Exercise 9.1

Q.1 Find the distance between the following pairs of points

Solution:

(a) $A(9,2), B(7,2)$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|7-9|^2 + |2-2|^2}$

$|AB| = \sqrt{(-2)^2 + (0)^2}$

$|AB| = \sqrt{4}$

$|AB| = 2$

(b) $A(2,-6), B(3,-6)$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|3-2|^2 + |-6-(-6)|^2}$

$|AB| = \sqrt{(1)^2 + (-6+6)^2}$

$|AB| = \sqrt{1 + (0)^2}$

$|AB| = \sqrt{1}$

$AB = 1$

(c) $A(-8,1), B(6,1)$

Distance $= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|A\,B| = \sqrt{|6-(-8)|^2 + |1-1|^2}$

$|A\,B| = \sqrt{(6+8)^2 + (0)^2}$

$|A\,B| = \sqrt{(14)^2}$

$|A\,B| = 14$

(d) $A(-4,\sqrt{2}), B(-4,-3)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|A\,B| = \sqrt{|-4-(-4)|^2 + |-3-\sqrt{2}|^2}$

$|A\,B| = \sqrt{(-4+4)^2 + \left(-(3+\sqrt{2})\right)^2}$

$|A\,B| = \sqrt{(0)^2 + (3+\sqrt{2})^2}$

$|A\,B| = \sqrt{(3+\sqrt{2})^2}$

$|A\,B| = 3+\sqrt{2}$

(e) $A(3,-11), B(3,-4)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|A\,B| = \sqrt{|3-3|^2 + |-4-(-11)|^2}$

$|A\,B| = \sqrt{(0)^2 + (-4+11)^2}$

$|A\,B| = \sqrt{(7)^2}$

$|A\,B| = 7$

(f) $A(0,0), B(0,-5)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|A\,B| = \sqrt{|0-0|^2 + |-5-0|^2}$

$|A\,B| = \sqrt{(-5)^2}$

$|A\,B| = \sqrt{25}$

$|A\,B| = 5$

Q.2 Let P be the print on x-axis with x-coordinate a and Q be the point on y-axis with y coordinate b as given below. Find the distance between P and Q

Solution:

(i) $a = 9, b = 7$

P is (9, 0) and Q (0, 7)

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|P\,Q| = \sqrt{|0-9|^2 + |7-0|^2}$

$$|PQ| = \sqrt{(-9)^2 + (7)^2}$$
$$|PQ| = \sqrt{81+49}$$
$$|PQ| = \sqrt{130}$$

(ii) $a = 2, b = 3$

$$P(2,0), Q(0,3)$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$|PQ| = \sqrt{|0-2|^2 + |3-0|^2}$$
$$|PQ| = \sqrt{(-2)^2 + (3)^2}$$
$$|PQ| = \sqrt{4+9}$$
$$|PQ| = \sqrt{13}$$

(iii) $a = -8, b = 6$

$$P(-8,0), Q(0,6)$$
$$|d| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$|PQ| = \sqrt{|0-(-8)|^2 + |6-0|^2}$$
$$|PQ| = \sqrt{(8)^2 + (6)^2}$$
$$|PQ| = \sqrt{64+36}$$
$$|PQ| = \sqrt{100}$$
$$|PQ| = 10$$

(iv) $a = -2, b = -3$

$$P(-2\ ,0)\ ,Q(0,-3)$$
$$|d| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$d = \sqrt{|0-(-2)|^2 + |-3-0|^2}$$
$$d = \sqrt{(2)^2 + (-3)^2}$$
$$d = \sqrt{4+9}$$
$$d = \sqrt{13}$$

(v) $a = \sqrt{2}, b = 1$

$$P(\sqrt{2},0), Q(0,1)$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$d = \sqrt{|0-\sqrt{2}|^2 + |1-0|^2}$$
$$d = \sqrt{(-\sqrt{2})^2 + (1)^2}$$
$$d = \sqrt{2+1}$$
$$d = \sqrt{3}$$

(vi) $a = -9, b = -4$

$$P(-9,0), Q(0,-4)$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$|PQ| = \sqrt{|0-(-9)|^2 + |-4-0|^2}$$
$$|PQ| = \sqrt{(9)^2 + (-4)^2}$$
$$|PQ| = \sqrt{81+16}$$
$$|PQ| = \sqrt{97}$$

Exercise 9.2

Q.1 **Show whether the points with vertices (5,-2),(5,4) and (-4,1) are the vertices of equilateral triangle or an isosceles triangle**

$P(5,-2), Q(5,4), R(-4,1)$

Solution:

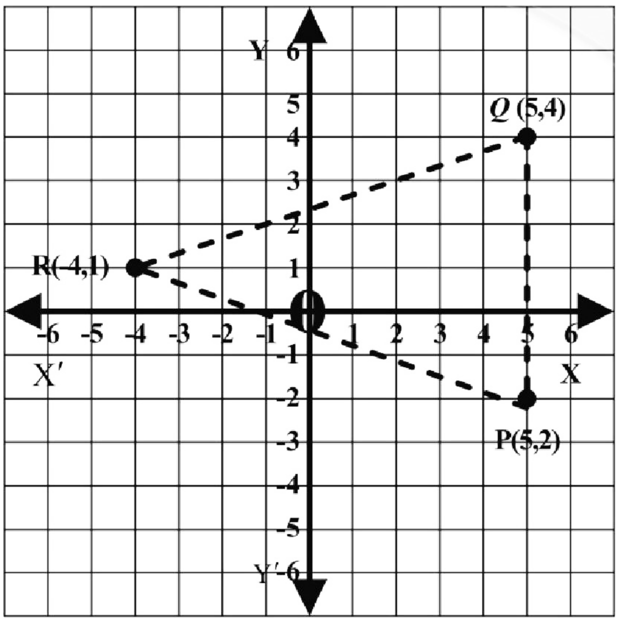
We know that the distance formula is

$$= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

We have $P(5,-2), Q(5,4)$

$$|PQ| = \sqrt{|5-5|^2 + |4-(-2)|^2}$$

$$|PQ| = \sqrt{(0)^2 + (4+2)^2}$$



$$|PQ| = \sqrt{(6)^2}$$

$$|PQ| = 6$$

$Q(5,4), R(-4,1)$

$$|QR| = \sqrt{|-4-5|^2 + |1-4|^2}$$

$$|QR| = \sqrt{(-9)^2 + (-3)^2}$$

$$|QR| = \sqrt{81+9}$$

$$|QR| = \sqrt{90}$$

$$|QR| = \sqrt{9\times 10} = 3\sqrt{10}$$

$R(-4,1), P(5,-2)$

$$|RP| = \sqrt{|5-(-4)|^2 + |-2-1|^2}$$

$$|RP| = \sqrt{(5+4)^2 + (-3)^2}$$

$$|RP| = \sqrt{(9)^2 + 9}$$

$$|RP| = \sqrt{81+9}$$

$$|RP| = \sqrt{90}$$

$$|RP| = \sqrt{9\times 10} = 3\sqrt{10}$$

$$|QR| = |PR|$$

Two lengths of triangle are equal

So it is a isosceles triangle

Q.2 **Show whether or not the points with vertices $(-1,1),(2,-2)$ and $(-4,1)$ form a Square**

Solution:

$P(-1,1)\,Q(5,4)\,R(2,-2)\,S(-4,1)$

$$\text{Distance} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|PQ| = \sqrt{\left|5-(-1)^2\right| + |4-1|^2}$$

$$|PQ| = \sqrt{|5+1|^2 + |3|^2}$$

$$|PQ| = \sqrt{6^2 + 9}$$

$$|PQ| = \sqrt{36+9}$$

$$|PQ| = \sqrt{45}$$

$$|PQ| = \sqrt{9\times 5}$$

$$|PQ| = 3\sqrt{5}$$

$$|QR| = \sqrt{|2-5|^2 + |-2-4|^2}$$

$|QR| = \sqrt{(-3)^2 + (6)^2}$

$|QR| = \sqrt{9+36}$

$|QR| = \sqrt{45}$

$|QR| = \sqrt{9\times 5}$

$|QR| = 3\sqrt{5}$

$|RS| = \sqrt{|-4-2|^2 + |1-(-2)|^2}$

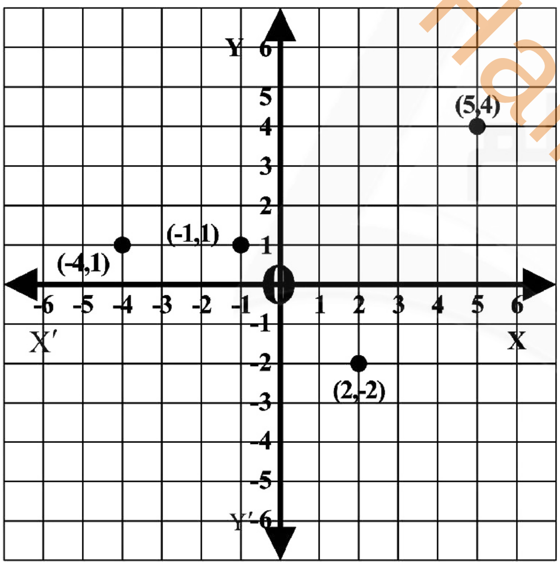
$|RS| = \sqrt{(-6)^2 + (1+2)^2} = \sqrt{36+(3)^2}$

$|RS| = \sqrt{36+9}$

$|RS| = \sqrt{45}$

$|RS| = \sqrt{9\times 5}$

$|RS| = 3\sqrt{5}$



$|SP| = \sqrt{|-4-(-1)|^2 + |1-1|^2}$

$|SP| = \sqrt{(-4+1)^2 + (0)^2}$

$|SP| = \sqrt{(-3)^2}$

$|SP| = \sqrt{9}$

$|SP| = 3$

If all the length are same then it will be a Square all the length are not equal so it is not square.

$|PQ| = |QR| = |RS| \neq |SP|$

Q.3 Show whether or not the points with coordinates $(1,3),(4,2)$ and $(-2,6)$ are vertices of a right triangle?

Solution:

$A(1,3), B(4,2), C(-2,6)$

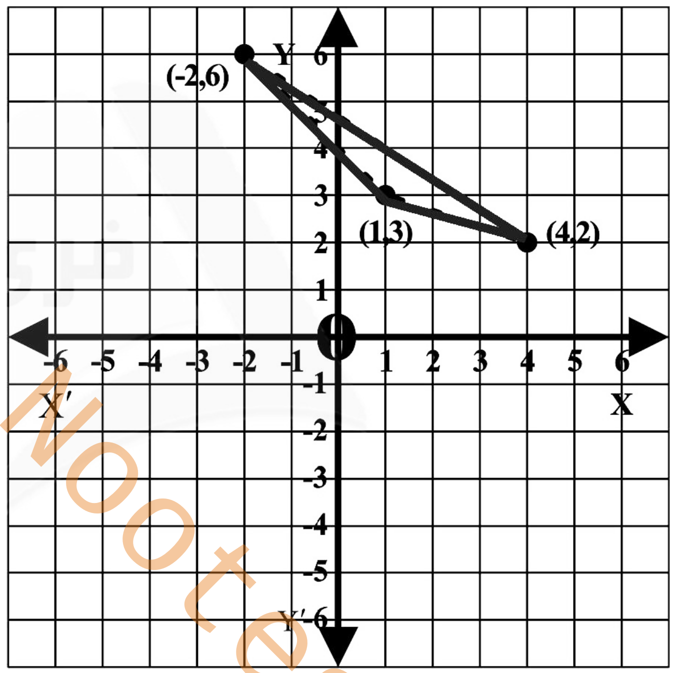
$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|4-1|^2 + |2-3|^2}$

$|AB| = \sqrt{(3)^2 + (-1)^2}$

$|AB| = \sqrt{9+1}$

$|AB| = \sqrt{10}$



$|BC| = \sqrt{|-2-4|^2 + |6-2|^2}$

$|BC| = \sqrt{(-6)^2 + (4)^2}$

$|BC| = \sqrt{36+16}$

$|BC| = \sqrt{52}$

$|CA| = \sqrt{|-2-1|^2 + |6-3|^2} = \sqrt{(-3)^2 + (3)^2}$

$|CA| = \sqrt{9+9}$

$|CA| = \sqrt{18}$

By Pythagoras theorem

$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$

$(\sqrt{52})^2 = (\sqrt{18})^2 + (\sqrt{10})^2$

$52 = 18 + 10$

$52 = 28$

Since $52 \neq 28$

So it not right angle triangle.

Q.4 Use distance formula to prove whether or not the points $(1,1), (-2,-8)$ and $(4,10)$ lie on a straight line?

Solution:

$A(1,1), B(-2,-8), C(4,10)$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|-2-1|^2 + |-8-1|^2}$$

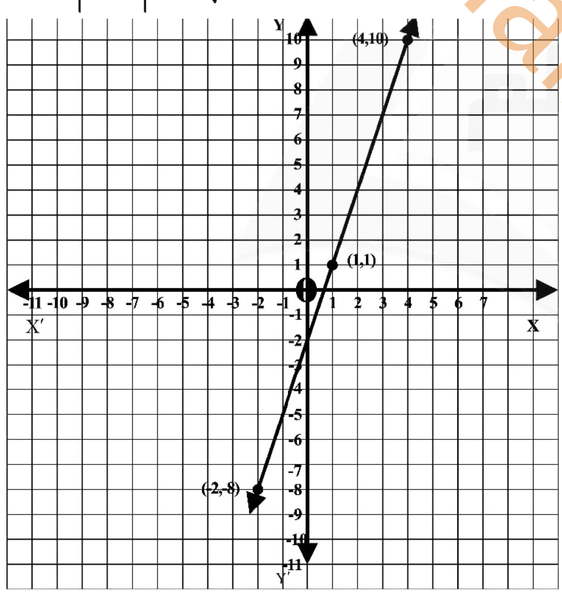
$$|AB| = \sqrt{(-3)^2 + (-9)^2}$$

$$|AB| = \sqrt{9+81}$$

$$|AB| = \sqrt{90}$$

$$|AB| = \sqrt{9 \times 10}$$

$$|AB| = 3\sqrt{10}$$



$$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$$

$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$

$$|BC| = \sqrt{(6)^2 + (18)^2}$$

$$|BC| = \sqrt{36+324}$$

$$|BC| = \sqrt{360}$$

$$|BC| = \sqrt{36 \times 10}$$

$$|BC| = 6\sqrt{10}$$

$$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$$

$$|AC| = \sqrt{(3)^2 + (9)^2}$$

$$|AC| = \sqrt{9+81}$$

$$|AC| = \sqrt{90}$$

$$|AC| = \sqrt{9 \times 10}$$

$$|AC| = 3\sqrt{10}$$

$$|AC| + |AB| = |BC|$$

$$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$$

$$6\sqrt{10} = 6\sqrt{10}$$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9,1)

Solution: $M(2,K), A(3,7)$ and $B(9,1)$

(3,7) A ———— (2,K) M ———— (9,1) B

$$|\overline{AM}| = |\overline{BM}|$$

$$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$$

$$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$$

Taking square on both Side

$$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$$

$$K^2 - 14K + 50 = 50 + K^2 - 2K$$

$$\cancel{K^2} - 14K + \cancel{50} - \cancel{50} - \cancel{K^2} + 2K = 0$$

$$-12K = 0$$

$$K = \frac{0}{-12}$$

$$K = 0$$

Q.6 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear.

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$$

$$|AB| = \sqrt{(3)^2 + (-12)^2}$$

$$|AB| = \sqrt{9+144}$$

$$|AB| = \sqrt{153}$$

$$|AB| = \sqrt{9 \times 17}$$

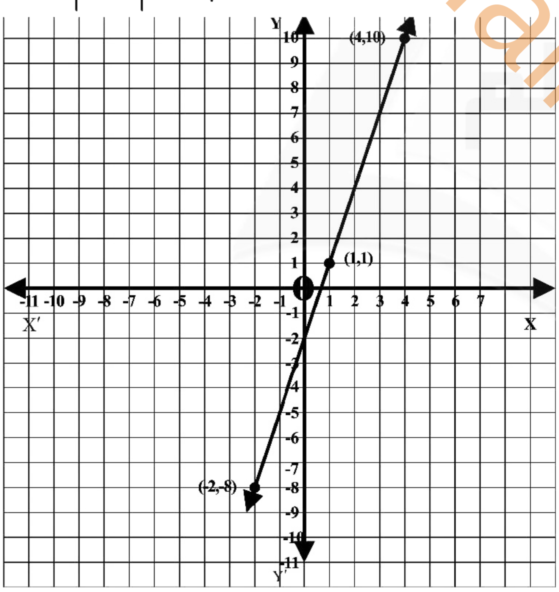
$$52 = 18 + 10$$
$$52 = 28$$
Since $52 \neq 28$

So it not right angle triangle.

Q.4 Use distance formula to prove whether or not the points $(1,1),(-2,-8)$ and $(4,10)$ lie on a straight line?

Solution:

$$A(1,1), B(-2,-8), C(4,10)$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$|AB| = \sqrt{|-2-1|^2 + |-8-1|^2}$$
$$|AB| = \sqrt{(-3)^2 + (-9)^2}$$
$$|AB| = \sqrt{9+81}$$
$$|AB| = \sqrt{90}$$
$$|AB| = \sqrt{9 \times 10}$$
$$|AB| = 3\sqrt{10}$$

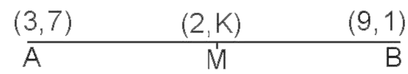


$$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$$
$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$
$$|BC| = \sqrt{(6)^2 + (18)^2}$$
$$|BC| = \sqrt{36+324}$$
$$|BC| = \sqrt{360}$$
$$|BC| = \sqrt{36 \times 10}$$
$$|BC| = 6\sqrt{10}$$
$$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$$
$$|AC| = \sqrt{(3)^2 + (9)^2}$$
$$|AC| = \sqrt{9+81}$$
$$|AC| = \sqrt{90}$$
$$|AC| = \sqrt{9 \times 10}$$
$$|AC| = 3\sqrt{10}$$
$$|AC| + |AB| = |BC|$$
$$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$$
$$6\sqrt{10} = 6\sqrt{10}$$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9,1)

Solution: $M(2,K), A(3,7)$ and $B(9,1)$



$$|\overline{AM}| = |\overline{BM}|$$
$$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$$
$$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$$

Taking square on both Side

$$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$$
$$K^2 - 14K + 50 = 50 + K^2 - 2K$$
$$\cancel{K^2} - 14K \cancel{+50} \cancel{-50} \cancel{-K^2} + 2K = 0$$
$$-12K = 0$$
$$K = \frac{0}{-12}$$
$$K = 0$$

Q.6 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear.

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$$
$$|AB| = \sqrt{(3)^2 + (-12)^2}$$
$$|AB| = \sqrt{9+144}$$
$$|AB| = \sqrt{153}$$
$$|AB| = \sqrt{9 \times 17}$$

$|AB| = 3\sqrt{17}$

$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$

$|BC| = \sqrt{(-5)^2 + (15+5)^2}$

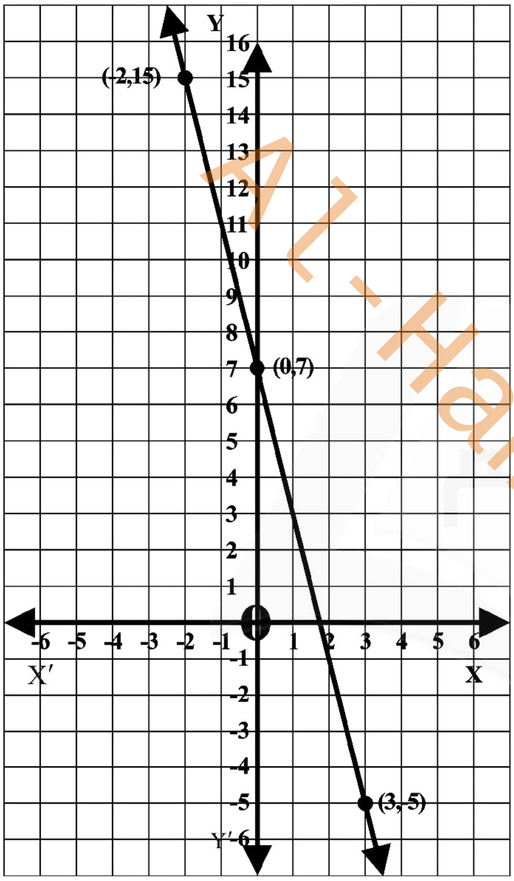
$|BC| = \sqrt{25 + (20)^2}$

$|BC| = \sqrt{25+400}$

$|BC| = \sqrt{425}$

$|BC| = \sqrt{25 \times 17}$

$|BC| = 5\sqrt{17}$



$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$

$|AC| = \sqrt{(-2)^2 + (8)^2}$

$|AC| = \sqrt{4+64}$

$|AC| = \sqrt{68}$

$|AC| = \sqrt{4 \times 17}$

$|AC| = 2\sqrt{17}$

$|AB| + |AC| = |BC|$

$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$

$5\sqrt{17} = 5\sqrt{17}$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)$ $A(\sqrt{3},1), B(\sqrt{3},-1)$ are the vertices of an equilateral triangle

Solution:

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|OA| = \sqrt{|\sqrt{3}-0|^2 + |0-1|^2}$

$|OA| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$|OA| = \sqrt{3+1}$

$|OA| = \sqrt{4}$

$|OA| = 2$

$|OB| = \sqrt{|\sqrt{3}-0|^2 + |-1-0|^2}$

$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$|OB| = \sqrt{3+1}$

$|OB| = \sqrt{4}$

$|OB| = 2$

$|AB| = \sqrt{|\sqrt{3}-\sqrt{3}|^2 + |-1-1|^2}$

$|AB| = \sqrt{0 + (-2)^2}$

$|AB| = \sqrt{4}$

$|AB| = 2$

All the sides are same in length so it is equilateral triangle

Q.8 Show that the points $A(-6,-5), B(5,-5), C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle find the length of its diagonals are equal

Solution:

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$A(-6,-5), B(5,-5)$

$|AB| = 3\sqrt{17}$

$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$

$|BC| = \sqrt{(-5)^2 + (15+5)^2}$

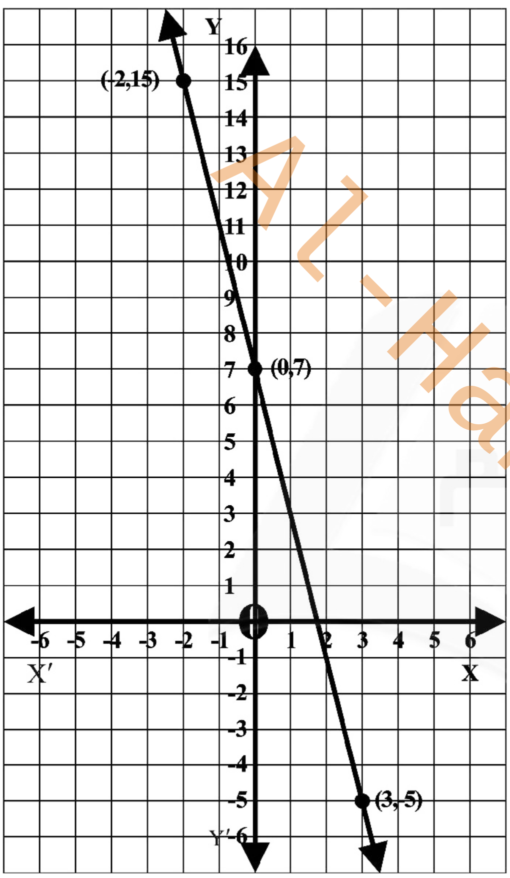
$|BC| = \sqrt{25+(20)^2}$

$|BC| = \sqrt{25+400}$

$|BC| = \sqrt{425}$

$|BC| = \sqrt{25\times 17}$

$|BC| = 5\sqrt{17}$



$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$

$|AC| = \sqrt{(-2)^2 + (8)^2}$

$|AC| = \sqrt{4+64}$

$|AC| = \sqrt{68}$

$|AC| = \sqrt{4\times 17}$

$|AC| = 2\sqrt{17}$

$|AB| + |AC| = |BC|$

$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$

$5\sqrt{17} = 5\sqrt{17}$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)$ $A(\sqrt{3},1), B(\sqrt{3},-1)$ are the vertices of an equilateral triangle

Solution:

$d = \sqrt{|x_2-x_1|^2 + |y_2-y_1|^2}$

$|OA| = \sqrt{|\sqrt{3}-0|^2 + |0-1|^2}$

$|OA| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$|OA| = \sqrt{3+1}$

$|OA| = \sqrt{4}$

$|OA| = 2$

$|OB| = \sqrt{|\sqrt{3}-0|^2 + |-1-0|^2}$

$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$|OB| = \sqrt{3+1}$

$|OB| = \sqrt{4}$

$|OB| = 2$

$|AB| = \sqrt{|\sqrt{3}-\sqrt{3}|^2 + |-1-1|^2}$

$|AB| = \sqrt{0+(-2)^2}$

$|AB| = \sqrt{4}$

$|AB| = 2$

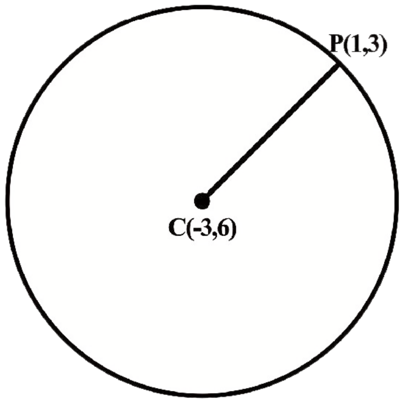
All the sides are same in length so it is equilateral triangle

Q.8 Show that the points $A(-6,-5), B(5,-5), C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle find the length of its diagonals are equal

Solution:

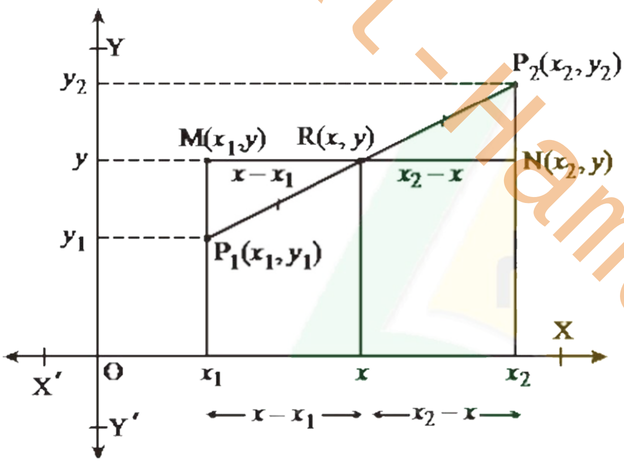
$d = \sqrt{|x_2-x_1|^2 + |y_2-y_1|^2}$

$A(-6,-5), B(5,-5)$



Recognition of the midpoint formula for any two points in the plane

Let $P_1(x,\ y)$ and $P_2(x_2, y_2)$ be any two points in the plane and $R(x,y)$ be midpoint of point P_1 and P_2 on the line segment P_1P_2 as shown in the figure.



If the line segment MN, parallel to x-*axis* has its midpoint $R(x,y)$,

then, $x_2 - x = x - x_1$

$$x_2 + x_1 = x + x$$

$$2x = x_1 + x_2 \Rightarrow x = \frac{x_1 + x_2}{2}$$

Similarly, $y = \frac{y_1 + y_2}{2}$

Thus the point R (x, y)

$= R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is the midpoint of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

Verification of the midpoint formula

$$|P_1R| = \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_1+x_2-2x_1}{2}\right)^2 + \left(\frac{y_1+y_2-2y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\frac{(x_2-x_1)^2}{4} + \frac{(y_2-y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2-x_1)^2 + (y_2-y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2-x_1)^2 + (y_2-y_1)^2}{2}}$$

OR

$$|P_1R| = \frac{1}{2}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \frac{1}{2}|P_1P_2|$$

and $|P_2R| = \sqrt{\left(\frac{x_1+x_2}{2} - x_2\right)^2 + \left(\frac{y_1+y_2}{2} - y_2\right)^2}$

$$|P_2R| = \sqrt{\left(\frac{x_1+x_2+2x_2}{2}\right)^2 + \left(\frac{y_1+y_2-2y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\left(\frac{x_1-x_2}{2}\right)^2 + \left(\frac{y_1-y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\frac{(x_1-x_2)^2}{4} + \frac{(y_1-y_2)^2}{4}}$$

$$|P_2R| = \sqrt{\frac{(x_1-x_2)^2 + (y_1-y_2)^2}{4}}$$

$$|P_2R| = \frac{\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}}{2}$$

OR

$$|P_2R| = \frac{1}{2}\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$\Rightarrow |P_2R| = |P_1R| = \frac{1}{2}|P_1R_2|$$

Thus it verifies that

$R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is the midpoint of the line segment P_1RP_2 which lies on the line segment since

$$|P_1R| + |P_2R| = |P_1P_2|$$

Exercise 9.3

Q.1 Find the midpoint of the line Segments joining each of the following pairs of points

Solution:

(a) $A(9,2), B(7,2)$

Let $M(x,y)$ the midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Midpoint formula

$$M(x,y)=M\left(\frac{9+7}{2},\frac{2+2}{2}\right)$$

$$=M\left(\frac{\cancel{16}^{8}}{\cancel{2}},\frac{\cancel{4}^{2}}{\cancel{2}}\right)$$

$$=M(8,2)$$

(b) $A(2,-6), B(3,-6)$

Let $M(x,y)$ the point of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Midpoint formula

$$M(x,y)=M\left(\frac{2+3}{2},\frac{-6-6}{2}\right)$$

$$M(x,y)=M\left(\frac{5}{2},\frac{-\cancel{12}^{6}}{\cancel{2}}\right)$$

$$M(x,y)=M(2.5,-6)$$

(c) $A(-8,1), B(6,1)$

Let $M(x,y)$ midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Formula

$$M(x,y)=M\left(\frac{-8+6}{2},\frac{1+1}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{2}}{\cancel{2}},\frac{\cancel{2}}{\cancel{2}}\right)$$

$$M(x,y)=M(-1,1)$$

(d) A (-4 , 9) , B (-4 , -3)

Let $M(x,y)$ midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) \text{ Formula}$$

$$M(x,y)=M\left(\frac{-4-4}{2},\frac{9-3}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{8}^{4}}{\cancel{2}},\frac{\cancel{6}^{3}}{\cancel{2}}\right)$$

$$M(x,y)=M(-4,3)$$

(e) $A(3,11), B(3,-4)$

Let $M(x,y)$ is the midpoint of AB

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{3+3}{2},\frac{-11-4}{2}\right)$$

$$M(x,y)=M\left(\frac{6}{2},\frac{-15}{2}\right)$$

$$M(x,y)=M(3,-7.5)$$

(f) A (0, 0) , B (0, -5)

Let $M(x,y)$ is the midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

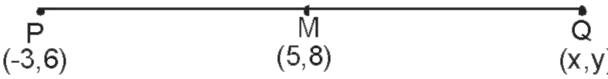
$$M(x,y)=M\left(\frac{0+0}{2},\frac{0-5}{2}\right)$$

$$M(x,y)=M\left(\frac{0}{2},\frac{-5}{2}\right)$$

$$=M(0,-2.5)$$

Q.2 The end point of line segment PQ is $(-3,6)$ and its midpoint is $(5,8)$ find the coordinates of the end point Q

Solution:



Let Q be the point (x,y), $M(5,8)$ is the midpoint of PQ

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$x=\frac{x_1+x_2}{2}$

$5=\frac{-3+x}{2}$

$5\times 2=-3+x$

$10+3=x$

$x=13$

$y=\frac{y_1+y_2}{2}$

$8=\frac{6+y}{2}$

$2\times 8=6+y$

$16-6=y$

$y=10$

Hence point Q is $(13,10)$

Q.3 Prove that midpoint of the hypotenuse of a right triangle is equidistance from it three vertices $P(-2,5), Q(1,3)$ and $R(-1,0)$

Solution:

$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

$P(-2,5), Q(1,3)$

$|PQ|=\sqrt{|-2-1|^2+|5-3|^2}$

$|PQ|=\sqrt{(-3)^2+(2)^2}$

$|PQ|=\sqrt{9+4}$

$|PQ|=\sqrt{13}$

$Q(1,3), R(-1,0)$

$|QR|=\sqrt{|1-(-1)|^2+|3-0|^2}$

$|QR|=\sqrt{(1+1)^2+(3)^2}$

$|QR|=\sqrt{(2)^2+9}=\sqrt{4+9}$

$|QR|=\sqrt{13}$

$P(-2,5), R(-1,0)$

$|PR|=\sqrt{|-2-(-1)|^2+|5-0|^2}$

$|PR|=\sqrt{|-2+1|^2+|5|^2}$

$|PR|=\sqrt{(-1)^2+(5)^2}=\sqrt{1+25}$

$|PR|=\sqrt{26}$

To find the length of hypotenuse and whether it is right angle triangle we use the Pythagoras theorem

$(PR)^2=(PQ)^2+(QR)^2$

$\left(\sqrt{26}\right)^2=\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2$

$26=13+13$

$26=26$

It is a right angle triangle and PR is hypotenuse

$P(-2,5), R(-1,0)$

Midpoint of PR

$M(x,y)=\left(\frac{-2-1}{2},\frac{5+0}{2}\right)$

$M(x,y)=\left(\frac{-3}{2},\frac{5}{2}\right)$

$MP=MR$

$M\left(\frac{-3}{2},\frac{5}{2}\right), P(-2,5), R(-1,0)$

$|MP|=|MR|$

(i)

$|MP|=\sqrt{\left|\frac{-3}{2}-(-2)\right|^2+\left|\frac{5}{2}-5\right|^2}$

$=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5-10}{2}\right)^2}$

$|MP|=\sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$

$=\sqrt{\left(\frac{1}{2}\right)^2+\frac{25}{4}}$

$|MP|=\sqrt{\frac{1}{4}+\frac{25}{4}}=\sqrt{\frac{1+25}{4}}$

$|MP|=\sqrt{\frac{26}{4}}$

$|MP|=\frac{\sqrt{26}}{2}$

(ii) $M\left(\frac{-3}{2},\frac{5}{2}\right), R(-1,0)$

$|MR|=\sqrt{\left|\frac{-3}{2}-(-1)\right|^2+\left|\frac{5}{2}-0\right|^2}$

$$|MR|=\sqrt{\left(\frac{-3}{2}+1\right)^2+\left(\frac{5}{2}\right)^2}$$

$$|MR|=\sqrt{\left(\frac{-3+2}{2}\right)^2+\frac{25}{4}}$$

$$=\sqrt{\left(\frac{-1}{2}\right)^2+\frac{25}{4}}$$

$$|MR|=\sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$|MR|=\sqrt{\frac{1+25}{4}}=\sqrt{\frac{26}{4}}$$

$$|MR|=\frac{\sqrt{26}}{2}$$

(iii) $M\left(\frac{-3}{2},\frac{5}{2}\right)$

$Q(1,3)$

$$|MQ|=\sqrt{\left(\frac{-3}{2}-1\right)^2+\left(\frac{5}{2}-3\right)^2}$$

$$=\sqrt{\left(\frac{-3-2}{2}\right)^2+\left(\frac{5-6}{2}\right)^2}$$

$$=\sqrt{\left(\frac{-5}{2}\right)^2+\left(\frac{-1}{2}\right)^2}$$

$$=\sqrt{\frac{25}{4}+\frac{1}{4}}=\sqrt{\frac{26}{4}}$$

Hence proved $MP=MR=|MQ|$

Q.4 If $O(0,0)$, $A(3,0)$ and $B(3,5)$ are three points in the plane find M_1 and M_2 as the midpoint of the line segments AB and OB respectively find $|M_1M_2|$

Solution:

M_1 is the midpoint of AB

$$M_1(x,y)=M_1\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$A(3,0),B(3,5)$

$$M_1\left(\frac{3+3}{2},\frac{0+5}{2}\right)$$

$$M_1\left(\frac{6}{2},\frac{5}{2}\right)$$

$$M_1\left(3,\frac{5}{2}\right)$$

M_2 is the midpoint of OB

$$M_2\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$0(0,0),B(3,5)$

$$M_2\left(\frac{0+3}{2},\frac{0+5}{2}\right)$$

$$M_2\left(\frac{3}{2},\frac{5}{2}\right)$$

$$M_1\left(3,\frac{5}{2}\right)M_2\left(\frac{3}{2},\frac{5}{2}\right)$$

$$|M_1M_2|=\sqrt{\left|\frac{3}{2}-3\right|^2+\left|\frac{5}{2}-\frac{5}{2}\right|^2}$$

$$|M_1M_2|=\sqrt{\left(\frac{3-6}{2}\right)^2+(0)^2}$$

$$=\sqrt{\left(\frac{-3}{2}\right)^2+0}$$

$$|M_1M_2|=\sqrt{\frac{9}{4}}$$

$$|M_1M_2|=\frac{3}{2}$$

Q.5 Show that the diagonals of the parallelogram having vertices $A(1,2),B(4,2),C(-1,-3)$ and $D(-4,-3)$ bisect each other.

Solution:

$ABCD$ is parallelogram which vertices are

$A(1,2),B(4,2),C(-1,-3)D(-4,-3)$

Let $\overline{BD}$ and $\overline{AC}$ the diagonals of parallelogram they intersect at point M

$A(1,2),C(-1,-3)$ midpoint of AC

Midpoint formula

$$M_1(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M_1(x,y) = M_1\left(\frac{1-1}{2}, \frac{2-3}{2}\right)$$

$$M_1(x,y) = M_1\left(\frac{0}{2}, \frac{-1}{2}\right) = \left(0, \frac{-1}{2}\right)$$

Midpoint of BD,

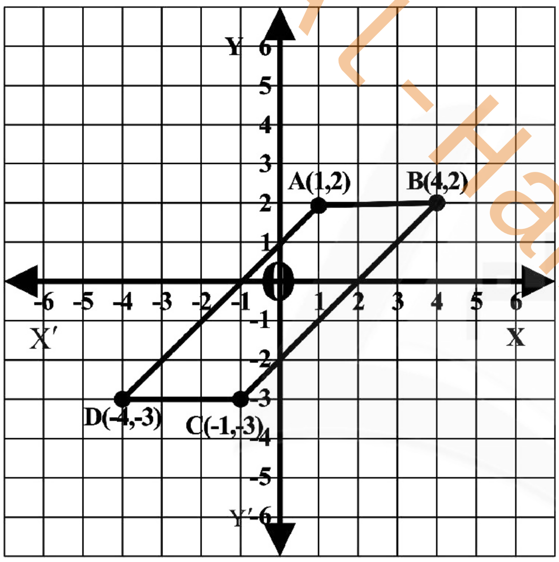
$$M_2(x,y) = M_2\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_2(x,y) = M_2\left(\frac{4-4}{2}, \frac{2-3}{2}\right)$$

$$M_2(x,y) = M_2\left(\frac{0}{2}, \frac{-1}{2}\right)$$

$$M_2(x,y) = M_2\left(0, \frac{-1}{2}\right)$$

As M_1 and M_2 Coincide the diagonals of the parallelogram bisect each other.



Q.6 The vertices of a triangle are $P(4,6), Q(-2,-4)$ and $R(-8,2)$. Show that the length of the line segment joining the midpoints of the line segments $\overline{PR}, \overline{QR}$ is $\frac{1}{2}\overline{PQ}$

Solution:

M_1 the midpoint of QR is

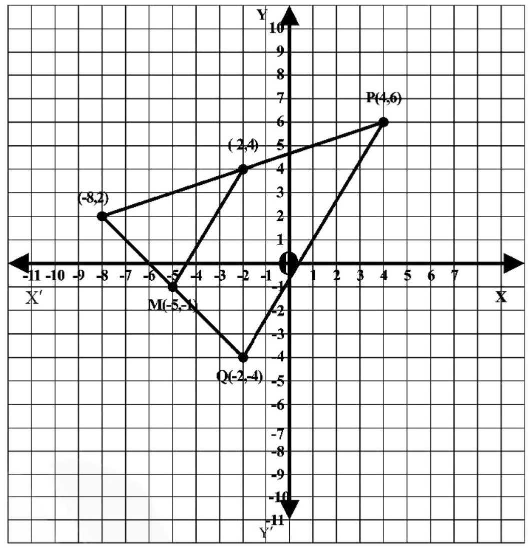
$$Q(-2,-4), R(-8,2)$$

$$M_1(x,y) = M_1\left(\frac{-2-8}{2}, \frac{-4+2}{2}\right)$$

$$= M_1\left(\frac{-10}{2}, \frac{-2}{2}\right)$$

$$= M_1(-5,-1)$$

$$M_1(-5,-1)$$



M_2 the midpoint of PR is

$$P(4,6), Q(-8,+2)$$

$$M_2(x,y) = M\left(\frac{4-8}{2}, \frac{6+2}{2}\right)$$

$$M_2(x,y) = M_2\left(\frac{-4}{2}, \frac{8}{2}\right)$$

$$M_2(x,y) = M_2(-2,4)$$

$$M_2(-2,4)$$

$$|M_1M_2| = \sqrt{|-5+2|^2 + |4+1|^2}$$

$$|M_1M_2| = \sqrt{(-3)^2 + (5)^2}$$

$$|M_1M_2| = \sqrt{9+25}$$

$$|M_1M_2| = \sqrt{34}$$

$$|PQ| = \sqrt{|4+2|^2 + |6+4|^2}$$

$$|PQ| = \sqrt{(6)^2 + (10)^2} = \sqrt{36+100}$$

$$|PQ| = \sqrt{136}$$

$$|P\ Q| = \sqrt{4 \times 34}$$

$$|P\ Q| = 2\sqrt{34}$$

$$\frac{|P\ Q|}{2} = \sqrt{34}$$

OR

$$\frac{1}{2}|PQ| = \sqrt{34}$$

Hence we proved that

$$|M_1M_2| = \frac{1}{2}|PQ|$$

Review Exercise 9

Q.1 Choose the Correct answer

(i) Distance between point (0 , 0) and (1, 1) is

(a) 0 **(b)** 1

(c) 2 **(d)** $\sqrt{2}$

(ii) Distance between the point (1 , 0) and (0 ,1) is

(a) 0 **(b)** 1

(c) $\sqrt{2}$ **(d)** 2

(iii) Midpoint of the (2, 2) and (0, 0) is

(a) (1, 1) **(b)** (1, 0)

(c) (0, 1) **(d)** (-1, -1)

(iv) Midpoint of the points (2, -2) and (-2 , 2) is

(a) (2, 2) **(b)** (-2, -2)

(c) (0 , 0) **(d)** (1, 1)

(v) A triangle having all sides equal is called

(a) Isosceles **(b)** Scalene

(c) Equilateral **(d)** None of these

(vi) A triangle having all sides different is called

(a) Isosceles **(b)** Scalene

(c) Equilateral **(d)** None of these

ANSWER KEYS

i	ii	iii	iv	v	vi
d	c	a	c	c	b

Q.2 Answer the following which is true and which is false

(i) A line has two end points (False)
(ii) A line segment has one end point (False)
(iii) A triangle is formed by the three collinear points (False)
(iv) Each side of triangle has two collinear vertices. (True)
(v) The end points of each side of a rectangle are Collinear (True)
(vi) All the points that lie on the x-axis are Collinear (True)
(vii) Origin is the only point Collinear with the points of both axis separately (True)

Q.3 Find the distance between the following pairs of points

Solution:

(i) $(6,3)(3,-3)$

$A(6,3), B(3,-3)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

`$|AB| = \sqrt{|3-6|^2 + |-3-3|^2}$

$|AB| = \sqrt{(-3)^2 + (-6)^2}$

$|AB| = \sqrt{9+36}$

$|AB| = \sqrt{45}$

$|AB| = \sqrt{9\times 5}$

$|AB| = 3\sqrt{5}$

(ii) $(7,5),(1,-1)$

$A(7,5), B(1,-1)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|7-1|^2 + |5-(-1)|^2}$

$|AB| = \sqrt{(6)^2 + (5+1)^2}$

$|AB| = \sqrt{36+(6)^2} = \sqrt{36+36}$

$|AB| = \sqrt{72} = \sqrt{36\times 2}$

$|AB| = 6\sqrt{2}$

(iii) $(0,0),(-4,-3)$

$A(0,0), B(-4,-3)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|0-4|^2 + |0-(-3)|^2}$

$|AB| = \sqrt{(-4)^2 + (3)^2}$

$|AB| = \sqrt{16+9}$

$|AB| = \sqrt{25}$

$|AB| = 5$

Q.4 Find the midpoint between following pairs of points

Solution:

(i) $(6,6),(4,-2)$

$M(x,y) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M(x,y) = M\left(\frac{6+4}{2}, \frac{6-2}{2}\right)$

$M(x,y) = M\left(\frac{10}{2}, \frac{4}{2}\right)$

$M(x,y) = M(5,2)$

(ii) $(-5,-7),(-7,-5)$

$M(x,y) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M(x,y) = M\left(\frac{-5-7}{2}, \frac{-7-5}{2}\right)$

$M(x,y) = M\left(\frac{-12}{2}, \frac{-12}{2}\right)$

$M(x,y) = M(-6,-6)$

(iii) $(8,0),(0,-12)$

$M(x,y) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M(x,y) = M\left(\frac{8+0}{2}, \frac{0-12}{2}\right)$

$M(x,y) = M\left(\frac{8}{2}, \frac{-12}{2}\right)$

$M(x,y) = M(4,-6)$

Q.5 Define the following

Solution:

(i) Co-ordinate Geometry:-

Co-ordinate geometry is the study of geometrical shapes in the Cartesian plane (or coordinate plane)

(ii) Collinear:-

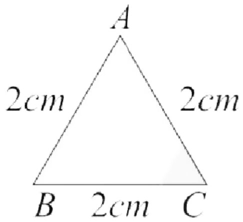
Two or more than two points which lie on the same straight line are called collinear points with respect to that line.

(iii) Non- Collinear:-

The points which do not lie on the same straight line are called non-collinear.

(iv) Equilateral Triangle:-

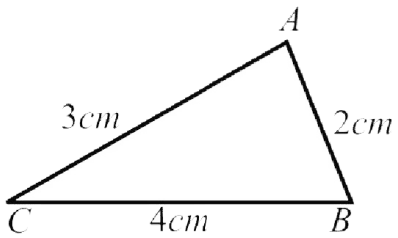
If the length of all three sides of a triangle are same then the triangle is called an equilateral triangle.



ΔABC is an equilateral triangle.

(v) Scalene Triangle:-

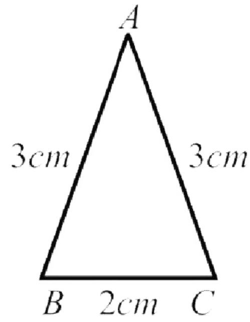
A triangle is called a scalene triangle if measure of all sides are different.



ΔABC is a Scalene triangle.

(vi) Isosceles Triangle:-

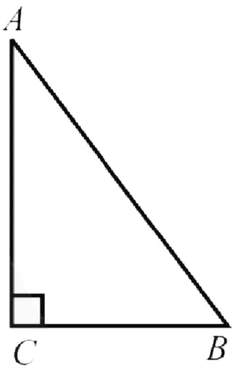
An isosceles triangles is a triangle which has two of its sides with equal length while the third side has different length.



ΔABC is an isosceles triangle

(vii) Right Triangle:-

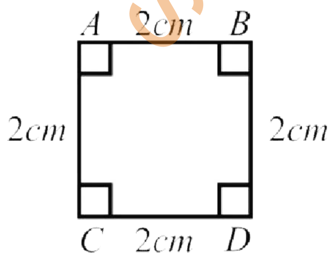
A triangle in which one of the angles has measure equal to 90° is called a right triangle.



ΔABC is a right angled triangle.

(viii) Square:-

A Square is closed figure formed by four non- collinear points such that lengths of all sides are equal and measure of each angles is 90°.



$ABCD$ is a square.

Unit 9: Introduction to Coordinate Geometry

Overview

Coordinate Geometry:

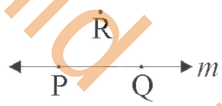
The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane).

Collinear Points:

Two or more than two points which lie on the same straight line are called collinear points with respect to that line.

Non-collinear points:

Tow or more points which does not lie on the same straight line are called non-collinear points.



Equilateral Triangle:

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle.

An Isosceles Triangle:

An isosceles triangle PQR is a triangle which has two of its sides with equal length while the third side has a different length.

Right Angle Triangle

A triangle in which one of the angles has measure equal to 90° is called a right angle triangle.

Scalene Triangle:-

A triangle is called a scalene triangle if measure of all sides are different.

Square:-

A Square is closed figure formed by four non- collinear points such that lengths of all sides are equal and measure of each angles is 90°.

Rectangle

A figure formed in the plane by four non-collinear points is called a rectangle if,

(i) Its opposite sides are equal in length

(ii) The angle at each vertex is of measure 90°

Parallelogram

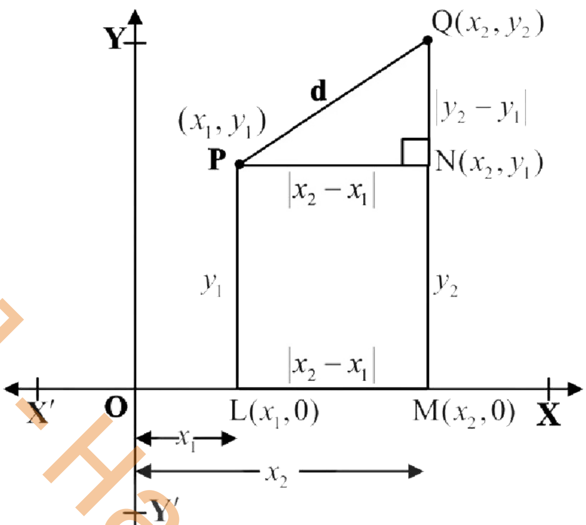
A figure formed by four non-collinear points in the plane is called a parallelogram if

(i) Its opposite sides are of equal length

(ii) Its opposite sides are parallel

(iii) Meausre of none of the angles is 90°.

Finding distance between two points.



Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the coordinate plane where d is the length of the line segment PQ i,e, $|\overline{PQ}| = d$

The line segments MQ and LP parallel to y-axis meet x-axis at point M and L respectively with coordinates M(x_2 , o) and L (x_1 , o)

The line segment PN is parallel to x-axis

In the right triangle PNQ $|\overline{NQ}| = |y_2 - y_1|$ and $|\overline{PN}| = |x_2 - x_1|$

Using Pythagoras theorem

$$(\overline{PQ})^2 = (\overline{PN})^2 + (\overline{QN})^2$$

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

Taking under root on both side

$$\sqrt{d^2} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Since d > 0 always