Exercise 7.1

Q.1 Solve the following equations

(i) $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Solution: $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

$\frac{4x-3x}{6}=\frac{6x+1}{6}$

$x=6\frac{(6x+1)}{6}$

$x=6x+1$

$-6x+x=1$

$-5x=1$

$x=\frac{1}{-5}$

$x=-\frac{1}{5}$

To check

Substitution $x=-\frac{1}{5}$

$\frac{2}{3}\times\frac{-1}{5}-\frac{1}{2}\times\frac{-1}{5}=\frac{-1}{5}+\frac{1}{6}$

$\frac{-2}{15}+\frac{1}{10}=\frac{-6+5}{30}$

$\frac{-2\times2+1\times3}{30}=\frac{-1}{30}$

$\frac{-4+3}{30}=\frac{-1}{30}$

$\frac{-1}{30}=\frac{-1}{30}$

Solution Set $=\left\{-\frac{1}{5}\right\}$

(ii) $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Solution $\frac{x-3}{3}-\frac{x-2}{2}=-1$

By taking L.C.M

$\frac{2(x-3)-3(x-2)}{6}=-1$

$2x-6-3x+6=-6$

$-x=-6$

Multiplying both sets by –1

$-1\times -x=-1\times-6$

$x=6$

To check

$\frac{x-3}{3}-\frac{x-2}{2}=-1$

When $x=6$

$\frac{6-3}{3}-\frac{6-2}{2}=-1$

$\frac{3}{3}-\frac{4}{2}=-1$

$\frac{6-12}{6}=-1$

$\frac{\cancel{-6}}{\cancel{6}}=-1$

$-1=-1$

Solution Set $=\{6\}$

(iii) $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Solution $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Taking L.C.M of brackets

$\frac{1}{2}\left(\frac{6x-1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1-6x}{2}\right)$

$\frac{6x-1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1-6x}{6}$

$\frac{6x-1+8}{12}=\frac{5+1-6x}{6}$

$\frac{\cancel{6}(6x+7)}{\cancel{12}^{2}}=6-6x$

$\frac{6x+7}{2}=6-6x$

$6x+7=2(6-6x)$

$6x+7=12-12x$

$6x+12x=12-7$

$18x=5$

$x=\frac{5}{18}$

To check

$$\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$$

When $x=\frac{5}{18}$

$$\frac{1}{2}\left[\frac{15}{18}-\frac{1}{6}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{1}{2}-3\left(\frac{5}{18}\right)\right]$$

$$\frac{1}{2}\left[\frac{5-3}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{1}{2}-\frac{5}{6}\right]$$

$$\frac{1}{2}\left[\frac{\cancel{2}}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{3-5}{6}\right]$$

$$\frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{-\cancel{2}^{1}}{\cancel{6}^{3}}\right]$$

$$\frac{1}{18}+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{-1}{3}\right]$$

$$\frac{1+12}{18}=\frac{5}{6}-\frac{1}{9}$$

$$\frac{13}{18}=\frac{15-2}{18}$$

$$\frac{13}{18}=\frac{13}{18}$$

Solution Set $=\left\{\frac{5}{18}\right\}$

(iv) $x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$

Solution $x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$

$$\frac{3x+1}{3}=2\left[\frac{3x-2}{3}\right]-6x$$

$$\frac{3x+1}{3}=\frac{6x-4}{3}-6x$$

Taking L.C.M of right side

$$\frac{3x+1}{3}=\frac{6x-4-18x}{3}$$

$$\frac{3x+1}{\cancel{3}}=\frac{(-12x-4)}{\cancel{3}}$$

$$3x+1=-12x-4$$

$$3x+12x=-4-1$$

$$15x=-5$$

$$x=\frac{-5}{15}$$

$$x=\frac{-1}{3}$$

To check

$$x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$$

When $x=\frac{-1}{3}$

$$\frac{-\cancel{1}}{\cancel{3}}+\frac{\cancel{1}}{\cancel{3}}=2\left[\frac{-1}{3}-\frac{2}{3}\right]-6\left(\frac{-1}{3}\right)$$

$$0=2\left[\frac{-1-2}{3}\right]+\frac{\cancel{6}^{2}}{\cancel{3}}$$

$$0=2\left[\frac{-\cancel{3}}{\cancel{3}}\right]+2$$

$$0=2(-1)+2$$

$$0=-\cancel{2}+\cancel{2}$$

$$0=0$$

Solution Set $=\left\{\frac{-1}{3}\right\}$

(v) $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Solution $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

$$\frac{5x-15-6x}{6}=\frac{9-x}{9}$$

$$\frac{-15-x}{6}=\frac{9-x}{9}$$

$$9(-15-x)=6(9-x)$$

$$-135-9x=54-6x$$

$$-135-54=-6x+9x$$

$$-189=3x$$

$$\frac{-189}{3}=x$$

$$x=-63$$

To check

$$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$$

When $x=-63$

$$\frac{5(-63-3)}{6}-(-63)=1-\frac{(-\cancel{63}^{7})}{\cancel{9}}$$

$$\frac{5(-\cancel{66}^{11})}{\cancel{6}}+63=1+7$$

$$-55+63=8$$
$$8=8$$

Solution Set $=\{-63\}$

(vi) $\dfrac{x}{3x-6}=2-\dfrac{2x}{x-2}, x\neq 2$

Solution $\dfrac{x}{3x-6}=2-\dfrac{2x}{x-2}, x\neq 2$

$$\frac{x}{3(x-2)}=\frac{2(x-2)-2x}{x-2}$$
$$\frac{x}{3(x-2)}=\frac{2x-4-2x}{x-2}$$
$$\frac{x}{3(x-2)}=\frac{-4}{x-2}$$
$$x(x-2)=-4\times 3(x-2)$$
$$x(x-2)=-12(x-2)$$
$$x(x-2)+12(x-2)=0$$
$$(x-2)(x+12)=0$$
$$x-2=0, \text{or } x+12=0$$
$$x=2, \text{or } x=-12$$

$x=2$ (Rejected because $x\neq 2$)

Hence $x=-12$

To check

$$\frac{x}{3x-6}=2-\frac{2x}{x-2}$$

When $x=-12$

$$\frac{-12}{3(-12)-6}=2-\frac{2(-12)}{-12-2}$$
$$\frac{-12}{-36-6}=2+\frac{24}{-14}$$
$$\frac{-12}{-42}=2-\frac{24}{14}$$
$$\frac{12}{42}=2-\frac{12}{7}$$
$$\frac{2}{7}=\frac{14-12}{7}$$
$$\frac{2}{7}=\frac{2}{7}$$

Solution Set $=\{-12\}$

(vii) $\dfrac{2x}{2x+5}=\dfrac{2}{3}-\dfrac{5}{4x+10}$

Solution $\dfrac{2x}{2x+5}=\dfrac{2}{3}-\dfrac{5}{4x+10}$

$$\frac{2x}{2x+5}=\frac{2(4x+10)-3\times 5}{3(4x+10)}$$
$$\frac{2x\times 3(4x+10)}{2x+5}=8x+20-15$$
$$\frac{6x\times 2\cancel{(2x+5)}}{\cancel{(2x+5)}}=8x+5$$
$$12x=8x+5$$
$$12x-8x=5$$
$$4x=5$$
$$x=\frac{5}{4}$$

To check

$$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$$

When $x=\dfrac{5}{4}$

$$\frac{2\left(\frac{5}{4}\right)}{2\left(\frac{5}{4}\right)+5}=\frac{2}{3}-\frac{5}{4\left(\frac{5}{4}\right)+10}$$
$$\frac{\frac{5}{2}}{\frac{5}{2}+5}=\frac{2}{3}-\frac{5}{5+10}$$
$$\frac{\frac{5}{2}}{\frac{5+10}{2}}=\frac{2}{3}-\frac{5}{15}$$
$$\frac{\frac{5}{2}}{\frac{15}{2}}=\frac{2}{3}-\frac{1}{3}$$
$$\frac{\cancel{5}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{15}^{3}}=\frac{2-1}{3}$$
$$\frac{1}{3}=\frac{1}{3}$$

Solution Set $=\left\{\dfrac{5}{4}\right\}$

(viii) $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1} \qquad x\neq 1$

Solution $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1} \qquad x\neq 1$

$$\frac{3\times 2x+1(x-1)}{3(x-1)}=\frac{5(x-1)+2\times 6}{6(x-1)}$$

$$\frac{6x+x-1}{3(x-1)}=\frac{5x-5+12}{6(x-1)}$$

$$\frac{7x-1}{3(x-1)}=\frac{5x-5+12}{6(x-1)}$$

$$7x-1=\frac{5\cancel{(x-1)}(5x+7)}{\cancel{(x-1)}}$$

$$2(7x-1)=5x+7$$

$$14x-2=5x+7$$

$$14x-5x=4+2$$

$$9x=9$$

$$x=\frac{9}{9}$$

$$x=1$$

No solution because $x \neq 1$.

(ix) $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1} \qquad x\neq \pm 1$

Solution $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1} \qquad x\neq \pm 1$

$$\frac{2}{(x-1)(x+1)}-\frac{1}{x+1}=\frac{1}{x+1}$$

$$\frac{2-(x-1)}{(x-1)(x+1)}=\frac{1}{x+1}$$

$$\frac{2-(x-1)}{(x-1)(x+1)}=\frac{1}{x+1}$$

$$2-x+1=\frac{(x-1)(x+1)}{(x+1)}$$

$$3-x=x-1$$

$$1+3=x+x$$

$$4=2x$$

$$\frac{4}{2}=x$$

$$x=2$$

To check

$$\frac{2}{2^2-1}-\frac{1}{2+1}=\frac{1}{2+1}$$

$$\frac{2}{4-1}-\frac{1}{3}=\frac{1}{3}$$

$$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

$$\frac{2-1}{3}=\frac{1}{3}$$

$$\frac{1}{3}=\frac{1}{3}$$

Solution Set $=\{2\}$

(x) $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$

Solution $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$

$$\frac{2}{3(x+2)}=\frac{1}{6}-\frac{1}{2(x+2)}$$

$$\frac{2}{3(x+2)}=\frac{x+2-3}{6(x+2)}$$

$$\frac{2\times 6(x+2)}{3(x+2)}=x-1$$

$$4=x-1$$

$$4+1=x$$

$$x=5$$

Check

$$\frac{2}{3(5)+6}=\frac{1}{6}-\frac{1}{2(5)+4}$$

$$\frac{2}{15+6}=\frac{1}{6}-\frac{1}{10+4}$$

$$\frac{2}{21}=\frac{1}{6}-\frac{1}{14}$$

$$\frac{2}{21}=\frac{7-3}{42}$$

$$\frac{2}{21}=\frac{\cancel{4}^{2}}{\cancel{42}_{21}}$$

$$\frac{2}{21}=\frac{2}{21}$$

Solution Set $=\{5\}$

Q.2 Check each equation and check for extraneous solution, if any

(i) $\sqrt{3x+4}=2$

Solution: $\sqrt{3x+4}=2$

Taking square on both side

$\left(\sqrt{3x+4}\right)^2=(2)^2$

$3x+4=4$

$3x=4-4$

$3x=0$

$x=\frac{0}{3}$

$x=0$

To check

$\sqrt{3x+4}=2$

When $x=0$

$\sqrt{3(0)+4}=2$

$\sqrt{4}=2$

$2=2$

L.H.S = R.H.S

Solution Set $=\{0\}$

(ii) $\sqrt[3]{2x-4}-2=0$

Solution: $\sqrt[3]{2x-4}-2=0$

$\sqrt[3]{2x-4}=2$

Taking cube on both sides

$\left(\sqrt[3]{2x-4}\right)^3=(2)^3$

$2x-4=8$

$2x=8+4$

$2x=12$

$x=\frac{12}{2}$

$x=6$

To check

$\sqrt[3]{2x-4}-2=0$

When $x=6$

$\sqrt[3]{2x-4}-2=0$

$\sqrt[3]{2(6)-4}-2=0$

$\sqrt{12-4}-2=0$

$\sqrt[3]{8}-2=0$

$\sqrt[3]{2^3}-2=0$

$2-2=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{6\}$

(iii) $\sqrt{x-3}-7=0$

Solution: $\sqrt{x-3}-7=0$

$\sqrt{x-3}=7$

Taking square on both side

$\left(\sqrt{x-3}\right)^2=(7)^2$

$x-3=49$

$x=49+3$

$x=52$

To check

$\sqrt{x-3}-7=0$

When $x=52$

$\sqrt{52-3}-7=0$

$\sqrt{49}-7=0$

$7-7=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{52\}$

(iv) $2\sqrt{t+4}=5$

Solution: $2\sqrt{t+4}=5$

Taking square on both side

$\left(2\sqrt{t+4}\right)^2=(5)^2$

$4(t+4)=25$

$t+4=\frac{25}{4}$

$t=\frac{25}{4}-4$

$t=\frac{25-16}{4}$

$t=\frac{9}{4}$

To check

$2\sqrt{t+4}=5$

When $t=\frac{9}{4}$

$2\sqrt{\frac{9}{4}+4}=5$

$2\sqrt{\frac{9+16}{4}}=5$

$2\sqrt{\frac{25}{4}}=5$

$2\times\frac{5}{2}=5$

$5 = 5$

L.H.S = R.H.S

Solution Set $= \left\{\frac{9}{4}\right\}$

(v) $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Solution: $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Taking cube on both sides

$\left(\sqrt[3]{2x+3}\right)^3 = \left(\sqrt[3]{x-2}\right)^3$

$2x+3 = x-2$

$2x - x = -2-3$

$x = -5$

To check

$\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

When $x = -5$

$\sqrt[3]{2(-5)+3} = \sqrt[3]{-5-2}$

$\sqrt[3]{-10+3} = \sqrt[3]{-7}$

$\sqrt[3]{-7} = \sqrt[3]{-7}$

L.H.S = R.H.S

Solution Set $= \{-5\}$

(vi) $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Solution: $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Taking cube on both sides

$\left(\sqrt[3]{2-t}\right)^3 = \left(\sqrt[3]{2t-28}\right)^3$

$2-t = 2t-28$

$2+28 = 2t+t$

$30 = 3t$

$\frac{30}{3} = t$

$t = 10$

To check

$\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

When $t = 10$

$\sqrt[3]{2-10} = \sqrt[3]{2(10)-28}$

$\sqrt[3]{-8} = \sqrt[3]{20-28}$

$\sqrt[3]{-8} = \sqrt[3]{-8}$

L.H.S = R.H.S

Solution Set $= \{10\}$

(vii) $\sqrt{2t+6} - \sqrt{2t-5} = 0$

Solution: $\sqrt{2t+6} - \sqrt{2t-5} = 0$

$\sqrt{2t+6} = \sqrt{2t-5}$

Taking square on both side

$\left(\sqrt{2t+6}\right)^2 = \left(\sqrt{2t-5}\right)^2$

$2t+6 = 2t-5$

$2t-2t = -5-6$

$0 = -11$

Solution is not possible

Solution Set $= \{\}$ or ϕ

(viii) $\sqrt{\frac{x+1}{2x+5}} = 2 \quad x \neq \frac{-5}{2}$

Solution: $\sqrt{\frac{x+1}{2x+5}} = 2 \quad x \neq \frac{-5}{2}$

Taking square on both side

$\left(\sqrt{\frac{x+1}{2x+5}}\right)^2 = (2)^2$

$\frac{x+1}{2x+5} = 4$

$x+1 = 4(2x+5)$

$x+1 = 8x+20$

$1-20 = 8x-x$

$-19 = 7x$

$-\frac{19}{7} = x$

Or, $x = \frac{-19}{7}$

To check

$\sqrt{\frac{x+1}{2x+5}} = 2$

When $x = \frac{-19}{7}$

$\sqrt{\left(\frac{-19}{7}+1\right) \div \left[2 \times \frac{-19}{7} + 5\right]} = 2$

$\sqrt{\frac{-19+7}{7} \div \left[\frac{-38}{7} + 5\right]} = 2$

$\sqrt{\frac{-12}{7} \div \left[\frac{-38+35}{7}\right]} = 2$

$\sqrt{\frac{-12}{7} \div \frac{-3}{7}} = 2$

$$\sqrt{\frac{-\cancel{12}^{4}}{\cancel{7}} \times \frac{\cancel{7}}{-\cancel{3}}} = 2$$

$$\sqrt{4} = 2$$

$$2 = 2$$

L.H.S = R.H.S

Solution Set $= \left\{ \frac{-19}{7} \right\}$

Exercise 7.2

Q1) Identify the following statements as true or

(i)	$\|x\| = 0$ has only one solution	True
(ii)	All absolute value equations have two solutions	False
(iii)	The equation $\|x\| = 2$ is equivalent to $x = 2$ *or* $x = -2$	True
(iv)	The equation $\|x-4\| = -4$ has no solution	True
(v)	The equation $\|2x-3\| = 5$ is equivalent to $2x-3=5$ *or* $2x+3=5$	False

Q2)

(i) $|3x-5|=4$

Solution $|3x-5|=4$

$3x-5 = \pm 4$

$3x-5=4$

$3x=4+5$

$3x=9$

$x=\frac{9}{3}$

$x=3$

To check

$x=3$

$|3(3)-5|=4$

$|9-5|=4$

$4=4$

$3x-5=-4$

$3x=-4+5$

$3x=1$

$x=\frac{1}{3}$

To check

$x=\frac{1}{3}$

$\left|3\times\frac{1}{3}-5\right|=4$

$|1-5|=4$

$|-4|=4$

$4=4$

Solution Set $=\left\{3,\frac{1}{3}\right\}$

(ii) $\frac{1}{2}|3x+2|-4=11$

Solution $\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}|3x+2|=11+4$

$\frac{1}{2}|3x+2|=15$

$|3x+2|=2\times 15$

$|3x+2|=30$

$3x+2=\pm 30$

$3x+2=30$

$3x=30-2$

$3x=28$

$x=\frac{28}{3}$

Check

$\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}\left|3\times\frac{28}{3}+2\right|-4=11$

$\frac{1}{2}|28+2|-4=11$

$\frac{1}{2}\times 30-4=11$

$15-4=11$

$11=11$

$3x+2=-30$

$3x=-30-2$

$3x=-32$

$x=\frac{-32}{3}$

$\frac{1}{2}\left|3\times-\frac{32}{3}+2\right|-4=11$

$\frac{1}{2}|-32+2|-4=11$

$\frac{1}{2}|-30|-4=11$

$\frac{1}{2}(30)-4=11$

$15-4=11$

$11=11$

Solution Set $=\left\{\frac{28}{3},\frac{-32}{3}\right\}$

(iii) $|2x+5|=11$

Solution $|2x+5|=11$

$2x+5=\pm 11$

$2x+5=11$

$2x=11-5$

$2x=6$

$x=\frac{6}{2}$

$x=3$

$2x+5=-11$

$2x=-11-5$

$2x=-16$

$x=\frac{-16}{2}$

$x=-8$

To check

$|2x+5|=11$ $\qquad$ $|2(-8)-8+5|=11$

$|2\times 3+5|=11$ $\qquad$ $|-16+5|=11$

$6+5 \;=11$ $\qquad$ $|-11|=11$

$11=11$ $\qquad$ $11=11$

Solution Set $=\{-8,3\}$

(iv) $|3+2x|=|6x-7|$

Solution $|3+2x|=|6x-7|$

$3+2x=\pm(6x-7)$

$3+2x=6x-7$ $\qquad$ $3+2x=-(6x-7)$

$3+7=6x-7$ $\qquad$ $3+2x=-6x+7$

$10=4x$ $\qquad$ $2x+6x=7-3$

$\frac{10}{4}=x$ $\qquad$ $\frac{4}{8}=x$

$x=\frac{5}{2}$ $\qquad$ $x=\frac{1}{2}$

To check

$|3+2x|=|6x-7|$ $\qquad$ $|3+2x|=|6x-7|$

$\left|3+2\left(\frac{5}{2}\right)\right|=\left|\cancel{6}^{3}\left(\frac{5}{\cancel{2}}\right)-7\right|$

$\left|3+2\times\frac{1}{2}\right|=\left|\cancel{6}^{3}\times\frac{1}{\cancel{2}}-7\right|$

$|3+5|=|15-7|$ $\qquad$ $|3+1|=|3-7|$

$|8|=|8|$ $\qquad$ $|4|=|-4|$

$8=8$ $\qquad$ $4=4$

Solution Set $=\left\{\frac{5}{2},\frac{1}{2}\right\}$

(v) $|x+2|-3=5-|x+2|$

Solution $|x+2|-3=5-|x+2|$

$|x+2|+|x+2|=5+3$

$2|x+2|=8$

$|x+2|=\frac{8}{2}$

$|x+2|=4$

$x+2=\pm 4$

$x+2=4$ $\qquad$ $x+2=-4$

$x=4-2$ $\qquad$ $x=-4-2$

$x=2$ $\qquad$ $x=-6$

To check

$|x+2|-3=5-|x+2|$ $\qquad$ $|x+2|-3=5-|x+2|$

$|2+2|-3=5-|2+2|$ $\qquad$ $|-6+2|-3=5-|-6+2|$

$14-3=5-|4|$ $\qquad$ $|-4|-3=5-|-4|$

$4-3=5-4$ $\qquad$ 4-3=5-4

$1=1$ $\qquad$ 1=1

Solution Set $=\{-6,2\}$

(vi) $\frac{1}{2}|x+3|+21=9$

Solution $\frac{1}{2}|x+3|+21=9$

$\frac{1}{2}|x+3|=9-21$

$\frac{1}{2}|x+3|=-12$

$|x+3|=-12\times 2$

$|x+3|=-24$

Value of absolute in never negative so solution is not possible

Solution Set $=\{\ \}$

(vii) $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

Solution $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{2}{3}+\frac{1}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{2+1}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{3}{3}$

$\left|\frac{3-5x}{4}\right|=1$

$\frac{3-5x}{4}=\pm 1$

$\frac{3-5x}{4}=1$ and $\frac{3-5x}{4}=-1$

$3-5x=4$ $\qquad$ $3-5x=-4$

$-5x=4-3$ $\qquad$ $-5x=-4-3$

$-5x=1$ $\qquad$ $-5x=-7$

$$x=\frac{1}{-5}$$
$$x=-\frac{1}{5}$$
$$\left|\frac{3-5\times\left(-\frac{1}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3+1}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3}$$

$$x=\frac{-7}{-5}$$
$$x=\frac{7}{5}$$
$$\left|\frac{3-5\times\left(+\frac{7}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3-7}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{-4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$|-1|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3}$$

Solution Set $=\left\{\frac{-1}{5},\frac{7}{5}\right\}$

(viii) $\left|\frac{x+5}{2-x}\right|=6$

Solution $\left|\frac{x+5}{2-x}\right|=6$

$$\frac{x+5}{2-x}=\pm 6$$
$$\frac{x+5}{2-x}=6$$
$$x+5=6(2-x)$$
$$x+5=12-6x$$
$$x+6x=12-5$$
$$7x=7$$
$$x=\frac{7}{7}$$
$$x=1$$

$$\frac{x+5}{2-x}=-6$$
$$x+5=-6(2-x)$$
$$x+5=-12+6x$$
$$5+12=6x-x$$
$$17=5x$$
$$\frac{17}{5}=x$$
$$x=\frac{17}{5}$$

To check

$$\left|\frac{x+5}{2-x}\right|=6$$
$$\left|\frac{1+5}{2-1}\right|=6$$
$$\left|\frac{6}{1}\right|=6$$
$$6=6$$
$$\left|\left(\frac{17}{5}+5\right)\div\left(2-\frac{17}{5}\right)\right|=6$$
$$\left|\frac{17+25}{5}\div\frac{10-17}{5}\right|=6$$
$$\left|\frac{42}{5}\div\frac{-7}{5}\right|=6$$
$$|-6|=6$$
$$6=6$$

Solution Set $=\left\{1,\frac{17}{5}\right\}$

Exercise 7.3

Q1) Solve the following inequalities.

(i) $3x+1<5x-4$

Solution: $3x+1<5x-4$

$3x<5x-4-1$

$3x-5x<-5$

$-2x<-5$

Case-I When negative is eliminated from both sides of inequality the symbol will be change.

Case-II When negative is transferred from variable to constant side, symbol will also change.

$x>\frac{-5}{-2}$

$x>\frac{5}{2}$

Solution Set $=\left\{x \mid x>\frac{5}{2}\right\}$

(ii) $4x-10.3\le 21x-1.8$

Solution: $4x-10.3\le 21x-1.8$

$4x-21x\le -8.5+10.3$

$-17x\le 8.5$

When negative value is shifted to other side its symbol changes.

$x\ge \frac{8.5}{-17}$

$x\ge -\frac{8.5}{17}$

$x\ge -0.5$

Solution Set $=\{x \mid x\ge -0.5\}$

(iii) $4-\frac{1}{2}x\ge -7+\frac{1}{4}x$

Solution: $4-\frac{1}{2}x\ge -7+\frac{1}{4}x$

$-\frac{1}{2}x-\frac{1}{4}\ge -7-4$

$\frac{-2x-x}{4}\ge -11$

$-3x\ge -44$

When negative value is shifted the symbol changes

$x\le \frac{-44}{-3}$

$x\le \frac{44}{3}$

Solution Set $=\{x \mid x\le \frac{44}{3}\}$

(iv) $x-2(5-2x)\ge 6x-3\frac{1}{2}$

Solution: $x-2(5-2x)\ge 6x-3\frac{1}{2}$

$x-10+4x\ge 6x-\frac{7}{2}$

$5x-6x\ge -\frac{7}{2}+10$

$-1x\ge \frac{-7+20}{2}$

$-x\ge -\frac{13}{2}$

When negative is shifted other side symbol changes

$x\le \frac{13}{-1\times 2}$

$x\le -\frac{13}{2}$

$x\le -6.5$

Solution Set $=\{x \mid x\le -6.5\}$

(v) $\frac{3x+2}{9}-\frac{2x+1}{3}>-1$

Solution: $\frac{3x+2}{9}-\frac{2x+1}{3}>-1$

$\frac{3x+2-3(2x+1)}{9}>-1$

$3x+2-6x-3>-9$

$-3x>-9+1$

$3x > -8$

Negative value is shifted to other side its symbols changes

$$x < \frac{-8}{-3}$$

$$x < \frac{8}{3}$$

Solution Set $= \left\{ x \mid x < \frac{8}{3} \right\}$

(vi) $3(2x+1) - 2(2x+5) < 5(3x-2)$

Solution: $3(2x+1) - 2(2x+5) < 5(3x-2)$

$$6x + 3 - 4x - 10 < 15x - 10$$

$$2x - 7 - 15x < -10$$

$$-13x < -10 + 7$$

$$-13x < -3$$

The value is negative when shifted to other side it changes its symbols

$$x > \frac{-3}{-13}$$

$$x > \frac{3}{13}$$

Solution Set $= \left\{ x \mid x > \frac{3}{13} \right\}$

(vii) $3(x-1) - (x-2) > -2(x+4)$

Solution: $3(x-1) - (x-2) > -2(x+4)$

$$3x - 3 - x + 2 > -2x - 8$$

$$2x - 1 > -2x - 8$$

$$2x + 2x > -8 + 1$$

$$4x > -7$$

$$x > \frac{-7}{4}$$

Solution Set $= \left\{ x \mid x > \frac{-7}{4} \right\}$

(viii) $2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$

Solution: $2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$

$$\frac{8}{3}x + \frac{10x-8}{3} > -\frac{(8x+7)}{3}$$

$$\frac{8x+10x-8}{3} > -\frac{8x+7}{3}$$

Multiplying both side by 3

$$\cancel{3} \times \frac{18x-8}{\cancel{3}} > -\cancel{3} \times \frac{8x+7}{\cancel{3}}$$

$$18x - 8 > -(8x+7)$$

$$18x - 8 > -8x - 7$$

$$18x + 8x > -7 + 8$$

$$26x > 1$$

$$x > \frac{1}{26}$$

Solution Set $= \left\{ x \mid x > \frac{1}{26} \right\}$

Q2) Solve the following inequalities

(i) $-4 < 3x + 5 < 8$

Solution: $-4 < 3x + 5 < 8$

$-4 < 3x + 5$	and	$3x + 5 < 8$
$-4 - 5 < 3x$		$3x < 8 - 5$
$-9 < 3x$		$3x < 3$
$\frac{-9}{3} < x$		$x < \frac{3}{3}$
$-3 < x$		$x < 1$

$-3<x<1$

Solution Set $= \{ x \mid -3 < x < 1 \}$

(ii) $-5 \leq \frac{4-3x}{2} < 1$

Solution: $-5 \leq \frac{4-3x}{2} < 1$

$-5 \leq \frac{4-3x}{2}$	and	$\frac{4-3x}{2} < 1$
$-10 \leq 4 - 3x$		$4 - 3x < 2$
$3x - 10 \leq 4$		$-3x < 2 - 4$
$3x \leq 4 + 10$		$-3x < -2$
$3x \leq 14$		$x > \frac{-2}{-3}$
$x \leq \frac{14}{3}$		$x > \frac{2}{3}$

$$\frac{2}{3} < x$$

$\frac{2}{3} < x \leq \frac{14}{3}$

Solution Set $= \{x \mid \frac{2}{3} < x \leq \frac{14}{3}\}$

(iii) $-6 < \frac{x-2}{4} < 6$

Solution: $-6 < \frac{x-2}{4} < 6$

$-6 < \frac{x-2}{4}$

$-24 < x - 2$

$-24 + 2 < x$

$-22 < x$

and

$\frac{x-2}{4} < 6$

$x - 2 < 24$

$x < 24 + 2$

$x < 26$

$-22 < x < 26$

Solution Set $= \{x \mid -22 < x < 26\}$

(iv) $3 \geq \frac{7-x}{2} \geq 1$

Solution: $3 \geq \frac{7-x}{2} \geq 1$

$3 \geq \frac{7-x}{2}$

$6 \geq 7 - x$

$6 - 7 \geq -x$

$-1 \geq -x$

Negative sign change the symbols

$1 \leq x$

and

$\frac{7-x}{2} \geq 1$

$7 - x \geq 2$

$-x \geq 2 - 7$

$-x \geq -5$

$x \leq 5$

$1 \leq x \leq 5$

Solution Set $= \{x \mid 1 \leq x \leq 5\}$

(v) $3x - 10 \leq 5 < x + 3$

Solution $3x - 10 \leq 5 < x + 3$

$3x - 10 \leq 5$ and $5 < x + 3$

$3x \leq 5 + 10$ $\quad$ $5 - 3 < x$

$3x \leq 15$ $\quad$ $2 < x$

$\frac{3x}{3} \leq \frac{15}{3}$

$x \leq 5$

$2 < x \leq 5$

Solution Set $= \{x \mid 2 < x \leq 5\}$

(vi) $-3 \leq \frac{x-4}{-5} < 4$

Solution $-3 \leq \frac{x-4}{-5} < 4$

$-3 \leq \frac{x-4}{-5}$ and $\frac{x-4}{-5} < 4$

$-3 \times -5 \geq x - 4$

$x - 4 > 4(-5)$

$15 \geq x - 4$ $\quad$ $x > -20 + 4$

$15 + 4 \geq x$ $\quad$ $x > -16$

$19 \geq x$ $\quad$ $-16 < x$

$x \leq 19$

$-16 < x \leq 19$

Solution Set $= \{x \mid -16 < x \leq 19\}$

(vii) $1 - 2x < 5 - x \leq 25 - 6x$

Solution: $1 - 2x < 5 - x \leq 25 - 6x$

$1 - 2x < 5 - x$ and

$5 - x \leq 25 - 6x$

$-x + 6x \leq 25 - 5$ $\quad$ $6x - x \leq 20$

$1 - 2x + x < 5$ $\quad$ $5x \leq 20$

$-x < 5 - 1$ $\quad$ $x \leq \frac{20}{5}$

$-x < 4$ $\quad$ $x \leq 4$

Due negative sign

Symbol change

$-4 < x$

$-4 < x \leq 4$

Solution Set $= \{x \mid -4 < x \leq 4\}$

(viii) $3x - 2 < 2x + 1 < 4x + 17$

Solution: $3x - 2 < 2x + 1 < 4x + 17$

$3x - 2 < 2x + 1$ $\quad$ $2x + 1 < 4x + 17$

$3x - 2x - 2 < +1$ $\quad$ $2x - 4x < 17 - 1$

$x < 1 + 2$ $\quad$ $-2x < 16$

$x < 3$

$$x > \frac{16}{-2}$$
$$x > -8$$
$$-8 < x$$

$-8 < x < 3$

Solution Set $= \{x \mid -8 < x < 3\}$

Review Exercise 7

Q.1 Choose the correct answer

(i) Which of the following is the solution of the inequality $3-4x \leq 11$?

(a) -8 **(b)** -2

(c) $-\frac{14}{4}$ **(d)** None of these

(ii) A statement involving any of the symbols $<$, $>$, $\leq$or $>$, is called------------

(a) Equation **(b)** Identity

(c) Inequality **(d)** Linear equation

(iii) x = ------- is a solution of the inequality $-z < x > \frac{3}{2}$

(a) -5 **(b)** 3

(c) 0 **(d)** $\frac{3}{2}$

(iv) If x is no larger than 10, then ----------

(a) $x \leq 8$ **(b)** $x \geq 10$

(c) $x < 10$ **(d)** $x > 10$

(v) If the capacity $<$ of an elevator is at most 1600 pounds then ----------

(a)$c < 1600$ **(b)**$c \geq 1600$

(c) $c \leq 1600$ **(d)** $c > 1600$

(vi) x = 0 is a solution of the inequality -----------

(a) $x > 0$ **(b)** $3x+5 < 0$

(c) $x+\frac{z}{2} < 0$ **(d)** $x-2 < 0$

ANSWER KEY

i	ii	iii	iv	v	vi
b	c	c	b	c	d

Q.2 Identify the following statement as true or false

(i) The equation $3x-5=7-x$ is a linear equation. (True)

(ii) The equation $x-0.3x=0.7x$ is an identity (True)

(iii) The equation $-2x+3=8$ is equivalent to $-2x=11$ (False)

(iv) To eliminate fractions we multiply each side of an equation by the L.C.M of denominators (True)

(v) $4(x+3)=x+3$ is a conditional equations (True)

(vi) The equation $2(3x+5)=6x+12$ is an in consistent equation (True)

(vii) To solve $\frac{2}{3}x=12$, we should multiply each side by $\frac{2}{3}$ (False)

(viii) Equations having exactly the same solution are called equivalent equations. (True)

(ix) A solution that does not satisfy the original equation is called extra solution (True)

Q.3 Answer the following short question.

(i) Define a linear inequality in one variable

Ans A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form $ax+b<0, a\neq 0$

(ii) State the trichotomy and transitive properties of in equalities

Ans **Trichotomy Property**

For any $a,b\in R$ one and only one of the following statements in true. $a<b$ or $a=b$, or $a>b$

Transitive Property

Let $a,b,c\in R$.

(a) If $a>b$ and $b>c$, then $a>c$

(b) If $a<b$ and $b<c$, then $a<c$

(iii) The formula relating degree Fahrenheit to degree Celsius is F= $\frac{9}{5}c+32$ for what value of c is F< O was

Ans $F=\frac{9}{5}c+32$

$\frac{9}{5}c+32=F$

Since $F<0$

So $\frac{9}{5}c+32<0$

$\frac{9c+160}{5}<0$

Or $9c+160<0\times 5$

Or $9c+160<0$

Or $9c<-160$

Or $c<-\frac{160}{9}$

(iv) Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relation ship

Solution: Let the integer $=y$

Sum of integer and $12=y+12$

Seven times sum of integer and $12=7(y+12)$

According to condition

$50\le 7(y+12)\le 60$

$\frac{50}{7}\le 7\frac{(y+12)}{7}\le\frac{60}{7}$

$\frac{50}{7}\le y+12\le\frac{60}{7}$

$\frac{50}{7}-12\le y+\cancel{12}-\cancel{12}\le\frac{60}{7}-12$

$$\frac{50-84}{7} \le y \le \frac{60-84}{7}$$

$$\frac{-34}{7} \le y \le \frac{-24}{7}$$

Solution Set $= \left\{ y \mid \frac{-34}{7} \le y - \frac{24}{7} \right\}$

Q.4 Solve each of the following and check for extraneous solution if any

(i) $\sqrt{2t+4} = \sqrt{t-1}$

Solution: $\sqrt{2t+4} = \sqrt{t-1}$

Taking square on both side

$$\left(\sqrt{2t+4}\right)^2 = \left(\sqrt{t-1}\right)^2$$

$$2t+4 = t-1$$

$$2t-t = -1-4$$

$$t = -5$$

To check

$$\sqrt{2t+4} = \sqrt{t-1}$$

When $t = -5$

$$\sqrt{2(-5)+4} = \sqrt{t-5-1}$$

$$\sqrt{-10+4} = \sqrt{-6}$$

$$\sqrt{-6} = \sqrt{-6}$$

L.H.S = R.H.S

Solution Set $= \{-5\}$

(ii) $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

Solution: $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

$$\sqrt{3x-1} = 2\sqrt{8-2x}$$

Taking square on both side

$$\left(\sqrt{3x-1}\right)^2 = \left(2\sqrt{8-2x}\right)^2$$

$$3x-1 = 4(8-2x)$$

$$3x-1 = 32-8x$$

$$3x+8x = 32+1$$

$$11x = 33$$

$$x = \frac{33}{11}$$

$$x = 3$$

To check

$$\sqrt{3x-1} - 2\sqrt{8-2x} = 0$$

When $x=3$

$\sqrt{3(3)-1}-2\sqrt{8-2(3)}=0$

$\sqrt{9-1}-2\sqrt{8-6}=0$

$\sqrt{8}-2\sqrt{2}=0$

$2\sqrt{2}-2\sqrt{2}=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{3\}$

Q.5 Solve for x

(i) $|3x+14|-2=5x$

Solution: $|3x+14|-2=5x$

$|3x+14|=5x+2$

$3x+14=\pm(5x+2)$

$3x+14=5x+2$

$14-2=5x-3x$

$12=2x$

$\frac{12}{2}=x$

$x=6$

To check

$|3x+14|-2=5x$

When $x=6$

$|3(6)+14|-2=5(6)$

$|18+14|-2=30$

$32-2=30$

$30=30$

Solution Set $=\{6\}$

$3x+14=-(5x+2)$

$3x+14=-5x-2$

$3x+5x=-2-14$

$8x=\frac{-16}{8}$

$x=-2$

$|3x+14|-2=5x$

when $x=-2$

$|3(-2)+14|-2=5(-2)$

$|-6+14|-2=-10$

$8-2=-10$

$6=-10$

(ii) $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

Solution $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

$\frac{2}{3}|x-3|=|x+2|$

$\frac{2}{3}=\frac{|x+2|}{|x-3|}$

$\frac{x+2}{x-3}=\pm\frac{2}{3}$

$$\frac{x+2}{x-3}=\frac{2}{3}$$

$$3(x+2)=2(x-3)$$

$$3x+6=2x-6$$

$$3x-2x=-6-6$$

$$x=-12$$

To check

$$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$$

When $x=-12$

$$\frac{1}{3}|-12-3|=\frac{1}{2}|-12+2|$$

$$\frac{1}{3}|-15|=\frac{1}{2}|-10|$$

$$\frac{1}{\cancel{3}}\left(\cancel{15}^{5}\right)=\frac{1}{\cancel{2}}\left(\cancel{10}^{5}\right)$$

$$5=5$$

and

$$\frac{x+2}{x-3}=-\frac{2}{3}$$

$$3(x+2)=-2(x-3)$$

$$3x+6=-2x+6$$

$$3x+2x=+6-6$$

$$5x=0$$

$$x=\frac{0}{5}\quad\Rightarrow x=0$$

$$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$$

when $x=0$

$$\frac{1}{3}|0-3|=\frac{1}{2}|0+2|$$

$$\frac{1}{3}|-3|=\frac{1}{2}|2|$$

$$\frac{1}{\cancel{3}}\left(\cancel{3}^{1}\right)=\frac{1}{\cancel{2}}\left(\cancel{2}^{1}\right)$$

$$\frac{1}{3}(3)=1$$

$$1=1$$

Solution Set $=\{-12,0\}$

Q.6 Solve the following inequality

(i) $-\frac{1}{3}x+5\le 1$

Solution $-\frac{1}{3}x+5\le 1$

$$-\frac{1}{3}x\le 1-5$$

$$-\frac{1}{3}x\le -4$$

$$x\ge -4\times(-3)$$

$$x\ge 12$$

Solution Set $=\{x\mid x\ge 12\}$

(ii) $-3<\frac{1-2x}{5}<1$

Solution $-3<\frac{1-2x}{5}<1$

$$-3<\frac{1-2x}{5} \qquad\qquad \frac{1-2x}{5}<1$$

$$-15 < 1 - 2x \qquad 1 - 2x < 5$$
$$-15 - 1 < -2x \qquad -2x < 5 - 1$$
$$-16 < -2x \qquad -2x < 4$$
$$\frac{-16}{-2} > x \qquad x > \frac{4}{-2}$$
$$8 > x \qquad x > -2$$
$$x < 8 \qquad -2 < x$$
$$-2 < x < 8$$

Solution Set $= \{x \mid -2 < x < 8\}$

Unit 7: Linear Equations and Inequalities

Overview

Linear Equation:

A linear equation in one unknown variable x is an equation of the form $ax + b = 0$, where $a, b \in R$ and $a \neq 0$.

Example:

(i) $5x - 3 = 0$

(ii) $\frac{1}{2}x + 18 = 0$

Radical equations:

When the variable in an equation occurs under a radical the equation is called a radical equation.

Example:

(i) $\sqrt{2x-3} - 7 = 0$

Absolute value:

The absolute value of a real number 'a' denoted by lal, is defined as

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

$|6| = 6,$

e.g., $|0| = 0$

$|-6| = -(-6) = 6$

Extraneous Roots:

If the solutions (roots) obtained from the equation does not satisfy the original equations are called extraneous roots.

Linear inequality:

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form. $ax + b < 0,\ a \neq 0\ a, b \in R$ we may replace the symbol <by>, $\leq$ or $\geq$ also.

Inconsistent equation:

An inconsistent equation is that whose solution set is ϕ.