\times 2\times 2\times 2\times 2}}$$
$$= \frac{\sqrt[5]{3}}{\sqrt[5]{(2)^5}}$$
$$= \frac{\sqrt[5]{3}}{2} \text{ } \textbf{Ans}$$

(iv) $\sqrt[3]{-\frac{8}{27}}$

Solution:

$$= \sqrt[3]{-\frac{8}{27}}$$
$$= \sqrt[3]{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)}$$
$$= \sqrt[3]{\left(-\frac{2}{3}\right)^3}$$
$$= -\frac{2}{3} \text{ } \textbf{Ans}$$

Exercise 2.4

Q.1 Use laws of exponents to simplify.

(i) $\dfrac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

Solution: $\dfrac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

$=\dfrac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

$=\dfrac{(3^5)^{-\frac{2}{3}}\times(2^5)^{-\frac{1}{5}}}{\sqrt{[(14)^2]^{-1}}}$

$=\dfrac{(3)^{-\frac{10}{3}}\times 2^{-1}}{\sqrt{[(14)^{-1}]^2}}$

$=\dfrac{(3)^{-\frac{10}{3}}\times 2^{-1}}{(14)^{-1}}$

$=\dfrac{14}{(3)^{\frac{10}{3}}\times 2}$

$=\dfrac{7}{3^{\frac{10}{3}}}$

$=\dfrac{7}{\sqrt[3]{3^{10}}}$

$=\dfrac{7}{\sqrt[3]{3\times3\times3\times3\times3\times3\times3\times3\times3\times3}}$

$=\dfrac{7}{\sqrt[3]{3^3\times3^3\times3^3\times3}}$

$=\dfrac{7}{\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3}}$

$=\dfrac{7}{3\times3\times3\times\sqrt[3]{3}}$

$=\dfrac{7}{27\sqrt[3]{3}}$ **Ans**

(ii) $(2x^5y^{-4})(-8x^{-3}y^2)$

Solution: $(2x^5y^{-4})(-8x^{-3}y^2)$

$=-16x^{5-3}y^{-4+2}$

$=-16x^2y^{-2}$

$=\dfrac{-16x^2}{y^2}$ **Ans**

(iii) $\left[\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^{0}}\right]^{-3}$

Solution: $\left[\dfrac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^{0}}\right]^{-3}$

$=\left[x^{-2-4}y^{-1+3}z^{-4-0}\right]^{-3}$

$=(x^{-6}y^{+2}z^{-4})^{-3}$

$=(x^{-6})^{-3}(y^2)^{-3}(y^{-4})^{-3}$

$=x^{18}y^{-6}z^{12}$

$=\dfrac{x^{18}z^{12}}{y^6}$ **Ans**

(iv) $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

Solution: $\dfrac{(81)^n.3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

$=\dfrac{(3^4)^n.3^5-3^{4n}.3^{-1}.3^5}{(3^2)^{2n}.3^3}$

$=\dfrac{3^{4m}.3^5-3^{4n}.3^{-1+5}}{3^{4n}.3^3}$

$=\dfrac{3^{4n}.3^5-3^{4n}.3^4}{3^{4n}.3^3}$

$$= \frac{3^{4n}.3^{4}(3-1)}{3^{4n}.3^{3}}$$

$$= 3^{4n-4n}.3^{4-3}.(2)$$

$$= 3^{0}.3^{1}.2$$

$$= 1 \times 3 \times 2$$

$= 6$ **Ans**

Q.2 **Show that**

$$\left[\frac{x^a}{x^b}\right]^{a+b} \times \left[\frac{x^b}{x^c}\right]^{b+c} \times \left[\frac{x^c}{x^a}\right]^{c+a} = 1$$

Proof:

L.H.S

$$= \left[\frac{x^a}{x^b}\right]^{a+b} \times \left[\frac{x^b}{x^c}\right]^{b+c} \times \left[\frac{x^c}{x^a}\right]^{c+a}$$

$$= \left(x^{a-b}\right)^{a+b} \times \left(x^{b-c}\right)^{b+c} \times \left(x^{c-a}\right)^{c+a}$$

$$= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{a^2}+\cancel{c^2}-\cancel{a^2}}$$

$$= x^{\circ}$$

$$= 1$$

$1 =$ R.H.S **Ans**

Q.3 **Simplify**

(i) $$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

Solution: $$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}} \times \left(3^{\cancel{3}}\right)^{\frac{1}{\cancel{3}}} \times (2 \times 2 \times 3 \times 5)^{\frac{1}{2}}}{(2 \times 2 \times 3 \times 3 \times 5)^{\frac{1}{2}} \times \left(2^{2}\right)^{-\frac{1}{3}} \times \left(3^{\cancel{2}}\right)^{\frac{1}{\cancel{2}}}}$$

$$= \frac{2^{\frac{1}{3}} \times 3\left(2^{2}\right)^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{\left(2^{2}\right)^{\frac{1}{2}} \times \left(3^{2}\right)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}$$

$$= \frac{2^{\frac{1}{3}} \times 3 \times 2 \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2 \times 3 \times 5^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}$$

$$= 2^{\frac{1}{3}} \times 2^{+1} \times 2^{-1} \times 2^{\frac{+2}{3}} \times 3^{1} \times 3^{\frac{1}{2}} \times 3^{-1} \times 3^{\frac{-1}{2}} \times 5^{\frac{1}{2}} \times 5^{\frac{-1}{2}}$$

$$= 2^{\frac{1}{3}\cancel{+1}\cancel{-1}+\frac{2}{3}} \times 3^{\cancel{1}+\cancel{\frac{1}{2}}-\cancel{1}-\cancel{\frac{1}{2}}} \times 5^{\cancel{\frac{1}{2}}\cancel{\frac{-1}{2}}}$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 3^{0} \times 5^{0}$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 1 \times 1$$

$$= 2^{\frac{\cancel{3}}{\cancel{3}}}$$

$= 2$ **Ans**

(ii) $$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

Solution: $$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

$$= \sqrt{\frac{\left(6^{3}\right)^{\frac{2}{3}} \times \left(5^{2}\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^{2} \times 5}{\left(\frac{\overset{25}{\cancel{100}}}{\cancel{4}}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^{2} \times 5}{(25)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^{2} \times 5}{\left(5^{\cancel{2}}\right)^{\frac{1}{\cancel{2}}}}}$$

$$= \sqrt{\frac{6^{2} \times 5}{5}}$$

$$= \sqrt{6^{2} \times 5^{+1} \times 5^{-1}}$$

$$= \sqrt{6^{2} \times 5^{\cancel{+1}\cancel{-1}}}$$

$$= \sqrt{6^{2} \times 5^{0}}$$

$$= \sqrt{6^{2} \times 1}$$

$$= \sqrt{6^{2}}$$

$= 6$ **Ans**

(iii) $5^{2^3} \div \left(5^2\right)^3$

Solution: $5^{2^3} \div \left(5^2\right)^3$

$$= 5^8 \div 5^6$$
$$= 5^{8-6}$$
$$= 5^2$$
$$= 25 \text{ Ans}$$

(iv) $\left(x^3\right)^2 \div x^{3^2}, x \neq 0$

Solution: $\left(x^3\right)^2 \div x^{3^2}, x \neq 0$

$$= x^6 \div x^9$$
$$= x^{6-9}$$
$$= x^{-3}$$
$$= \frac{1}{x^3} \textbf{ Ans}$$

Exercise 2.5

Q.1 Evaluate

(i) i^7

Solution:

$= i^7$

$= i^6 . i$

$= (i^2)^3 . i$

$= (-1)^3 . i$

$= -1 \times i$

$= -i$

$= -i$ **Ans**

(ii) i^{50}

Solution: i^{50}

$= (i^2)^{25}$

$= (-1)^{25}$

$= -1$ **Ans**

(iii) i^{12}

Solution:

i^{12}

$= (i^2)^6$

$= (-1)^6$

$= 1$ Ans

(iv) $(-i)^8$

Solution:

$(-i)^8$

$= i^8$

$= (i^2)^4$

$= (-1)^4$

$= 1$ **Ans**

(v) $(-i)^5$

Solution:

$(-i)^5$

$= -i^5$

$= -i^4 . i$

$= -(i^2)^2 . i$

$= -(-1)^2 . i$

=-$(1)(i)$

$-i$ **Ans**

(vi) i^{27}

Solution: i^{27}

$= i^{26} . i$

$= (i^2)^{13} . i$

$= (-1)^{13} . i$

$= -1 . i$

$= -i$ **Ans**

Q.2 Write the conjugate of the following numbers.

(i) $2 + 3i$

$= 2 - 3i$

(ii) $3 - 5i$

$= 3 + 5i$

(iii) $-i$

$= i$

(iv) $-3 + 4i$

$= -3 - 4i$

(v) $-4 - i$

$= -4 + i$

(vi) $i - 3$

$= -i - 3$

Q.3 Write the real and imaginary part of the following numbers.

(i) $1 + i$

Real = 1

Imaginary = 1

(ii) $-1 + 2i$

Real = -1

Imaginary = 2

(iii) $-3i+2$

Real = 2

Imaginary = - 3

(iv) $-2-2i$

Real = -2

Imaginary = - 2

(v) $-3i$

Real = 0

Imaginary = - 3

(vi) $2+0i$

Real = 2

Imaginary = 0

Q.4 Find the value of x and y if

$$x+iy+1=4-3i$$

Solution: Given that

$$x+iy+1=4-3i$$

$$x+iy=4-3i-1$$

$$x+iy=3-3i$$

$$x=3 \quad y=-3$$

$x=3, y=-3$ **Ans**

Exercise 2.6

Q.1 Identify the following statement as true or false.

(i) $\sqrt{-3}\sqrt{-3}=3$ **False**

(ii) $i^{73}=-i$ **False**

(iii) $i^{10}=-1$ **True**

(iv) Complex conjugate of $\left(-6i+i^2\right)$ is $\left(-1+6i\right)$ **True**

(v) Difference of a complex number $z=a+bi$ and its conjugate is a real number. **False**

(vi) If $(a-1)-(b+3)i=5+8i$, then $a=6$ and $b=-11$. **True**

(vii) Product of a complex number and its conjugate is always a non-negative real number. **True**

Q.2 Express the each complex number in the standard form $a+bi$, where a and b are real number.

(i) $(2+3i)+(7-2i)$

Solution:

$=2+3i+7+2i$

$=2+7+3i-2i$

$=9+i$ **Ans**

(ii) $2(5+4i)-3(7+4i)$

Solution: $2(5+4i)-3(7+4i)$

$=10+8i-21-12i$

$=10-21+8i-12i$

$=-11-4i$ **Ans**

(iii) $(-3+5i)-(4+9i)$

Solution: $(-3+5i)-(4+9i)$

$=+3-5i-4-9i$

$=3-4-5i-9i$

$=-1-14i$ **Ans**

(iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

Solution: $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

$=2(-1)+6i^2.i+3(i^2)^8-6(i^2)^9.i+4(i^2)^{12}.i$

$=-2+6(-1)i+3(-1)^8-6(-1).i+4(-1)^{12}.i$

$=-2-6i+3-6(-1)i+4(+1)i$

$=1-\cancel{6i}+\cancel{6i}+4i$

$=1+4i$ **Ans**

Q.3 Simplify and write your answer in the form $a+bi$

(i) $(-7+3i)(-3+2i)$

Solution: $(-7+3i)(-3+2i)$

$=-7(-3+2i)+3i(-3+2i)$

$=21-14i-9i+6i^2$

$=21-23i+6(-1)$

$=21-23i-6$

$=21-6-23i$

$=15-23i$ **Ans**

(ii) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

Solution: $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

$=\left(2-\sqrt{4\times -1}\right)\left(3-\sqrt{4\times -1}\right)$

$=\left(2-\sqrt{4i^2}\right)\left(3-\sqrt{4i^2}\right)$

$=(2-2i)(3-2i)$

$=2(3-2i)-2i(3-2i)$

$=6-4i-6i+4i^2$

$=6-10i+4(-1)$

$=6-10i-4$

$=2-10i$ **Ans**

(iii) $\left(\sqrt{5}-3i\right)^2$

Solution: $\left(\sqrt{5}-3i\right)^2$

$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$

$=5+9i^2-6\sqrt{5}i$

$=5+9(-1)-6\sqrt{5}i$

$=5-9-6\sqrt{5}i$

$=-4-6\sqrt{5}i$ **Ans**

(iv) $(2-3i)\left(\overline{3-2i}\right)$

Solution: $(2-3i)\left(\overline{3-2i}\right)$

$=(2-3i)(3+2i)$

$=2(3+2i)-3i(3+2i)$

$=6+4i-9i-6i^2$

$=6-5i-6(-1)$

$=6-5i+6$

$=6+6-5i$

$=12-5i$ **Ans**

Q.4 Simplify and write your answer in the form $a+bi$.

(i) $\dfrac{-2}{1+i}$

Solution: $\dfrac{-2}{1+i}$

$=\dfrac{-2}{1+i}\times\dfrac{1-i}{1-i}$

$=\dfrac{-2(1-i)}{(1)^2-(i)^2}$

$=\dfrac{-2+2i}{1-i^2}$

$=\dfrac{-2+2i}{1-(-1)}$

$=\dfrac{-2+2i}{1+1}$

$=\dfrac{-2+2i}{2}$

$=-\dfrac{2}{2}+\dfrac{2i}{2}$

$=-1+i$ **Ans**

(ii) $\dfrac{2+3i}{4-i}$

Solution: $\dfrac{2+3i}{4-i}$

$=\dfrac{2+3i}{4-i}\times\dfrac{4+i}{4+i}$

$=\dfrac{(2+3i)(4+i)}{(4)^2-(i)^2}$

$=\dfrac{2(4+i)+3i(4+i)}{16-(-1)}$

$=\dfrac{8+2i+12i+3i^2}{16+1}$

$=\dfrac{8+4i+3(-1)}{17}$

$=\dfrac{8+14i-3}{17}$

$=\dfrac{8-3+14i}{17}$

$=\dfrac{5+14i}{17}$

$=\dfrac{5}{17}+\dfrac{14}{17}i$ **Ans**

(iii) $\dfrac{9-7i}{3+i}$

Solution: $\dfrac{9-7i}{3+i}$

$=\dfrac{9-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{(9-7i)(3-i)}{(3)^2-(i)^2}$

$=\dfrac{9(3-i)-7i(3-i)}{9-(-1)}$

$=\dfrac{27-9i-21i+7i^2}{9+1}$

$\dfrac{27-30i+7(-1)}{10}$

$=\dfrac{27-30i-7}{10}$

$=\dfrac{27-7-30i}{10}$

$=\dfrac{20-30i}{10}$

$=\dfrac{20}{10}-\dfrac{30i}{10}$

$=2-3i$ **Ans**

(iv) $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

Solution: $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

$=\dfrac{2-6i-(4+i)}{3+i}$

$=\dfrac{2-6i-4-i}{3+i}$

$=\dfrac{2-4-6i-i}{3+i}$

$=\dfrac{-2-7i}{3+i}$

$=\dfrac{-2-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{-2-7i}{3+i}\times\dfrac{3-i}{3-i}$

$=\dfrac{-2(3-i)-7i(3-i)}{(3)^2-(i)^2}$

$=\dfrac{-6+2i-21i+7i^2}{9-(-1)}$

$=\dfrac{-6-19i+7(-1)}{9+1}$

$=\dfrac{-6-19i-7}{10}$

$=\dfrac{-6-7-19i}{10}$

$=\dfrac{-13-19i}{10}$

$=\dfrac{-13}{10}-\dfrac{19i}{10}$ **Ans**

(v) $\left[\dfrac{1+i}{1-i}\right]^2$

Solution: $\left[\dfrac{1+i}{1-i}\right]^2$

$=\dfrac{(1+i)^2}{(1-i)^2}$

$=\dfrac{(1)^2+(i)^2+2ab}{(1)^2+(i)^2-2ab}$

$=\dfrac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$

$=\dfrac{1+(-1)+2i}{1+(-1)-2i}$

$=\dfrac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$

$=\dfrac{\cancel{2i}}{-\cancel{2i}}=-1$

$=-1$

$=-1+0i$ **Ans**

(vi) $\dfrac{1}{(2+3i)(1-i)}$

Solution: $\dfrac{1}{(2+3i)(1-i)}$

$=\dfrac{1}{2(1-i)+3i(1-i)}$

$=\dfrac{1}{2-2i+3i-3i^2}$

$$= \frac{1}{2+i-3(-1)}$$

$$= \frac{1}{2+i+3}$$

$$= \frac{1}{2+3+i}$$

$$= \frac{1}{5+i}$$

$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{1(5-i)}{(5)^2-(i)^2}$$

$$= \frac{5-i}{25-(-1)}$$

$$= \frac{5-i}{25+1}$$

$$= \frac{5-i}{26}$$

$$= \frac{5}{26} - \frac{1i}{26}$$ **Ans**

Q.5 Calculate $(a)\,\overline{z}\,(b)\,z+\overline{z}\,(c)\,z-\overline{z}\,(d)\,z\overline{z}$ **for each of the following.**

(i) $z = -i$

Solution: $z = -i$

(a) $\overline{z} = +i$

(b) $z + \overline{z} = -i + i$

$= 0$

(c) $z - \overline{z} = (-i) - (i)$

$= -2i$

(d) $z\overline{z} = (-i)(i)$

$= -i^2$

$= -(-1)$

$= 1$ **Ans**

(ii) $z = 2+i$

Solution: $z = 2+i$

$z + 2i$

(a) $\overline{z} = 2-i$

(b) $z + \overline{z} = (2+i)+(2-i)$

$= 2 + \not{i} + 2 - \not{i}$

$= 2+2$

$= 4$

(c) $z - \overline{z} = (2+i)-(2-i)$

$= \not{2} + i - \not{2} + i$

$= i + i$

$= 2i$

(d) $z\overline{z} = (2+i)(2-i)$

$= (2)^2 - (i)^2$

$= 4 - i^2$

$= 4 - (-1)$

$= 4 + 1$

$= 5$ **Ans**

(iii) $z = \frac{1+i}{1-i}$

Solution: $z = \frac{1+i}{1-i}$

$$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{1(1+i)+i(1+i)}{(1-i)(1+i)}$$

$$= \frac{1+i+i+(-1)}{(1)^2-(i)^2}$$

$$= \frac{1+2i+(-1)}{1-(-1)}$$

$$= \frac{\not{1}+2i-\not{1}}{1+1}$$

$$= \frac{\not{2}i}{\not{2}}$$

$= i$

$z = i$

(a) $\overline{z} = -i$

(b) $z + \overline{z} = i + (-i)$

$= \not{i} \not{-i}$

$= 0$

(c) $z - \overline{z} = i - (-i)$

$= i + i$

$= 2i$

(d) $z\bar{z} = (i)(-i)$

$= -i^2$

$= -(-1)$

$= +1$ **Ans**

(iv) $z = \dfrac{4-3i}{2+4i}$

Solution: $z = \dfrac{4-3i}{2+4i}$

$z = \dfrac{4-3i}{2+4i} \times \dfrac{2-4i}{2-4i}$

$= \dfrac{4(2-4i)-3i(2-4i)}{(2+4i)(2-4i)}$

$= \dfrac{8-16i-6i+12i^2}{(2)^2-(4i)^2}$

$= \dfrac{8-22i+12(-1)}{4-16i^2}$

$= \dfrac{8-22i-12}{4-16(-1)}$

$= \dfrac{8-12-22i}{4+16}$

$= \dfrac{-4-22i}{20}$

$= \dfrac{-4}{20} - \dfrac{22}{20}i$

$= -\dfrac{1}{5} - \dfrac{11}{10}i$

(a) $\bar{z} = \dfrac{-1}{5} + \dfrac{11}{10}i$

(b)

$z + \bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right) + \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\dfrac{1}{5} - \dfrac{11}{10}i - \dfrac{1}{5} + \dfrac{11}{10}i$

$= -\dfrac{1}{5} - \dfrac{1}{5}$

$= \dfrac{-1-1}{5}$

$= -\dfrac{2}{5}$

(c)

$z - \bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right) - \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\dfrac{1}{5} - \dfrac{11}{10}i + \dfrac{1}{5} - \dfrac{11}{10}i$

$= -\dfrac{11}{10}i - \dfrac{11}{10}i = \dfrac{-11i-11i}{10}$

$= -\dfrac{22}{10}i$

$= -\dfrac{11}{5}i$

(d) $z\bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right)\left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= \left(-\dfrac{1}{5}\right)^2 - \left(\dfrac{11}{10}i\right)^2$

$= \dfrac{1}{25} - \dfrac{121}{100}i^2$

$= \dfrac{1}{25} - \dfrac{121}{100}(-1)$

$= \dfrac{1}{25} + \dfrac{121}{100}$

$= \dfrac{4+121}{100}$

$= \dfrac{125}{100}$

$= \dfrac{5}{4}$ **Ans**

Q.6 **If** $z = 2+3i$ **and show that.**

(i) $\overline{z+w} = \bar{z} + \bar{w}$

Solution: $\overline{z+w} = \bar{z} + \bar{w}$

$z + w = 2+3i+5-4i$

$= 2+5+3i-4i$

$= 7-i$

L.H.S $= \overline{z+w}$

$= \overline{7-i}$

$= 7+i$ … (i)

R. H. S $= \bar{z} + \bar{w}$

$= (\overline{2+3i}) + (\overline{5-4i})$

$= 2-3i+5+4i$

$= 2+5-3i+4i$

$= 7 + i$ ...

(ii)

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z+w} = \bar{z} + \bar{w}$

Hence proved

(ii) $\overline{z-w} = \bar{z} - \bar{w}$

Solution: $\overline{z-w} = \bar{z} - \bar{w}$

$z - w = (2+3i) - (5-4i)$

$= 2 + 3i - 5 + 4i$

$= 2 - 5 + 3i + 4i$

$= -3 + 7i$

L.H.S $= \overline{z-w}$

$= \overline{-3+7i}$

$= -3 - 7i$ ...(i)

R.H.S $= \bar{z} - \bar{w}$

$= \left(\overline{2+3i}\right) - \left(\overline{5-4i}\right)$

$= 2 + 3i - (5 + 4i)$

$= 2 - 3i - 5 - 4i$

$= -3 - 7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z-w} = \overline{z-w}$

Hence proved

(iii) $\overline{zw} = \bar{z}\,\bar{w}$

Solutions: $\overline{zw} = \bar{z}\,\bar{w}$

$zw = (2+3i)(5+4i)$

$= 2(5-4i) + 3i(5-4i)$

$= 10 - 8i + 15i - 12i^2$

$= 10 + 7i - 12(-1)$

$= 10 + 7i + 12$

$= 22 + 7i$

L.H.S $= \overline{zw}$

$= \overline{22+7i}$

$= 22 - 7i$

R.H.S $= \overline{zw}$

$= \left(\overline{2+3i}\right)\left(\overline{5-4i}\right)$

$= (2-3i)(5+4i)$

$= 2(5+4i) - 3i(5+4i)$

$= 10 + 8i - 15i - 12i^2$

$= 10 - 7i - 12(-1)$

$= 10 - 7i + 12$

$= 22 - 7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{zw} = \overline{zw}$

Hence proved

(iv) $\overline{\left[\frac{z}{w}\right]} = \frac{\bar{z}}{\bar{w}}$, where $\neq 0$

Solutions: $\overline{\left[\frac{z}{w}\right]} = \frac{\bar{z}}{\bar{w}}$

$\frac{z}{w} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i}$

$= \frac{2(5+4i) + 3i(5+4i)}{(5-4i)(5+4i)}$

$= \frac{10 + 8i + 15i + 12i^2}{(5)^2 - (4i)^2}$

$= \frac{10 + 23i + 12(-1)}{25 - 16i^2}$

$= \frac{10 + 23i - 12}{25 - (6(-1))}$

$= \frac{10 + 23i - 12}{25 + 16}$

$= \frac{-2 + 23i}{41}$

L.H.S= $\overline{\left(\frac{z}{w}\right)}$

$= \overline{\left(\frac{-2+23i}{41}\right)}$

$= \frac{-2}{41} - \frac{23}{41}i$ ... (i)

R.H.S $= \frac{\bar{z}}{\bar{w}}$

$= \frac{\left(\overline{2+3i}\right)}{\left(\overline{5-4i}\right)}$

$$=\frac{2-3i}{5+4i}$$
$$=\frac{2-3i}{5+4i}\times\frac{5-4i}{5-4i}$$
$$=\frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)}$$
$$=\frac{10-8i-15i+12i^2}{(5)^2-(4i)^2}$$
$$=\frac{10-23i+12(-1)}{25-16i^2}$$
$$=\frac{10-23i+12(-1)}{25-16(-1)}$$
$$=\frac{10-23i-12}{25+16}$$
$$=\frac{-2-23i}{41}$$
$$=\frac{-2}{41}-\frac{23}{41}i \quad \ldots\text{(ii)}$$

From (i) and (ii) we get
L.H.S=R.H.S
Hence Proved

$$\overline{\left[\frac{z}{w}\right]}=\frac{\bar{z}}{\bar{w}}$$

(v) $\frac{1}{2}(z+\bar{z})$ **is the real part of** z**.**

Solution: $\frac{1}{2}(z+\bar{z})$

$$=\frac{1}{2}\left[(2+3i)+\left(\overline{2+3i}\right)\right]$$
$$=\frac{1}{2}\left[(2+3i)+(2-3i)\right]$$
$$=\frac{1}{2}\left[2+\cancel{3i}+2-\cancel{3i}\right]$$
$$=\frac{1}{2}[2+2]$$
$$=\frac{1}{\cancel{2}}\left[\cancel{4}^{2}\right]$$
$$=2=\operatorname{Re}(z)$$

$\frac{1}{2}(z+\bar{z})$ is the real part of z. **Ans**

(vi) $\frac{1}{2}(z-\bar{z})$ **is the imaginary part of** z**.**

Solution: $\frac{1}{2}(z-\bar{z})$

$$\frac{1}{2}(z-\bar{z})=$$
$$=\frac{1}{2}\left[(2+3i)-\left(\overline{2+3i}\right)\right]$$
$$=\frac{1}{2}\left[(2+3i)-(2-3i)\right]$$
$$=\frac{1}{2}\left[\cancel{2}+3i-\cancel{2}+3i\right]$$
$$=\frac{1}{\cancel{2}}\left[{}^{3}\cancel{6}i\right]$$
$$=3i$$
$$=\text{Imaginary}(z)$$

$\frac{1}{2}(z-\bar{z})$ is the imaginary part of z. **Ans**

Q.7 Solve the following equations for real x **and** y**.**

(i) $(2-3i)(x+yi)=4+i$

Solution: $(2-3i)(x+yi)=4+i$

$$x+yi=\frac{4+i}{2-3i}$$
$$x+yi=\frac{4+i}{2-3i}\times\frac{2+3i}{2+3i}$$
$$=\frac{4(2+3i)+i(2+3i)}{(2-3i)(2+3i)}$$
$$=\frac{8+12i+2i+3i^2}{(2)^2-(3i)^2}$$
$$=\frac{8+14i+3(-1)}{4-9i^2}$$
$$=\frac{8+14i-3}{4-9(-1)}$$
$$=\frac{8-3+14i}{4+9}$$

$$= \frac{5+14i}{13}$$

$$x + yi = \frac{5}{13} + \frac{14}{13}i$$

$$x = \frac{5}{13}, y = \frac{14}{13}$$ **Ans**

(ii) $(3-2i)(x+yi) = 2(x-2yi)+2i-1$

Solution:

$$(3-2i)(x+yi) = 2(x-2yi)+2i-1$$

$$3(x+yi)-2i(x+yi) = 2x-4yi+2i-1$$

$$3x+3yi-2xi-2yi^2 = (2x-1)+i(2-4y)$$

$$3x+(3x-2x)i-2y(-1) = (2x-1)+i(2-4y)$$

$$3x+(3y-2x)i+2y = (2x-1)+i(2-4)$$

$$(3x+2y)+(3y-2x)i = (2x-1)+(2-4y)i$$

Comparing the real and imaginary parts.

$$3x+2y = 2x-1 \quad ,$$
$$3y-2x = 2-4y$$
$$3x-2x+2y = -1 \quad ,$$
$$3y-2x = 2-4y$$
$$x+2y = -1 \quad ,$$

$$-2x+3y+4y = 2$$

$$-2x+7y = 2$$

$x+2y=-1$ ____________ (i)

$-2x+7y=2$ ____________ (ii)

Multiply equation (i) with (2)

$$2(x+2y) = -1\times 2$$

$2x+4y=-2$ ____________ (iii)

$$\cancel{2x}+4y = \cancel{-2}$$

$$\cancel{-2x}+7y = \cancel{2}$$

$$11y = 0$$

$$y = \frac{0}{11}$$

$$y = 0$$

Putting $y=0$ in equation (i)

$$x+2y = -1$$

$$x+2(0) = -1$$

$$x+0 = -1$$

$$x = -1+0$$

$x = -1$ **Ans**

(iii) $(3+4i)^2 - 2(x-yi) = x+yi$

Solution: $(3+4i)^2 - 2(x-yi) = x+yi$

$$(3+4i)^2 - 2(x-yi) = x+yi$$

$$9+24i+16i^2-2x+2yi = x+yi$$

$$9+24i+16(-1)-2x+2yi = x+yi$$

$$9+24i-16-2x+2yi = x+yi$$

$$9+24i-16-2x = x+2yi-yi = 0$$

$$9+24i-16-3x+yi = 0$$

$$-3x+yi = -9-24i+16$$

$$-3x+yi = 16-9-24i$$

$$-3x+yi = 7-24i$$

Comparing the real and imaginary parts.

$-3x = 7$

$y = -24$

$x = \frac{-7}{3}$

$y = -24$ **Ans**

Review Exercise 2

Q.1 Multiple choice questions. Choose the correct answer.

(i) $\left(27x^{-1}\right)^{-\frac{2}{3}}$ ________

(a) $\frac{\sqrt[3]{x^2}}{9}$ **(b)** $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$ **(d)** $\frac{\sqrt{x^3}}{8}$

(ii) **Write** $\sqrt[7]{x}$ **in the exponential form** ________

(a) x **(b)** x^7

(c) $x^{\frac{1}{7}}$ **(d)** $x^{\frac{7}{2}}$

(iii) **Write** $4^{\frac{2}{3}}$ **with radical sing** ________

(a) $\sqrt[3]{4^2}$ **(b)** $\sqrt[2]{4^3}$

(c) $\sqrt[2]{4^3}$ **(d)** $\sqrt{4^6}$

(iv) **In** $\sqrt[3]{35}$ **the radicand is;**

(a) 3 **(b)** $\frac{1}{3}$

(c) 35 **(d)** None

(v) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} =$ ________

(a) $\frac{5}{4}$ **(b)** $\frac{4}{5}$

(c) $-\frac{5}{4}$ **(d)** $-\frac{4}{5}$

(vi) **The conjugate of** $5+4i$ **is** ________

(a) $-5+4i$ **(b)** $-5-4i$

(c) $5-4i$ **(d)** $5+4i$

(vii) **The value of** i^9 **is;**

(a) 1 **(b)** -1

(c) i **(d)** $-i$

(viii) **Every real number is ___________**

(a) Positive integer
(b) A rational number
(c) A negative integer
(d) A complex number

(ix) **Real point of** $2ab\left(i+i^2\right)$ **is ___________**

(a) $2ab$
(b) $-2ab$
(c) $2abi$
(d) $-2abi$

(x) **Imaginary part of** $-i(3i+2)$ **is ___________**

(a) -2
(b) 2
(c) 3
(d) -3

(xi) **Which of the following sets have the closure property w.r.t addition ___________**

(a) $\{0\}$
(b) $\{0,1\}$
(c) $\{0,1\}$
(d) $\left\{1,\sqrt{2},\frac{1}{2}\right\}$

(xii) **Name the property of real number used in** $\left[-\frac{\sqrt{5}}{2}\right]\times 1=-\frac{\sqrt{5}}{2}$ ___________

(a) Additive identity
(b) Additive inverse
(c) Multiplicative identity
(d) Multiplicative inverse

(xiii) **If** $x,y,z\in R, z<0,$ **then** $x<y\Rightarrow> \ldots$

(a) $xz<yz$
(b) $xz>yz$
(c) $xz=yz$
(d) None of these

(xiv) **IF** $a,b\in\ \in\ R,$ **only one of** $a=b$ **or** $a<b$ or $a>b$ **hold is called ___________**

(a) Trichotomy property
(b) Transitive property
(c) Additive property
(d) Multiplicative property

(xv) **A non-terminating, non-recurring decimal represents …**

(a) A natural number
(b) A rational number
(c) An irrational number
(d) A prime number

ANSWER KEY

i	a	vi	c	xi	a
ii	c	vii	c	xii	c
iii	a	viii	d	xiii	b
iv	c	ix	b	xiv	a
v	b	x	a	xv	c

Q.2 True or False? Identity

(i) Division is not an associative operation. **True**
(ii) Every whole number is a natural number. **False**
(iii) Multiplicative inverse of 0.02 is 50. **True**
(iv) π is rational number. **False**
(v) Every integer is a rational number. **True**
(vi) Subtraction is a commutative operation. **False**
(vii) Every real number is a rational number. **False**
(viii) Decimal representation of a rational number is either terminating or recurring. **True**
(ix) $1.\bar{8} = 1 + \frac{8}{9}$ **True**

Q.3 Simplify the following

(i) $\sqrt[4]{81y^{-12}x^{-8}}$

Solution:

$$= \left(3^4 y^{12} x^{-8}\right)^{\frac{1}{4}}$$

$$= 3^{4\times\frac{1}{4}} y^{-12^{3}\times\frac{1}{4}} x^{-8^{2}\times\frac{1}{4}}$$

$$= 3y^{-3}x^{-2}$$

$$\sqrt[4]{81y^{-12}x^{-8}} = \frac{3}{y^3x^2} \quad \textbf{Ans}$$

(ii) $\sqrt{25x^{10}y^{8m}}$

Solution:

$$= \sqrt{25x^{10n}y^{8m}}$$

$$= \left(5^2 x^{10n} y^{8m}\right)^{\frac{1}{2}}$$

$$= 5^{2\times\frac{1}{2}}.x^{10n^{5}\times\frac{1}{2}}.y^{8m^{4}\times\frac{1}{2}}$$

$$\sqrt{25x^{10}y^{8m}} = 5x^{5n}.y^{4m} \quad \textbf{Ans}$$

(iii) $\left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}}$

Solution:

$$= \left(x^{3+2}.y^{4+1}.z^{5+5}\right)^{\frac{1}{5}}$$

$$= \left(x^5y^5z^{10}\right)^{\frac{1}{5}}$$

$$= x^{5\times\frac{1}{5}} \times y^{5\times\frac{1}{5}} \times z^{10^{2}\times\frac{1}{5}}$$

$$\left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}} = x.y.z^2 \quad \textbf{Ans}$$

(iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$

Solution:

$$= \left(\frac{2^5x^{-4}y^{-4}z}{5^4x^4yz^{-4}}\right)^{\frac{2}{5}}$$

$$= \left[\frac{2^5z^{1+4}}{5^4x^{4+6}\times y^{1+4}}\right]^{\frac{2}{5}}$$

$$= \left[\frac{2^5z^5}{5^4x^{10}y^5}\right]^{\frac{2}{5}}$$

$$= \frac{2^{5\times\frac{2}{5}} \times z^{5\times\frac{2}{5}}}{5^{4\times\frac{2}{5}} \times x^{10^{2}\times\frac{2}{5}} \times y^{5\times\frac{2}{5}}}$$

$$= \frac{2^2 \times z^2}{5^{\frac{8}{5}} \times x^4 \times y^2}$$

$$= \frac{4z^2}{5^{\frac{5}{5}+\frac{3}{5}} \times x^4y^2}$$

$$= \frac{4z^2}{5^{1+\frac{3}{5}} \times x^4y^2}$$

$$\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}} = \frac{4z^2}{5\times5^{\frac{3}{5}}x^4y^2} \quad \textbf{Ans}$$

Q.4 **Simplify** $\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

Solution: $\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

$$=\sqrt{\frac{(6^3)^{\frac{2}{3}}\times(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}}$$

$$=\sqrt{\frac{6^2\times5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$$

$$=\sqrt{\frac{6^2\times5}{(5^2)^{\frac{3}{2}}}}$$

$$=\sqrt{\frac{6^2\times5}{(5)^3}}$$

$$=\sqrt{\frac{6^2}{5^3\times5^{-1}}}$$

$$=\sqrt{\frac{6^2}{5^{3-1}}}$$

$$=\sqrt{\frac{6^2}{5^2}}$$

$$=\sqrt{\left(\frac{6}{5}\right)^2}$$

$$=\left(\frac{6}{5}\right)^{\cancel{2}\times\frac{1}{\cancel{2}}}$$

$$=\frac{6}{5}\ \textbf{Ans}$$

Q.5 $\left(\frac{a^p}{a^q}\right)^{p+q}\times\left(\frac{a^q}{a^r}\right)^{q+r}\div5\left(a^q a^r\right)^{p-r}$

Solution:

$$=\frac{\left(a^{p-q}\right)^{p+q}\left(a^{q-r}\right)^{q+r}}{5\left(a^{p+r}\right)^{p-r}}$$

$$=\frac{a^{(p-q)(p+q)}a^{(q-r)(q+r)}}{5a^{(p+r)(p-r)}}$$

$$=\frac{a^{p^2-q^2}a^{q^2-r^2}}{5a^{p^2-r^2}}$$

$$=\frac{a^{p^2-q^2+q^2-r^2}}{5a^{p^2-r^2}}$$

$$=\frac{a^{p^2-r^2-p^2+r^2}}{5}$$

$$=\frac{a^0}{5}$$

$$\left(\frac{a^p}{a^q}\right)^{p+q}\times\left(\frac{a^q}{a^r}\right)^{q+r}\div5\left(a^q a^r\right)^{p-r}$$

$$=\frac{1}{5}\ \textbf{Ans}$$

Q.6 **Simplify** $\left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+2}}\right)$

Solution:

$$=a^{2l-l-m}a^{2m-m-n}a^{2n-n-n}$$

$$=a^{l-m}a^{m-n}a^{n-l}$$

$$=a^{l-m+m-n+n-l}$$

$$=a^0$$

$$\left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+2}}\right)=1\ \textbf{Ans}$$

Q.7 **Simplify** $\sqrt[3]{\frac{a^l}{a^m}}\times\sqrt[3]{\frac{a^m}{a^n}}\times\sqrt[3]{\frac{a^n}{a^r}}$

Solution:

$$=\sqrt[3]{a^{l-m}}\sqrt[3]{a^{m-n}}\sqrt[3]{a^{n-l}}$$

$$=\left(a^{l-m}\right)^{\frac{1}{3}}\times\left(a^{m-n}\right)^{\frac{1}{3}}\times\left(a^{n-l}\right)^{\frac{1}{3}}$$

$$=a^{\frac{l-m}{3}}\times a^{\frac{m-n}{3}}\times a^{\frac{n-l}{3}}$$

$$=a^{\frac{l-m}{3}+\frac{m-n}{3}+\frac{n-l}{3}}$$

$$=a^{\frac{l-m+m-n+n-l}{3}}$$

$$=a^{\frac{0}{3}}$$

$$=a^0$$

$$\sqrt[3]{\frac{a^l}{a^m}}\times\sqrt[3]{\frac{a^m}{a^n}}\times\sqrt[3]{\frac{a^n}{a^r}}=1\ \textbf{Ans}$$

Unit 2: Real and Complex Numbers

Overview

Natural Numbers:

The numbers $1,2,3,\ldots$ which we use for counting certain objects are called natural numbers or positive integers. The set natural numbers is denoted by N.

i.e. $N = \{1, 2, 3, \ldots\}$

Whole Numbers:

If we include 0 in the set of natural number, the resulting set is the set of whole numbers, denoted by W,

i.e. $W = \{0, 1, 2, 3, \ldots\}$

Integers:

The set of integers consist of positive integers, 0 and negative integers and is denoted by Z i.e. $Z\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers:

All numbers of the form $\frac{p}{q}$ where p, q are integers and q is not zero are called rational numbers. The set of rational numbers is denoted by Q,

i.e. $Q = \left\{\frac{p}{q} \mid p, q \in Z \wedge q \neq 0, (p, q) = 1\right\}$

Irrational Numbers:

The numbers which cannot be expressed as quotient of integers are called irrational numbers. The set of irrational numbers is denoted by Q',

i.e. $Q' = \left\{x \mid x \neq \frac{p}{q}, p, q \in Z \wedge q \neq 0\right\}$

The union of the set of rational numbers and irrational numbers is known as the set of real numbers. It is denoted by R,

i.e. $R = Q \cup Q'$

Types of Rational Numbers:

(i) Terminating Decimal Fractions

The decimal fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction. For example $\frac{2}{5} = 0.4$ and $\frac{3}{8} = 0.375$.

(ii) Recurring and Non-terminating Decimal Fractions:

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called recurring decimal fraction.

For example $\frac{2}{9} = 0.2222\ldots$ and $\frac{4}{11} = 0.363636\ldots$

Concept of Radicals and Radicands:

In the radical $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called the radical sign, n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Base and Exponent:

In the exponential notation of (read as a to the nth power) we call 'a' as the base and 'n' as the exponent or the power t which the base is raised.

Definition of a Complex Number:

A number of the form $z = a + bi$ where a and b are real numbers and $i - \sqrt{-1}$, is called a complex number and is represented by z i.e., $z = a + ib$

Conjugate of a Complex Number:

If we change i to $-i$ in $z = a + bi$, we obtain another complex number $a - bi$ called the complex conjugate of z and is denoted by $\overline{z}$ (read z bar).