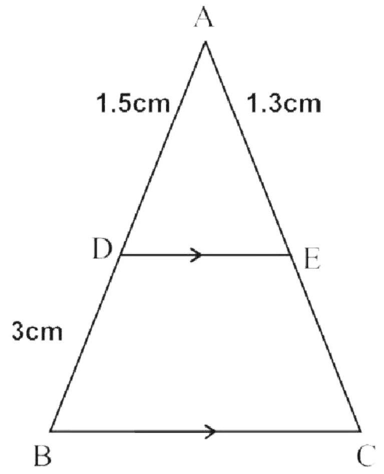
Exercise 14.1

Q.1 In ΔABC

$\overline{DE} \parallel \overline{BC}$



(i) **If** $\overline{AD} = 1.5\text{cm}$ $\overline{BD} = 3\text{cm}$

$\overline{AE} = 1.3\text{cm}$, then find $\overline{CE}$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By substituting the values of $\overline{AD}, \overline{BD}$ and $\overline{AE}$

So

$$\frac{1.5}{3} = \frac{1.3}{\text{EC}}$$

$$\overline{EC}(1.5) = 1.3 \times 3$$

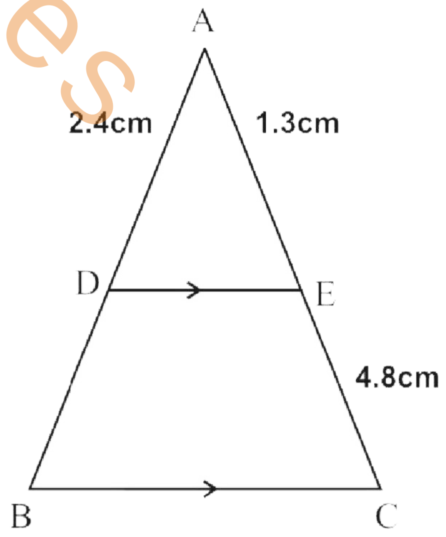
$$\overline{EC} = \frac{1.3 \times 3}{1.5}$$

$$\overline{EC} = \frac{3.9}{1.5}$$

$$\overline{EC} = 2.6\text{cm}$$

(ii) **If AD = 2.4cm** $\overline{AE} = 3.2\text{cm}$

$\overline{EC} = 4.8\text{cm}$ find AB



$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\overline{AC} = AE + EC$$

$$\overline{AC} = 3.2 + 4.8$$

$$\overline{AC} = 8cm$$

$$\therefore \frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

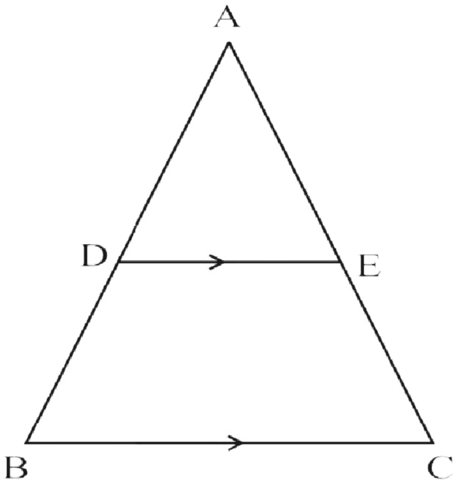
$$\frac{2.4}{\text{AB}} = \frac{3.2}{8}$$

$$2.4 \times 8 = (3.2)\overline{AB}$$

$$\frac{19.2}{3.2} = \overline{AB}$$

$$\overline{AB} = 6\text{cm}$$

(iii) **If** $\frac{\overline{AD}}{\overline{BD}} = \frac{3}{5}\ \overline{AC} = 4.8cm$ **find** $\overline{AE}$



$\overline{AC} = \overline{AE} + \overline{EC}$

$\overline{AC} = \overline{EC} + \overline{AE}$

$\overline{AE} = 4.8 - \overline{EC}$

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AC} - \overline{EC}}{\overline{EC}}$

$\frac{3}{5} = \frac{4.8 - \overline{EC}}{\overline{EC}}$

$3(\overline{EC}) = 5(4.8 - \overline{EC})$

$3(\overline{EC}) = 24 - 5(\overline{EC})$

$3(\overline{EC}) + 5(\overline{EC}) = 24$

$8(\overline{EC}) = 24$

$(\overline{EC}) = \frac{\cancel{24}^{3}}{\cancel{8}}$

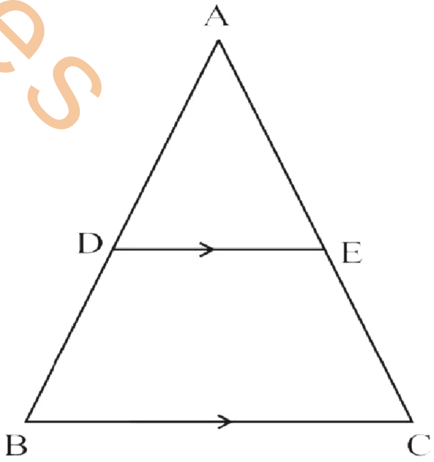
$\overline{EC} = 3cm$

$\overline{AE} = \overline{AC} - \overline{EC}$

$= 4.8 - 3$

$= 1.8cm$

(iv) **If** $\overline{AD} = 2.4cm\ \overline{AE} = 3.2cm\ \overline{DE} = 2cm\ \overline{BC} = 5cm$. **Find** $\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}$.



$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}}$

$\frac{2.4}{AB} = \frac{3.2}{AC} = \frac{2}{5}$

$\frac{2.4}{AB} = \frac{2}{5}$

$(2.4)5 = 2(AB)$

$\frac{12.0}{2} = AB$

$\overline{AB} = 6cm$

$\frac{3.2}{AC} = \frac{2}{5}$

$16.0 = 2(AC)$

$\frac{\cancel{16}^{8}}{\cancel{2}} = AC$

$\overline{AC} = 8cm$

$\overline{DB} = \overline{AB} - \overline{AD}$

$\overline{DB} = 6 - 2.4$

$\overline{BD} = 3.6\,\text{cm}$

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{6} = \frac{\overline{AE}}{8}$$

$$\overline{AE} = \frac{2.4}{6} \times 8$$

$$\overline{AE} = \frac{19.2}{6}$$

$\overline{AE} = 3.2cm$

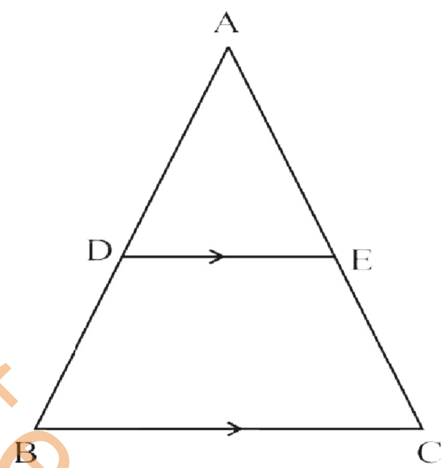
$\overline{CE} = \overline{AC} - \overline{AE}$

$\overline{CE} = 8 - 3.2$

$\overline{CE} = 4.8cm$

If $\overline{AD} = 4x - 3$ $\overline{AE} = 8x - 7$

$\overline{BD} = 3x - 1$ and $CE = 5x - 3$ Find the value of x



$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{EC}$$

By putting the value of $\overline{AD}, \overline{AE}, \overline{BD}$ and $\overline{CE}$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

By cross multiplying

$(4x - 3)(5x - 3) = (8x - 7)(3x - 1)$

$20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$0 = 24x^2 - 20x^2 - 29x + 27x + 7 - 9$

$4x^2 - 2x - 2 = 0$

$2(2x^2 - x - 1) = 0$

$$2x^2 - 2x + 1x - 1 = \frac{0}{2}$$

$2x(x-1) + 1(x-1) = 0$

$(x-1)(2x + 1) = 0$

$x - 1 = 0$ $\quad$ $2x + 1 = 0$

$x = 1$ $\quad$ $2x = -1$

$$x = -\frac{1}{2}$$

Distance is not taken in negative it is always in positive so the value of x = 1.

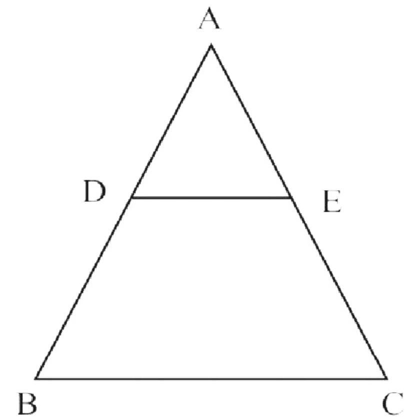
Q.2 **In ΔABC is an isosceles triangle ∠A is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that**

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

Prove that ΔADE is also an isosceles triangle.

Given:

ΔABC is an isosceles triangle, $\angle A$ is vertex and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$.



$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$$

To Prove

$$m\overline{AD} = m\overline{AE}$$

Proof

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

Or $\frac{\overline{BD}}{\overline{AD}} = \frac{\overline{EC}}{\overline{AE}}$

Or $\frac{\overline{AD}+\overline{BD}}{\overline{AD}} = \frac{\overline{AE}+\overline{EC}}{\overline{EC}}$

As we know

$$\overline{AB} = \overline{AD}+\overline{BD}$$

$$\overline{AC} = \overline{AE}+\overline{EC}$$

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

From this

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

$$\overline{AD} = \overline{AE}$$

$\overline{AB} = \overline{AC}$ **(Given)**

Q.3 In an equilateral triangle ABC shown in the figure $m\overline{AE}:m\overline{AC} = m\overline{AD}:m\overline{AB}$ find all the three angles of ΔADE and name it also.

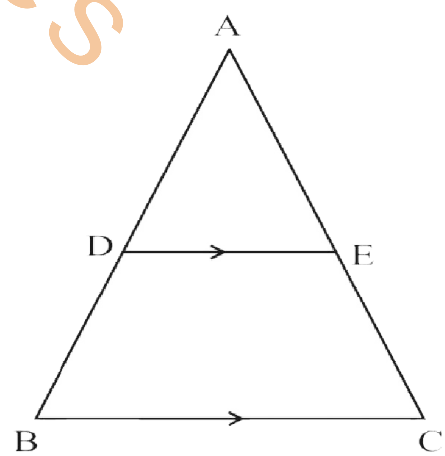
Given

ΔABC is equilateral triangle

To prove

To find the angles of ΔADE

Solution:



$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

All angles are equal as it is an equilateral triangle which are equal to 60° each

$\angle A = \angle B = \angle C$

$m\overline{BC} \| m\overline{DE}$

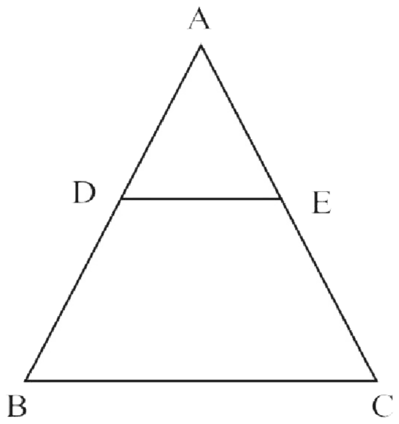
$\angle ADE = \angle ABC = 60^\circ$

$\angle AED = \angle ACB = 60^\circ$

$\angle A = 60^\circ$

ΔADE is an equilateral triangle

Q.4 Prove that line segment drawn through the midpoint of one side of a triangle and parallel to another side bisect the third side



Given

$\overline{AD} = \overline{BD}$

$\overline{DE} \| \overline{BC}$

To Prove

$\overline{AE} = \overline{EC}$

In ΔABC

$\overline{DE} \| \overline{BC}$

In theorem it is already discussed that

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

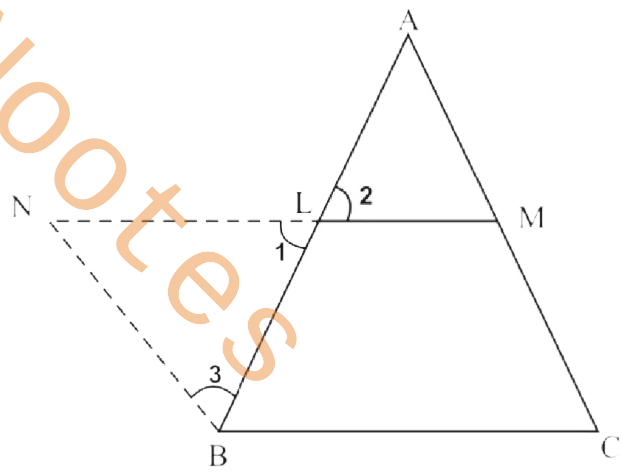
As we know $\overline{AD} = \overline{BD}$ or $\overline{BD} = \overline{AD}$

$$\frac{\cancel{\overline{AD}}}{\cancel{\overline{AD}}} = \frac{\overline{AE}}{\overline{EC}}$$

$$1 = \frac{\overline{AE}}{\overline{EC}}$$

$\overline{EC} = \overline{AE}$

Q.5 Prove that the line segment joining the midpoint of any two sides of a triangle is parallel to the third side



Given

ΔABC the midpoint of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

To Prove

$\overline{LM} \| \overline{BC}$ and $m\overline{LM} = \frac{1}{2}\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that

$\overline{ML} \cong \overline{LN}$

Join N to B and in the figure name the angles $\angle 1$, $\angle 2$, and $\angle 3$

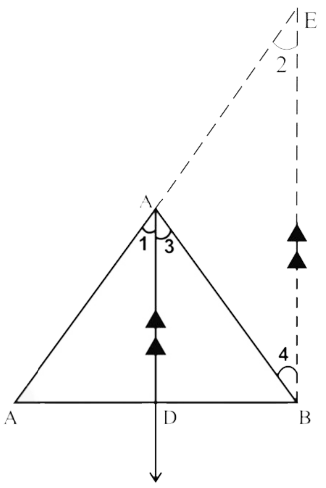
Proof

Statements	Reasons
$\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong AL$	Given
$\angle 2 = \angle 1$ or $\angle 1 = \angle 2$	Vertical angles
$\overline{NL} = \overline{ML}$	Construction
$\therefore\ \Delta BLN \cong \Delta ALM$	Corresponding angle of congruent triangles Given
$\therefore \angle A = \angle 3$	
And $\overline{NB} \cong \overline{AM}$	
$\overline{NB} \| \overline{AM}$	

$\overline{ML} = \overline{AM}$	Given
$\overline{NB} \cong \overline{ML}$	
$\overline{BC}\,\overline{MN}$ is parallelogram	
$\therefore\ \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	
$\overline{BC} \cong \overline{NM}$	(Opposite side of parallelogram BCMN)
$m\overline{LM} = \frac{1}{2} m\overline{NM}$	(Opposite side of parallelogram)
Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	

Theorem 14.1.3

The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.



Given

In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the points D.

To prove

$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction

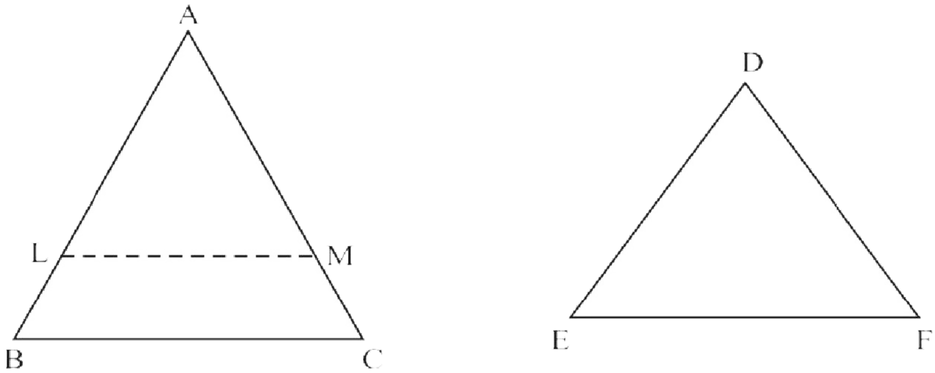
Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ Produced at E

Proof

Statements	Reasons
$\because \overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersect them	Construction
$m\angle 1 = m\angle 2$ ....(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them	
$\therefore\ m\angle 3 = m\angle 4$ ....(ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore\ m\angle 2 = m\angle 4$	From (i) and (ii)
And $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ, the sides opposite to congruent angles are also congruent
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional



Given

$\Delta ABC \sim \Delta DEF$

i.e $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction

(I) Suppose that $m\overline{AB} > m\overline{DE}$

(II) $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$

Join L and M by the line segment LM

Proof

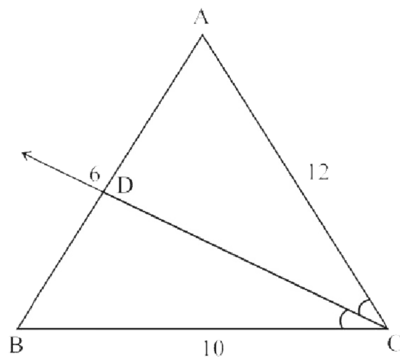
Statements	Reasons
In $\Delta ALM \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S Postulate
And $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$ ....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ ....(ii)	

Thus $\frac{m\overline{DE}}{m\overline{AB}}=\frac{m\overline{DF}}{m\overline{AC}}=\frac{m\overline{EF}}{m\overline{BC}}$	By (i) and (ii)
Or $\frac{m\overline{AB}}{m\overline{DE}}=\frac{m\overline{AC}}{m\overline{DF}}=\frac{m\overline{BC}}{m\overline{EF}}$	By taking reciprocals
If $m\overline{AB}=m\overline{DE}$	
Then in $\Delta ABC \leftrightarrow \Delta DEF$	
(II) If $m\overline{AB}<m\overline{DE}$, it can similarly be proved by taking intercepts on the sides of ΔDEF	
$\angle A \cong \angle D$	
$\angle B \cong \angle E$	
And $\overline{AB}\cong\overline{DE}$	
So $\Delta\ ABC \cong \Delta DEF$	A.S.A ≅ A.S.A
Thus $\frac{m\overline{AB}}{m\overline{DE}}=\frac{m\overline{AC}}{m\overline{DF}}=\frac{m\overline{BC}}{m\overline{EF}}=1$	$\overline{AC}\cong\overline{DF}$, $\overline{BC}\cong\overline{EF}$
Hence the result is true for all the cases.	

Exercise 14.2

Q.1 In ΔABC as shown in the figure $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D. $m\overline{BD}$ is equal to

(a) 5 **(b)** 16 **(c)** 10 **(d)** 18



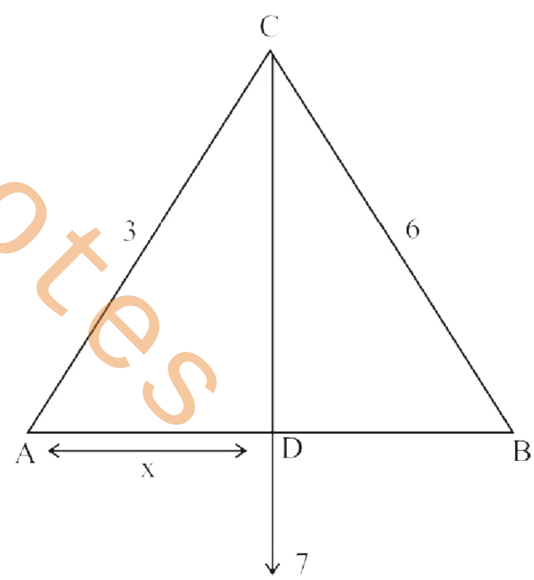
$$\frac{m\overline{BD}}{m\overline{DA}}=\frac{m\overline{BC}}{m\overline{CA}}$$

$$\frac{\overline{BD}}{6}=\frac{10}{12}$$

$$\overline{BD}=\frac{\cancel{10}^{5}\times\cancel{6}^{2}}{\cancel{12}_{\cancel{4}\,2}}\text{ or }\overline{BD}=\frac{10\times 6}{12}=\frac{\cancel{60}^{5}}{\cancel{12}}$$

$$\overline{BD}=5$$

Q.2 In ΔABC shown in the figure $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC}=3$, $\overline{CB}=6$ and $m\overline{AB}=7$ then find $m\overline{AD}$ and $\overline{DB}$



$$\overline{AB}=\overline{AD}+\overline{BD}$$

$$\overline{AD}=\overline{AB}-\overline{BD}$$

$$\overline{AD}=7-x$$

$$\frac{m\overline{AD}}{m\overline{BD}}=\frac{m\overline{AC}}{m\overline{CB}}$$

$$\frac{x}{7-x}=\frac{\cancel{3}^{1}}{\cancel{6}_{2}}$$

$$\frac{x}{7-x}=\frac{1}{2}$$

$2x = 7-x$

$2x + x = 7$

$3x = 7$

$$x=\frac{7}{3}\quad\text{or}\quad\overline{AD}=\frac{7}{3}$$

$$\overline{AB}=\overline{AD}+\overline{BD}$$

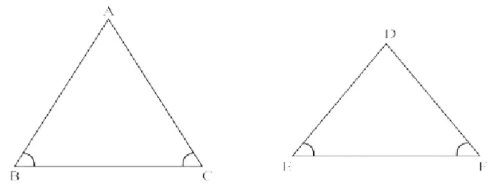
$$7=\frac{7}{3}+\overline{BD}$$

$$7-\frac{7}{3}=\overline{BD}$$

$$\frac{21-7}{3}=\overline{BD}$$

$$\overline{BD}=\frac{14}{3}$$

Q.3 **Show that in any corresponding of two triangles if two angles of one triangle are congruent to the corresponding angles of the other, then the triangle are similar**



Given

ΔABC and Δ DEF

$\angle B \cong \angle E$

$\angle C \cong \angle F$

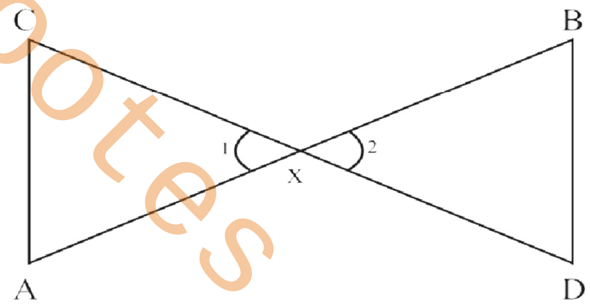
To Prove

$\Delta ABC \cong \Delta DEF$

Proof

Statements	Reasons
$\angle A + \angle B + \angle C = 180^\circ$ $\angle D + \angle E + \angle F = 180$ $\angle A \cong \angle D$ $\angle B = \angle E$ $\angle C = \angle F$ Hence Δ ABC $\cong \Delta DEF$	Sume of three angles of a triangle = 180°

Q.4 **If line segment** $\overline{AB}$ **and** $\overline{CD}$ **are intersecting at point** X **and** $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ **then show that** ΔAXC and ΔBXD **are similar**



Given

Line segment $\overline{AB}$ and $\overline{CD}$ intersect at X

$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$

To Prove

ΔCXA and ΔDXB are similar

Proof

Statements	Reasons
$\frac{\overline{AX}}{\overline{XB}} = \frac{\overline{CX}}{\overline{XD}}$	Given
$\angle 1 \cong \angle 2$	
$\overline{AC} \parallel \overline{BD}$	Vertical angles
$\angle A = m\angle B$	
$m\angle C = m\angle D$	Alternate angles
Hence proved the triangle are similar	

Review Exercise 14

Q.1 Which of the following are true which are false?

(i)	Congruent triangles are of same size and shape.	(True)
(ii)	Similar triangles are of same shape but different sizes.	(True)
(iii)	Symbol used for congruent is '~'	(False)
(iv)	Symbol used for similarity is ' ≅'	(False)
(v)	Congruent triangle are similar	(True)
(vi)	Similar triangles are congruent	(False)
(vii)	A line segment has only one midpoint	(True)
(viii)	One and only one line can be drawn through two points	(True)
(ix)	Proportion is non equality of two ratio	(False)
(x)	Ratio has no unit	(True)

Q.2 Define the following

(i) Ratio

The ratio between two a like quantities is defined as $a : b = \frac{a}{b}$ where a and are the elements of the ratio.

(ii) Proportion

Proportion is defined as the equality of two ratio i,e a : b = c : d

(iii) Congruent Triangles

Two triangles are said to be congruent (symbols ≅) if there emits a corresponding betweet them such that all the corresponding sides and angles are congruent.

(iv) Similar Triangles

If two triangles are similar then the measures of their corresponding sides are proportional.

Q.3 In ΔLMN shown in the figure $\overline{MN} || \overline{PQ}$

(i) If $m\overline{LM} = 5\text{cm}$, $m\overline{LP} = 2.5\text{cm}$

$m\overline{LQ} = 2.3$ cm then find LN

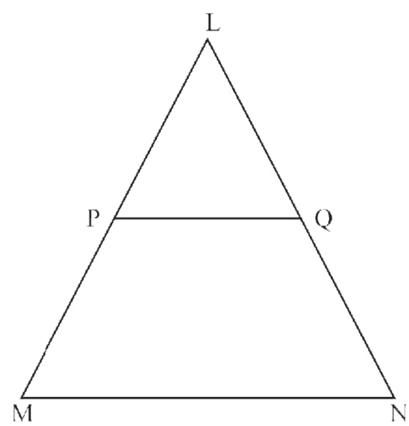
$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{2.5}{5} = \frac{2.3}{\overline{LN}}$$

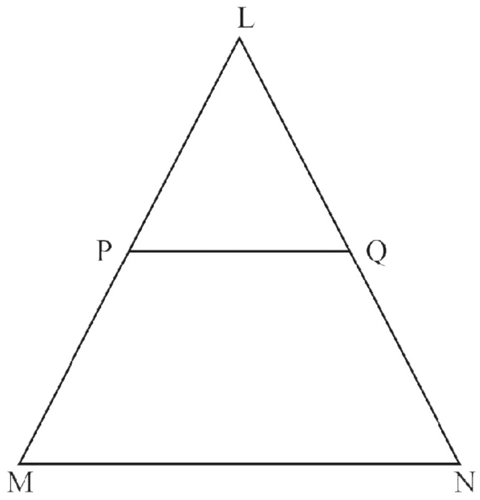
$$(2.5)\ \overline{LN} = 5 \times 2.3$$

$$\overline{LN} = \frac{11.5}{2.5}$$

$$\overline{LN} = 4.6\text{cm}$$



(ii) If mLM = 6cm , mLQ = 2.5cm
mQN = 5cm then find
mLP



$$\frac{m\overline{LP}}{m\overline{LM}}=\frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{LP}{6}=\frac{2.5}{LN}$$

$$\overline{LN}=\overline{LQ}+\overline{QN}$$

$$\overline{LN}=2.5+5$$

$$\overline{LN}=7.5cm$$

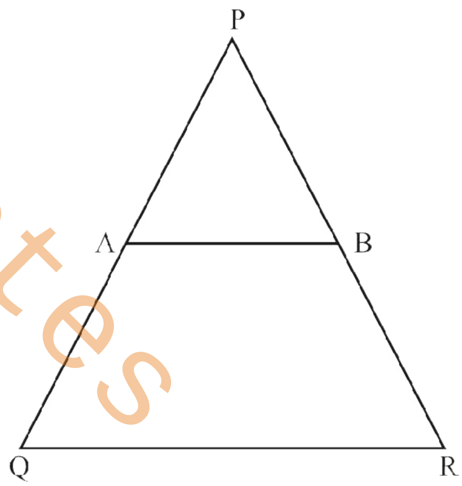
$$\frac{\overline{LP}}{6}=\frac{2.5}{7.5}$$

$$\overline{LP}=\frac{2.5\times 6}{7.5}$$

$$\overline{LP}=\frac{15}{7.5}$$

$$\overline{LP}=2cm$$

Q.4 **In the show figure let mPA = 8x – 7 mPB = 4x – 3** $m\overline{AQ}=5x-3$ $m\overline{BR}=3x-1$ **find the value of x if** $\overline{AB}||\overline{QR}$



$$\frac{mPA}{mAQ}=\frac{mBP}{mBR}$$

$$\frac{8x-7}{5x-3}=\frac{4x-3}{3x-1}$$

By cross multiplying

$$(8x-7)(3x-1)=(4x-3)(5x-3)$$

$$24x^2-8x-21x+7=20x^2-12x-15x+9$$

$$24x^2-29x+7=20x^2-27x+9$$

$$24x^2-20x^2-29x+27x+7-9=0$$

$$4x^2-2x-2=0$$

$$4x^2-4x+2x-2=0$$

$$4x(x-1)+2(x-1)=0$$

$$(x-1)(4x+2)=0$$

$$x-1=0$$

$$x=1$$

$$4x+2=0$$

$$4x=-2$$

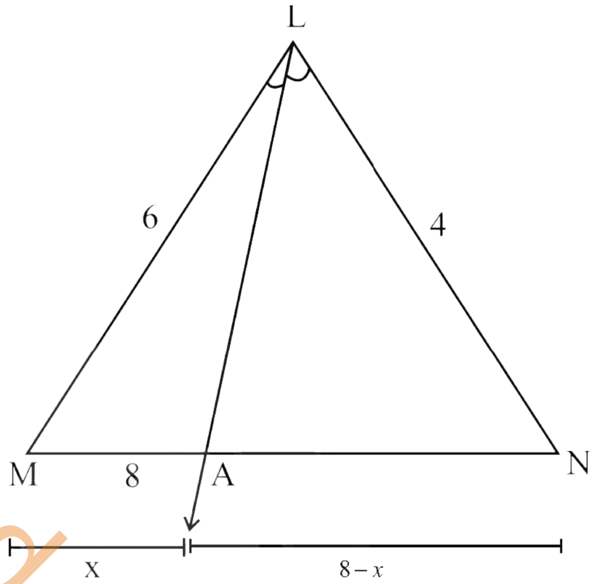
$$x=\frac{-\cancel{2}^{1}}{\cancel{4}_{2}}$$

$x = \frac{-1}{2}$

Length is always taken as positive not negative so value of $x = 1$

Q.5 In ΔLMN Shown in figure $\overrightarrow{LA}$ bisects ∠L. If $m\overline{LN} = 4m$ $m\overline{LM} = 6cm$ $m\overline{MN} = 8$ then find $m\overline{MA}$ and $m\overline{AN}$

$\frac{m\overline{MA}}{m\overline{AN}} = \frac{m\overline{LM}}{m\overline{LN}}$



$\overline{MA} = x$

$\overline{AN} = 8\text{-}x$

$\frac{x}{8-x} = \frac{6}{4}$

$4x = 6(8-x)$

$4x = 48 - 6x$

$4x + 6x = 48$

$10x = 48$

$x = \frac{48}{10}$

$x = 4.8cm$

$m\overline{MA} = 4.8cm$

$\overline{MN} = \overline{MA} + \overline{AN}$

$8 = 4.8 + \overline{AN}$

$8 - 4.8 = \overline{AN}$

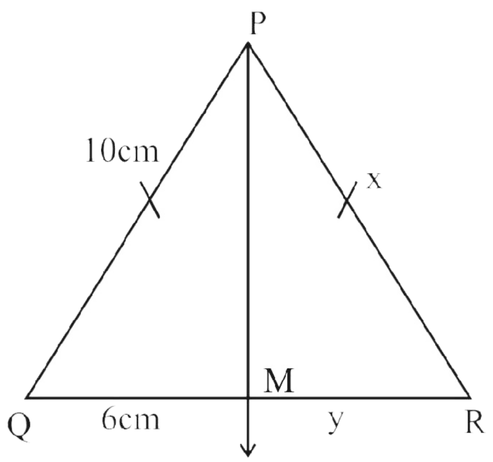
$\overline{AN} = 3.2cm$

Q.6 In Isosceles ΔPQR Shown in the figure, find the value of x and y

As we know that it is isosceles triangle

So

$\overline{PQ} = \overline{RP}$



$10 = x$

Or

$x = 10\text{cm}$

$\overline{PM} \perp \overline{QR}$

So it bisects the side and bisects the angle also

SO $\overline{QM} = \overline{MR}$

$6 = y$

Or

$y = 6\text{cm}$

Unit 14: Ratio and Proportion

Overview

Theorem 14.1.1

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.

Given:

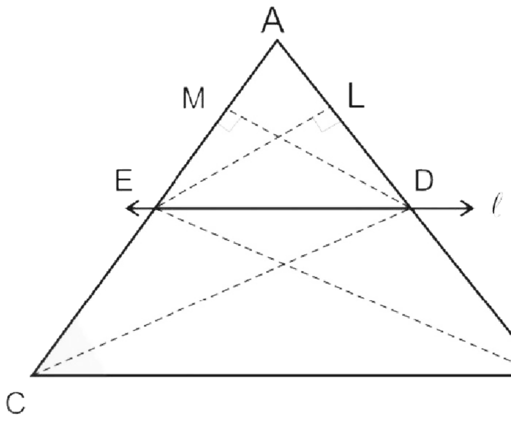
In ΔABC, the line ℓ is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$

To Prove

$m\overline{AD} : \overline{DB} = m\overline{AE} : m\overline{EC}$

Construction:

Join B to E and C to D .From D draw $\overline{DM} \perp \overline{AC}$ and from E draw $\overline{EL} \perp \overline{AB}$



Proof

Statements	Reasons
In triangles BED and AED, EL is the common perpendicular	
$\therefore$ Area of $\Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$.......(i)	Area of a $\Delta = \frac{1}{2}$ (base)(height)
and Area of $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL}$.......(ii)	
Thus Area of $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{DB}}{m\overline{AD}}$........(iii)	Dividing (i) by (ii)
Similarly $\frac{\text{Area of } \Delta CDE}{\text{Area of } \Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$........(iv)	
But $\Delta BED \cong \Delta CDE$	(Areas of triangles with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$, so altitudes are equal).
$\therefore$ From (iii) and (iv) We have $\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$ or	
$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Taking reciprocal of both sides.
Hence $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	

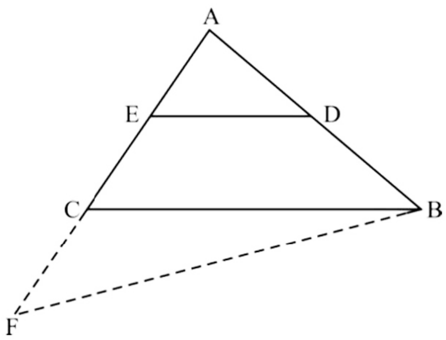
Theorem: 14.1.2 Converse of Theorem 14.1.1

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.

Given

In ΔABC, $\overline{ED}$ intersect $\overline{AB}$ and $\overline{AC}$ such that

$m\overline{AD} : \overline{DB} = m\overline{AE} : m\overline{EC}$



To Prove

$\overline{ED} \parallel \overline{CB}$

Construction

If $\overline{ED} \nparallel \overline{CB}$ then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ Produced at F

Proof

Statements	Reasons
In ΔABF	
$\overline{DE} \parallel \overline{BF}$	Construction
$\therefore \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EF}}$..........(i)	(A line parallel to one side of a triangle divides the other two sides proportionally Theorem 14.1.1)
But $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$..........(ii)	Given
$\therefore \frac{m\overline{AE}}{m\overline{EF}} = \frac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF} = m\overline{EC}$,	
This is possible only if point F is coincident with C.	(Property of real numbers)
$\therefore$ Our supposition is wrong	
Hence $\overline{ED} \parallel \overline{CB}$	