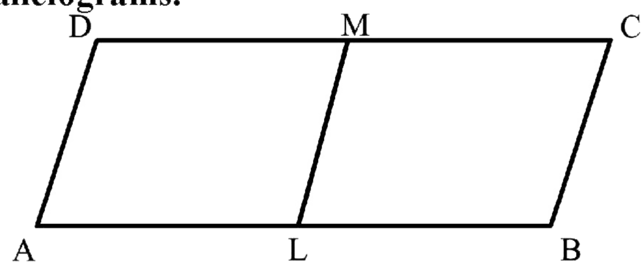
Exercise 16.1

Q.1 Show that the line segment joining the midpoint of opposite sides of a parallelogram divides it into two equal parallelograms.



Given

ABCD is a parallelogram. L is the midpoint of $\overline{AB}$ and M is the midpoint of $\overline{DC}$

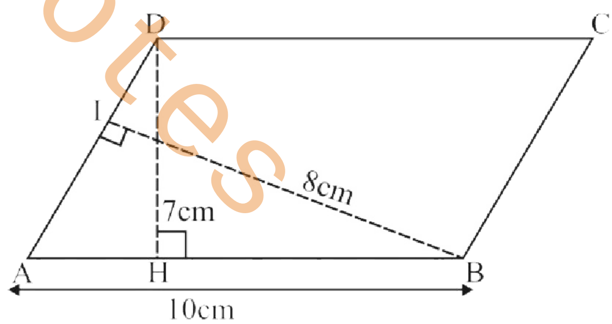
To prove

Area of parallelogram ALMD = area of parallelogram LBCM.

Proof

Statements	Reasons
$\overline{AB} \parallel \overline{DC}$	Opposite sides of parallelogram ABCD.
$\overline{AL} \cong \overline{LB}$ ...(i)	L is midpoint of $\overline{AB}$
The parallelograms ALMD and LBCM are on equal bases and between the same parallel lines $\overline{AB}$ and $\overline{DC}$	From equation (i)
Hence area of parallelogram ALMD= area of parallelogram LBCM.	They have equal areas

Q.2 In a parallelogram ABCD, m $\overline{AB}$ =10cm the altitudes Corresponding to Sides AB and AD are respectively 7cm and 8cm Find $\overline{AD}$



$\overline{AB}$ = 10 cm

$\overline{DH}$ = 7cm

$\overline{MB}$ = 8cm

$\overline{AD}$ =?

Formula

Area of parallelogram = base x altitude

$\overline{AB} \times \overline{DH} = \overline{AD} \times \overline{IB}$

$10 \times 7 = \overline{AD} \times 8$

$\frac{\cancel{70}^{35}}{\cancel{8}^{4}} = \overline{AD}$

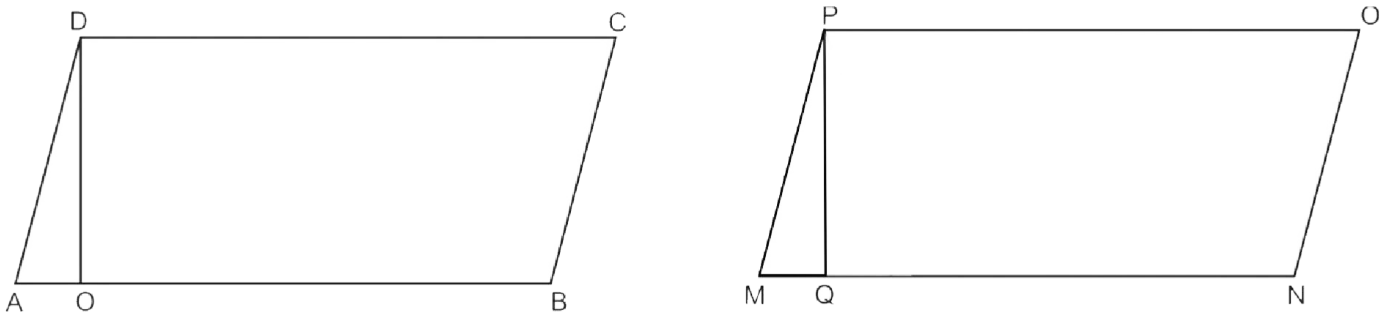
$\frac{35}{4} = \overline{AD}$

$\overline{AD} = \frac{35}{4}$

Or

$\overline{AD}$ = 8.75cm

Q.3 If two parallelograms of equal areas have the same or equal bases, their altitude are equal



In parallelogram opposite side and opponents angles are Congruent.

Given

Parallelogram ABCD and parallelogram MNOP

OD is altitude of parallelogram ABCD

PQ is altitude of parallelogram MNOP

Area of ABCD $\|^{gm}$ $\cong$ Area of MNOP $\|^{gm}$

To prove

$m\overline{OD} \cong m\overline{PQ}$

Proof

Statements	Reasons
Area of parallelogram ABCD= Area of parallelogram MNOP	Given
Area of parallelogram= base $\times$ height	Given
$\overline{AB} \times \overline{OD} = \overline{MN} \times \overline{PQ}$	
We know that	
$\overline{AB} = \overline{MN}$	
So	
$\frac{\cancel{AB}}{\cancel{AB}} \times \overline{OD} = \overline{PQ}$	Proved
$\overline{OD} = \overline{PQ}$	

Theorem 16.1.3

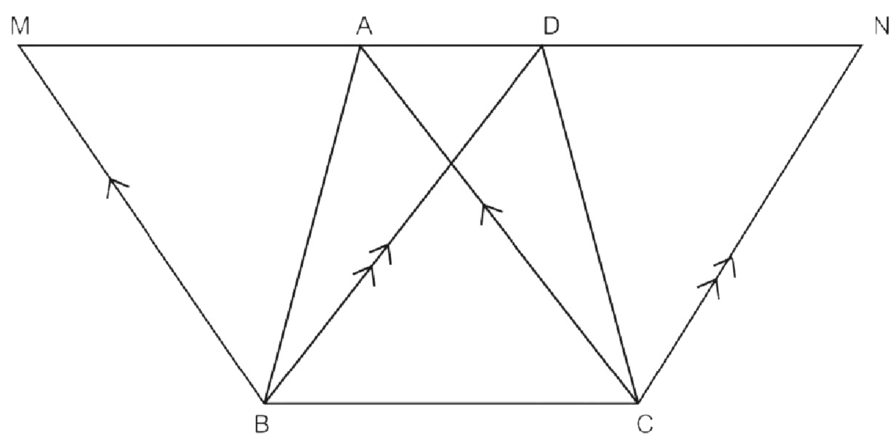
Triangle on the same base and of the same (i.e…equal) altitudes are equal in area

Given

Δ's ABC , DBC on the Same base $\overline{BC}$ and having equal altitudes

To prove

Area of (ΔABC) = area of (ΔDBC)



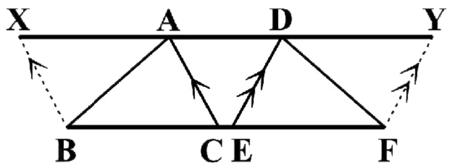
Construction:

Draw $\overline{BM} \parallel$ to $\overline{CA}$, $\overline{CN} \parallel$ to $\overline{BD}$ meeting $\overline{AD}$ produced in M.N.

Proof

Statements	Reasons
ΔABC and ΔDBC are between the same $\parallel^{s}$	Their altitudes are equal
Hence MADN is parallel to $\overline{BC}$	
$\therefore$ Area $\parallel^{gm}$ (BCAM)= Area $\parallel^{gm}$ (BCND)	These $\parallel^{gm}$ are on the same base $\overline{BC}$ and between the same $\parallel^{s}$
But $\Delta ABC = \frac{1}{2} \parallel^{gm}$ (BCAM)------(ii)	Each diagonal of a $\parallel^{gm}$
And $\Delta DBC = \frac{1}{2} \parallel^{gm}$ (BCND)-----(iii)	
Hence area (ΔABC) = Area(ΔDBC)	Bisects it into two congruent triangles From (i) (ii) and (iii)

Theorem 16.1.4



Triangles on equal bases and of equal altitudes are equal in area.

Given

Δs ABC, DEF on equal bases $\overline{BC}$, $\overline{EF}$ and having altitudes equal

To prove

Area (ΔABC) = Area (ΔDEF)

Construction:

Place the Δs ABC and DEF so that their equal bases $\overline{BC}$ and $\overline{EF}$ are in the same straight line BCEF and their vertices on the same side of it .Draw $\overline{BX} \parallel \overline{CA}$ and $\overline{FY} \parallel \overline{ED}$ meeting $\overline{AD}$ produced in X, Y respectively

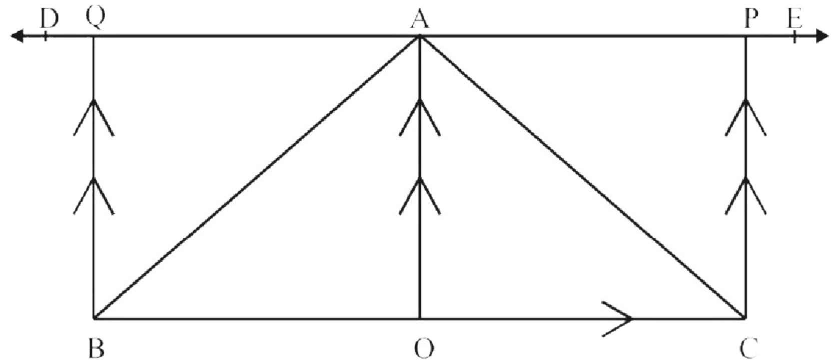
Proof

Statements	Reasons
ΔABC, ΔDEF are between the same parallels	Their altitudes are equal (given)

$\therefore$ XADY is $\|^{gm}$ to BCEF $\therefore$ area $\|^{gm}$ (BCAX) = A area $\|^{gm}$ (EFYD)----(i)	These $\|^{gm}$ are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2} \|^{gm}$ (BCAX)----(ii)	Diagonal of a $\|^{gm}$ bisect it
And area of $\Delta DEF = \frac{1}{2}$ area of $\|^{gm}$ (EFYD)__ (iii)	
$\therefore$ area (ΔABC) = area (ΔDEF)	From (i),(ii)and(iii)

Exercise 16.2

Q.1



Show that

Given

ΔABC,O is the mid point of $\overline{BC}$

$\overline{OB} \cong \overline{OC}$

To prove

Area ΔABO = area ΔACO

Construction

Draw $\overline{DE} \parallel \overline{BC}$

$\overline{CP} \parallel \overline{OA}$

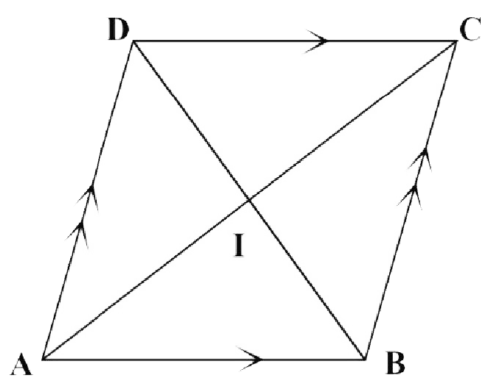
$\overline{BQ} \parallel \overline{OA}$

Proof

Statements	Reasons
$\overline{BQ} \parallel \overline{OA}$	Construction
$\overline{OB} \parallel \overline{AQ}$	Construction
$\parallel^{gm}$ BOAQ	Base of same
$\parallel^{gm}$ COAP	Parallel line of $\overleftrightarrow{DE}$
$\overline{OB} \cong \overline{OC}$	O is the mid point of $\overline{BC}$
Area of $\parallel^{gm}$ BOAQ= Area of $\parallel^{gm}$ COAP … (i)	
Area of $\Delta ABO = \frac{1}{2}$ Area of $\parallel^{gm}$ BOAQ	
Area of ΔACO $= \frac{1}{2}$ Area of $\parallel^{gm}$ COAP	
Area of ΔABO = Area of ΔACO	Dividing equation (i) both side by (ii)

So median of a triangle divides it into two triangles of equal area.

Q.2 **Prove that a parallelogram is divided by its diagonals into four triangles of equal area.**



Given:

In parallelogram ABCD, $\overline{AC}$ and $\overline{BD}$ are its diagonals, which meet at I

To prove:

Triangles ABI, BCI CDI and ADI have equal areas.

Proof:

Triangles ABC and ABD have the same base $\overline{AB}$ and are between the same parallel lines $\overline{AB}$ and $\overline{DC}$ $\therefore$ they have equal areas.

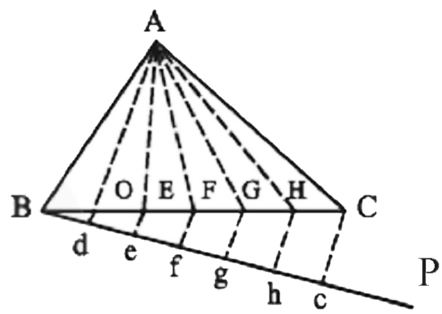
Or area of ΔABC =area of Δ ABD

Or area of Δ ABI + area of Δ BCI= area of Δ ABI+ area of Δ ADI
$\Rightarrow$ Area of Δ BCI = area of Δ ADI ... (i)
Similarly area of Δ ABC = area of Δ BCD
$\Rightarrow$ Area of Δ ABI +area of Δ BCI = area of Δ BCI + area of Δ CDI
$\Rightarrow$ Area of Δ ABI = area of Δ CDI… (ii)

As diagonals of a parallelogram bisect each other I is the midpoint of $\overline{AC}$ so $\overline{BI}$ is a median of Δ ABC
$\therefore$ Area of Δ ABI = area of Δ BCI… (iii)
$\Delta CDI \cong \Delta AOI$
$\overline{BI} \cong \overline{DI}$
Area of Δ ABI = area of Δ BCI = area of Δ CDI= area of Δ ADI

Q.3 Divide a triangle into six equal triangular parts

Given
ΔABC
To prove
To divide ΔABC into six equal part triangular parts
Construction
Take $\overrightarrow{BP}$ any ray making an acute angle with $\overline{BC}$ draw six arcs of the same radius on $\overrightarrow{BP}$ i.e $m\overline{Bd} = mde = mef = mfg = mgh = mhc$
Join c to C and parallel line segments as
$\overline{cC} \| \overline{hH} \| \overline{gG} \| \overline{fF} \| \overline{eE} \| \overline{do}$
Join A to O,E,F,G,H



Proof
Base $\overline{BC}$ of ΔABC has been divided to six equal parts.
We get six triangles having equal base and same altitude
Their area is equal
Hence ΔBOA=ΔOEA=ΔEFA=ΔFGA=ΔGHA=ΔHCA

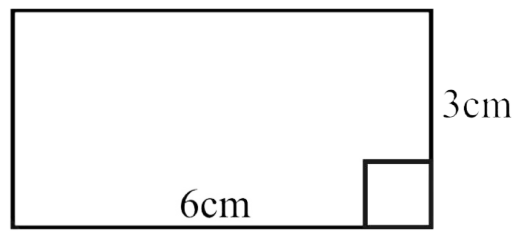
Review Exercise 16

Q.1 Which of the following are true and which are false?

(i) Area of a figure means region enclosed by bounding lines of closed figures. (True)
(ii) Similar figures have same area. (False)
(iii) Congruent figures have same area. (True)
(iv) A diagonal of a parallelogram divides it into two non-congruent triangles. (False)
(v) Altitude of a triangle means perpendicular from vertex to the opposite side (base). (True)
(vi) Area of a parallelogram is equal to the product of base and height. (True)

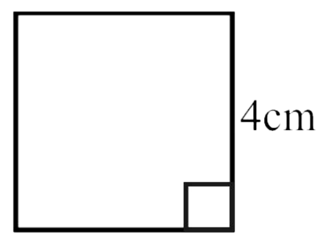
Q.2 Find the area of the following.

(i)



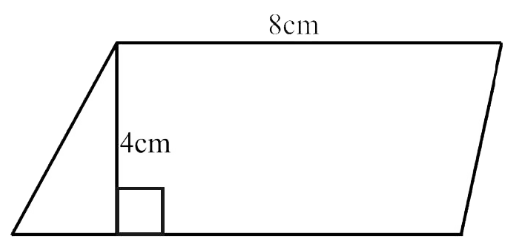
Given
Length of rectangle = ℓ = 3cm
Width of rectangle = w = 6cm
Required:
Area of rectangle =?
Solution:
Area of rectangle = length × width
= 3cm × 6cm
⇒ Area of rectangle = 18 cm^2

(ii)



Given
Length of square = ℓ = 4cm
Required:
Area of square =?
Solution:
Area of square = $\ell \times \ell$
$= \ell^2$
$= (4cm)^2$
⇒ Area of square = $16cm^2$

(iii)



Given
Height of parallelogram = 4cm
Base of parallelogram = 8cm
Required:
Area of parallelogram = ?
Solution:
Area of parallelogram = b × h
= 8cm × 4cm

$\Rightarrow$ area of parallelogram = 32 cm^2

(iv)

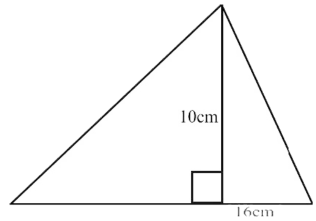
Given:

Height of triangle = h = 10 m

Base of triangle = b = 16cm

Required:

Area of triangle =?



Solution:

Area of triangle = $\frac{1}{2} \times b \times h$

$= \frac{1}{\not{2}} \times {}^{8}\not{16}\ \text{cm} \times 10\text{cm}$

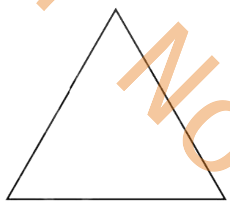
= 8cm × 10 cm

=80cm^2

Q.3 Define the following

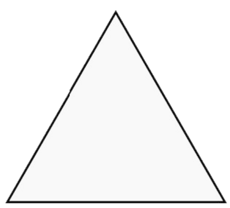
(i) Area of a figure

The region enclosed by the bounding lines of a closed figure is known as area of the figure.



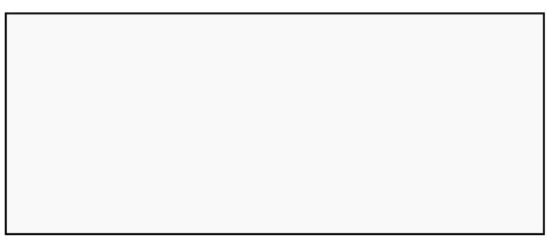
(ii) Triangular Region

A triangular region is the union of a triangle and its interior i-e three line segments forming the triangle and its interior



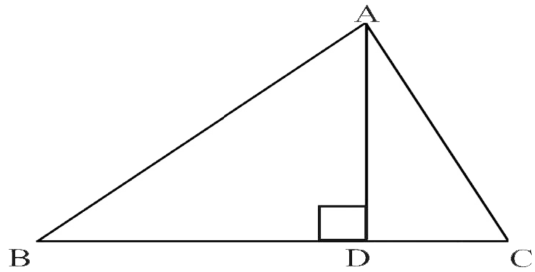
(iii) Rectangular Region

A rectangular region is the union of a rectangle and its interior. A rectangular region can be divided into two or more than two triangular regions in many ways.



(iv) Altitude or Height

If one side of a triangle is taken as its base, the perpendicular distance form one vertex opposite side is called altitude of triangle. $\overline{AD}$ is its altitude.



Unit 16: Theorems Related With Area

Overview

Theorem 16.1.1

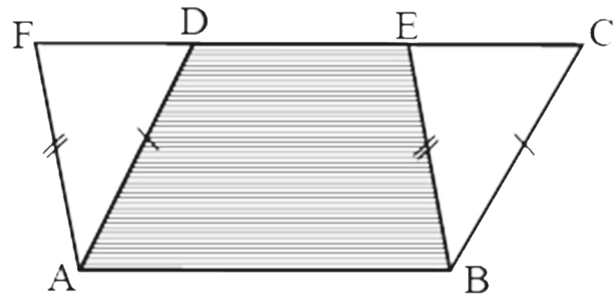
Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area

Given

Two parallelograms ABCD and ABEF having the same base $\overline{AB}$ between the same parallel lines $\overline{AB}$ and $\overline{DE}$

To prove

Area of parallelogram ABCD=area of parallelogram ABEF



Proof

Statements	Reasons
Area of (parallelogram ABCD) = Area of (Quad. ABED) + Area of (Δ CBE) … (1)	[Area addition axiom]
Area of (parallelogram ABEF) = Area of (Quad. ABED) + Area of (Δ DAF) … (2)	[Area addition axiom]
In Δ s CBE and DAF	
m $\overline{CB}$ = m $\overline{DA}$	[opposite sides of a Parallelogram]
m $\overline{BE}$ = m $\overline{AF}$	[opposite sides of a Parallelogram]
m $\angle$CBE = m $\angle$DAF	$\left[\because \overline{BC} \parallel \overline{AD}, \overline{BE} \parallel \overline{AF}\right]$
Δ CBE $\cong$ Δ DAF	[S.A.S Cong.axiom]
Area of (ΔCBE)= area of (ΔDAF)… (3)	[Cong. Area axiom]
Hence area of (Parallelogram ABCD) = area of (parallelogram ABEF)	From (1),(2) and (3)

Theorem 16.1.2

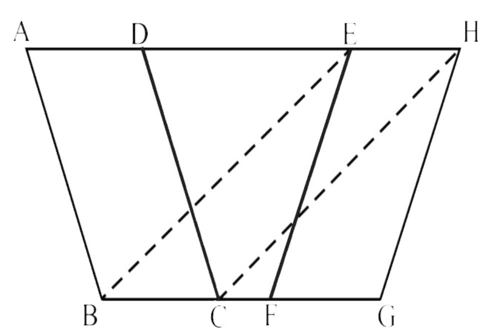
Parallelograms on equal bases and having the same (or equal) altitude area equal in area.

Given :

Parallelogram ABCD, EFGH are on equal base $\overline{BC}$, $\overline{FG}$ having equal altitudes

To prove

Area of (Parallelogram ABCD)= area of (parallelogram EFGH)



Construction

Place the parallelogram ABCD and EFGH So that their equal bases $\overline{BC}$, $\overline{FG}$ are in the straight line BCFG. Join $\overline{BE}$ and $\overline{CH}$

Proof

Statements	Reasons
The give 11^{mg} ABCD and EFGH are between the same parallels	
Hence ADEH is a straight line $\parallel$ to $\overline{BC}$	Their altitudes are equal (given)
$\therefore$ m$\overline{BC}$ = m$\overline{FG}$ =m$\overline{EH}$	
Now m$\overline{BC}$ =m$\overline{EH}$ and they are $\parallel$	Given
$\therefore$ $\overline{BE}$ and $\overline{CH}$ are both equal and $\parallel$	EFGH is a parallelogram
Hence EBCH is a Parallelogram	A quadrilateral with two opposite side congruent and parallel is a parallelogram
Now $\parallel^{gm}$ ABCD = $\parallel^{gm}$ EBCH –(i)	Being on the same base $\overline{BC}$ and between the same parallels
But $\parallel^{gm}$ EBCH = $\parallel^{gm}$ EFGH – (ii)	Being on the same base $\overline{EH}$ and between the same parallels
Hence area $\parallel^{gm}$ (ABCD)= Area $\parallel^{gm}$ (EFGH)	From (i) and (ii)