Exercise 5.1

Q.1 Factorize

(i) $2abc-4abx+2abd$

Solution: $2abc-4abx+2abd$

$=2ab(c-2x+d)$

(ii) $9xy-12x^2y+18y^2$

Solution: $9xy-12x^2y+18y^2$

$=3y(3x-4x^2+6y)$

(iii) $-3x^2y-3x+9xy^2$

Solution: $-3x^2y-3x+9xy^2$

$=-3x(xy+1-3y^2)$

(iv) $5ab^2c^3-10a^2b^3c-20a^3bc^2$

Solution: $5ab^2c^3-10a^2b^3c-20a^3bc^2$

$=5abc(bc^2-2ab^2-4a^2c)$

(v) $3x^3y(x-3y)-7x^2y^2(x-3y)$

Solution: $3x^3y(x-3y)-7x^2y^2(x-3y)$

$=(x-3y)(3x^3y-7x^2y^2)$

$=(x-3y)\ x^2y(3x-7y)$

$=x^2y(x-3y)(3x-7y)$

(vi) $2xy^3(x^2+5)+8xy^2(x^2+5)$

Solution: $2xy^3(x^2+5)+8xy^2(x^2+5)$

$=(x^2+5)(2xy^3+8xy^2)$

$=(x^2+5)2xy^2(y+4)$

$=2xy^2(x^2+5)(y+4)$

Q.2 Factorize

(i) $5ax-3ay-5bx+3by$

Solution: $5ax-3ay-5bx+3by$

$=5ax-5bx-3ay+3by$

$=5x(a-b)-3y(a-b)$

$=(a-b)(5x-3y)$

(ii) $3xy+2y-12x-8$

Solution: $3xy+2y-12x-8$

$=3xy-12x+2y-8$

$=3x(y-4)+2(y-4)$

$=(y-4)(3x+2)$

(iii) $x^3+3xy^2-2x^2y-6y^3$

Solution: $x^3+3xy^2-2x^2y-6y^3$

By cyclic order

$=x^3-2x^2y+3xy^2-6y^3$

$=x^2(x-2y)+3y^2(x-2y)$

$=(x-2y)(x^2+3y^2)$

(iv) $(x^2-y^2)z+(y^2-z^2)x$

Solution: $(x^2-y^2)z+(y^2-z^2)x$

$=\ x^2z-y^2z+xy^2-xz^2$

Arrange in cyclic order

$x^2z+\ xy^2-xz^2-y^2z$

$=x^2z+xy^2-y^2z-xz^2$

$=x(xz+y^2)-z(xz+y^2)$

$=(xz+y^2)(x-z)$

Q.3 Factorize

(i) $144a^2+24a+1$

Solution: $144a^2+24a+1$

By using formula

$(a+b)^2=a^2+2ab+b^2$

$=(12a)^2+2(12a)(1)+(1)^2$

$=(12a+1)^2$

(ii) $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

Solution: $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

Formula $a^2-2ab+b^2=(a-b)^2$

$=\left(\frac{a}{b}\right)^2-2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2$

$=\left(\frac{a}{b}-\frac{b}{a}\right)^2$

(iii) $(x+y)^2-14z(x+y)+49z^2$

Solution: $(x+y)^2-14z(x+y)+49z^2$

Formula $a^2-2ab+b^2=(a-b)^2$

$=(x+y)^2-2(x+y)(7z)+(7z)^2$

$=(x+y-7z)^2$

(iv) $12x^2-36x+27$

Solution: $12x^2-36x+27$

$=3(4x^2-12x+9)$

Formula $a^2-2ab+b^2=(a-b)^2$

$=3\left[(2x)^2-2(2x)(3)+(3)^2\right]$

$=3(2x-3)^2$

Q.4 Factorize

(i) $3x^2-75y^2$

Solution: $3x^2-75y^2$

$=3(x^2-25y^2)$

Formula $a^2-b^2=(a+b)(a-b)$

$=3\left[(x)^2-(5y)^2\right]$

$=3(x+5y)(x-5y)$

(ii) $x(x-1)-y(y-1)$

Solution: $x(x-1)-y(y-1)$

$=x^2-x-y^2+y$

Arranging in cyclic order

$=x^2-y^2-x+y$

Taking common

$=(x^2-y^2)-(x-y)$

$=\left[(x+y)(x-y)\right]-(x-y)$

$=(x-y)(x+y-1)$

(iii) $128am^2-242an^2$

Solution: $128am^2-242an^2$

$=2a(64m^2-121n^2)$

$=2a\left[(8m)^2-(11n)^2\right]$

$=2a(8m+11n)(8m-11n)$

(iv) $3x-243x^3$

Solution: $3x-243x^3$

$=3x(1-81x^2)$

$=3x\left[(1)^2-(9x)^2\right]$

$=3x(1+9x)(1-9x)$

Q.5 Factorize

(i) x^2-y^2-6y-9

Solution: x^2-y^2-6y-9

$=x^2-\left[y^2+6y+9\right]$

$=x^2-[(y)^2+2(y)(3)+(3)^2]$

$=x^2-(y+3)^2$

$=(x)^2-(y+3)^2$

$=(x+y+3)[x-(y+3)]$

$=(x+y+3)(x-y-3)$

(ii) x^2-a^2+2a-1

Solution: x^2-a^2+2a-1

$=x^2-\left[a^2-2a+1\right]$

$=x^2-(a-1)^2$

$=[x+(a-1)][x-(a-1)]$

$=(x+a-1)(x-a+1)$

(iii) $4x^2 - y^2 - 2y - 1$

Solution: $4x^2 - y^2 - 2y - 1$

$= 4x^2 - (y^2 + 2y + 1)$

$= 4x^2 - \left[(y)^2 + 2(y)(1) + (1)^2\right]$

$= 4x^2 - (y+1)^2$

$= (2x)^2 - (y+1)^2$

$= \left[2x + (y+1)\right]\left[2x - (y+1)\right]$

$= (2x+y+1)(2x-y-1)$

(iv) $x^2 - y^2 - 4x - 2y + 3$

Solution: $x^2 - y^2 - 4x - 2y + 3$

$= x^2 - 4x + 4 - y^2 - 2y - 1$

$= (x^2 - 4x + 4) - (y^2 + 2y + 1)$

$= \left[(x)^2 - 2(x)(2) + (2)^2\right]$

$- \left[(y)^2 + 2(y)(1) + (1)^2\right]$

$= (x-2)^2 - (y+1)^2$

$= (x - 2 + y + 1)\left[x - 2 - (y+1)\right]$

$= (x - 2 + y + 1)(x - 2 - y - 1)$

$= (x + y - 2 + 1)(x - y - 2 - 1)$

$= (x + y - 1)(x - y - 3)$

(v) $25x^2 - 10x + 1 - 36z^2$

Solution: $25x^2 - 10x + 1 - 36z^2$

$= (5x)^2 - 2(5x)(1) + (1)^2] - 36z^2$

$= (5x-1)^2 - (6z)^2$

$= \left[(5x-1) + 6Z\right]\left[(5x-1) - 6Z\right]$

$= (5x - 1 + 6z)(5x - 1 - 6z)$

(vi) $x^2 - y^2 - 4xz + 4z^2$

Solution: $x^2 - y^2 - 4xz + 4z^2$

$= x^2 - 4xz + 4z^2 - y^2$

$= \left[(x)^2 - 2(x)(2z) + (2z)^2\right] - y^2$

$= (x - 2z)^2 - (y)^2$

$= (x - 2z + y)(x - 2z - y)$

$= (x + y - 2z)(x - y - 2z)$

Exercise 5.2

Q.1 Factorize

(i) $x^4+\frac{1}{x^4}-3$

Solution: $x^4+\frac{1}{x^4}-3$

$=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2-3$

By adding and subtracting by 2

$=(x^2)^2+\left(\frac{1}{x^2}\right)^2+2-2-3$

$=\left[(x^2)^2+\left(\frac{1}{x^2}\right)^2-2\right]+2-3$

$=\left[(x^2)^2+\left(\frac{1}{x^2}\right)^2-2\right]-1$

$=\left(x^2-\frac{1}{x^2}\right)^2-(1)^2$

$=\left(x^2-\frac{1}{x^2}+1\right)\left(x^2-\frac{1}{x^2}-1\right)$

(ii) $3x^4+12y^4$

Solution: $3x^4+12y^4$

$=3\left(x^4+4y^4\right)$

By adding and subtracting by $2(x^2)(2y^2)$

$=3\left[\left(x^2\right)^2+\left(2y^2\right)^2+2\left(x^2\right)\left(2y^2\right)-2\left(x^2\right)\left(2y^2\right)\right]$

$=3\left[\left(x^2\right)^2+\left(2y^2\right)^2+2\left(x^2\right)\left(2y^2\right)-2\left(x^2\right)\left(2y^2\right)\right]$

$=3\left[\left(x^2+2y^2\right)^2-4x^2y^2\right]$

$=3\left[\left(x^2+2y^2\right)^2-\left(2xy\right)^2\right]$

$=3\left[\left(x^2+2y^2+2xy\right)\left(x^2+2y^2-2xy\right)\right]$

$=3\left[\left(x^2+2xy+2y^2\right)\left(x^2-2xy+2y^2\right)\right]$

(iii) $a^4+3a^2b^2+4b^4$

Solution: $a^4+3a^2b^2+4b^4$

$=\left(a^4+4b^4\right)+3a^2b^2$

$=(a^2)^2+(2b^2)^2+3a^2b^2$

By adding and subtracting by $2\left(a^2\right)\left(2b^2\right)$

$=\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$=\left[\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)\right]-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$=(a^2+2b^2)^2-a^2b^2$

$=(a^2+2b^2)^2-(ab)^2$

$=(a^2+2b^2+ab)(a^2+2b^2-ab)$

(iv) $4x^4+81$

Solution: $4x^4+81$

$=\left(2x^2\right)^2+\left(9\right)^2$

By adding and subtracting by $2\left(2x^2\right)\left(9\right)$

$=\left[\left(2x^2\right)^2+\left(9\right)^2+2\left(2x^2\right)\left(9\right)-2\left(2x^2\right)\left(9\right)\right]$

$=\left[\left(2x^2\right)^2+\left(9\right)^2+2\left(2x^2\right)\left(9\right)\right]-2\left(2x^2\right)\left(9\right)$

$=\left(2x^2+9\right)^2-36x^2$

$=\left(2x^2+9\right)^2-\left(6x\right)^2$

$=\left(2x^2+9+6x\right)\left(2x^2+9-6x\right)$

$=\left(2x^2+6x+9\right)\left(2x^2-6x+9\right)$

(v) x^4+x^2+25

Solution: x^4+x^2+25

$=\left(x^4+25\right)+x^2$

$=\left[\left(x^2\right)^2+\left(5\right)^2\right]+x^2$

By adding and subtracting by $2\left(x^2\right)\left(5\right)$

$=\left[\left(x^2\right)^2+\left(5\right)^2+2\left(x^2\right)\left(5\right)-2\left(x^2\right)\left(5\right)\right]+x^2$

$=\left[\left(x^2\right)^2+\left(5\right)^2+2\left(x^2\right)\left(5\right)\right]-2\left(x^2\right)\left(5\right)+x^2$

$=\left(x^2+5\right)^2-10x^2+x^2$

$=(x^2+5)^2-9x^2$

$=(x^2+5)^2-(3x)^2$

$=\left(x^2+5+3x\right)\left(x^2+5-3x\right)$

$=(x^2+3x+5)(x^2-3x+5)$

(vi) x^4+4x^2+16

Solution: x^4+4x^2+16

$=(x^2)^2+16+4x^2$

$=(x^2)^2+(4)^2+4x^2$

By adding and subtracting by $2\left(x^2\right)\left(4\right)$

$=\left(x^2\right)^2+\left(4\right)^2+2\left(x^2\right)\left(4\right)-2\left(x^2\right)\left(4\right)+4x^2$

$=(x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$

$=\left(x^2+4\right)^2-8x^2+4x^2$

$=(x^2+4)^2-4x^2$

$=(x^2+4)^2-(2x)^2$

$=\left(x^2+4+2x\right)\left(x^2+4-2x\right)$

$=\left(x^2+2x+4\right)\left(x^2-2x+4\right)$

Q.2 Factorize

(i) $x^2+14x+48$

Solution: $x^2+14x+48$

$=\ x^2+8x+6x+48$

$=x(x+8)+6(x+8)$

$=(x+8)(x+6)$

(ii) $x^2-21x+108$

Solution: $x^2-21x+108$

$=\ x^2-12x-9x+108$

$=x(x-12)-9(x-12)$

$=(x-9)(x-12)$

(iii) $x^2-11x-42$

Solution: $x^2-11x-42$

$=\ x^2-14x+3x-42$

$=x(x-14)+3(x-14)$

$=(x+3)(x-14)$

(iv) $x^2+x-132$

Solution: $x^2+x-132$

$=\ x^2+12x-11x-132$

$=x(x+12)-11(x+12)$

$=(x-11)(x+12)$

Q.3 Factorize

(i) $4x^2+12x+5$

Solution: $4x^2+12x+5$

$=\ 4x^2+2x+10x+5$

$=2x(2x+1)+5(2x+1)$

$=(2x+5)(2x+1)$

(ii) $30x^2+7x-15$

Solution: $30x^2+7x-15$

$=\ 30x^2+25x-18x-15$

$=5x(6x+5)-3(6x+5)$

$=(5x-3)(6x+5)$

(iii) $24x^2-65x+21$

Solution: $24x^2-65x+21$

$=24x^2-56x-9x+21$

$=8x(3x-7)-3(3x-7)$

$=(8x-3)(3x-7)$

(iv) $5x^2-16x-21$

Solution: $5x^2-16x-21$

$=\ 5x^2+5x-21x-21$

$=5x\ (x+1)-21(x+1)$

$=(5x-21)(x+1)$

(v) $4x^2-17xy+4y^2$

Solution: $4x^2-17xy+4y^2$

$=\ 4x^2-16xy-xy+4y^2$

$=4x(x-4y)-y(x-4y)$

$=(4x-y)(x-4y)$

(vi) $3x^2-38xy-13y^2$

Solution: $3x^2-38xy-13y^2$

$=\ 3x^2-39xy+xy-13y^2$

$=3x(x-13y)+y(x-13y)$

$=(3x+y)(x-13y)$

(vii) $5x^2+33xy-14y^2$

Solution: $5x^2+33xy-14y^2$

$= 5x^2 + 35xy - 2xy - 14y^2$
$= 5x(x + 7y) - 2y(x + 7y)$
$= (5x - 2y)(x + 7y)$

(viii) $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, x \neq 0$

Solution: $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4, x \neq 0$

$= \left(5x - \frac{1}{x}\right)^2 + 2\left(5x - \frac{1}{x}\right)(2) + (2)^2$

$= \left(5x - \frac{1}{x} + 2\right)^2$

$= \left(5x - \frac{1}{x} + 2\right)\left(5x - \frac{1}{x} + 2\right)$

Q.4

(i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Solution: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Suppose that
$x^2 + 5x = y$
So,
$(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$
$= (y + 4)(y + 6) - 3$
$= [y(y + 6) + 4(y + 6) - 3$
$= (y^2 + 6y + 4y + 24) - 3$
$= (y^2 + 10y + 24) - 3$
$= y^2 + 10y + 24 - 3$
$= y^2 + 10y + 21$
$= y^2 + 7y + 3y + 21$
$= y(y + 7) + 3(y + 7)$
$= (y + 3)(y + 7)$
We know that $y = x^2 + 5x$
$= (x^2 + 5x + 3)(x^2 + 5x + 7)$

(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Solution: $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Suppose that
$x^2 - 4x = y$
So,
$= (y)(y - 1) - 20$
$= (y^2 - y) - 20$
$= y^2 - y - 20$
$= y^2 - 5y + 4y - 20$
$= y(y - 5) + 4(y - 5)$
$= (y + 4)(y - 5)$
We know that $a = x^2 - 4x$
$= (x^2 - 4x + 4)(x^2 - 4x - 5)$
$= \left[(x)^2 - 2(x)(2) + (2)^2\right]\left[x^2 - 5x + x - 5\right]$
$= (x - 2)^2\left[x(x - 5) + 1(x - 5)\right]$
$= (x - 2)^2(x - 5)(x + 1)$
$= (x - 5)(x + 1)(x - 2)^2$

(iii) $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Solution: $(x + 2)(x + 3)(x + 4)(x + 5) - 15$
$= [(x + 2)(x + 5)][(x + 3)(x + 4)] - 15$
$= [x(x + 5) + 2(x + 5)][x(x + 4) + 3(x + 4)] - 15$
$= [x^2 + 5x + 2x + 10][x^2 + 4x + 3x + 12] - 15$
$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$
Suppose that
$x^2 + 7x = y$
So,
$(x^2 + 7x + 10)(x^2 + 7x + 12) - 15$
$= (y + 10)(y + 12) - 15$
$= [y(y + 12) + 10(y + 12)] - 15$
$= (y^2 + 12y + 10y + 120) - 15$
$= (y^2 + 22y + 120) - 15$
$= y^2 + 22y + 120 - 15$
$= y^2 + 22y + 105$
$= y^2 + 15y + 7y + 105$
$= y(y + 15) + 7(y + 15)$
$= y(y + 15) + 7(y + 15)$
$= (y + 7)(y + 15)$
We know that $y = x^2 + 7x$
$= (x^2 + 7x + 7)(x^2 + 7x + 15)$

(iv) $(x + 4)(x - 5)(x + 6)(x - 7) - 504$

Solution: $(x+4)(x-5)(x+6)(x-7)-504$

$=[(x+4)(x-5)][(x+6)(x-7)]-504$

$=[x(x-5)+4(x-5)][x(x-7)+6(x-7)]-504$

$=(x^2-5x+4x-20)(x^2-7x+6x-42)-504$

$=(x^2-x-20)(x^2-x-42)-504$

Suppose that

$x^2-x=y$

So,

$=(y-20)(y-42)-504$

$=[y(y-42)-20(y-42)]-504$

$=(y^2-42y-20y+840)-504$

$=y^2-62y+840-504$

$=y^2-62y+336$

$=y^2-56y-6y+336$

$=y(y-56)-6(y-56)$

$=(y-6)(y-56)$

We know that $a=x^2-x$

$=(x^2-x-6)(x^2-x-56)$

$=(x^2-3x+2x-6)(x^2-8x+7x-56)$

$=[x(x-3)+2(x-3)][x(x-8)+7(x-8)]$

$=(x+2)(x-3)(x+7)(x-8)$

(v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

Solution: $(x+1)(x+2)(x+3)(x+6)-3x^2$

$=[(x+1)(x+6)][(x+2)(x+3)]-3x^2$

$=[x(x+6)+1(x+6)][x(x+3)+2(x+3)]-3x^2$

$=(x^2+6x+x+6)(x^2+3x+2x+6)-3x^2$

$=(x^2+6+7x)(x^2+6+5x)-3x^2$

Suppose that

$x^2+6=y$

So,

$=(y+7x)(y+5x)-3x^2$

$=[y(y+5x)+7x(y+5x)]-3x^2$

$=(y^2+5xy+7xy+35x^2-3x^2)$

$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$

$=y(y+8x)+4x(y+8x)$

$=(y+4x)(y+8)$

We know that $y=x^2+6$

$=(x^2+6+4x)(x^2+6+8x)$

$=(x^2+4x+6)(x^2+8x+6)$

Q.5

(i) $x^3+48x-12x^2-64$

Solution: $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(4)+3(x)(4)^2-(4)^3$

$=(x-4)^3$

(ii) $8x^3+60x^2+150x+125$

Solution: $8x^3+60x^2+150x+125$

$a^3+3a^2b+3ab^2+b^3=(a+b)^3$

$=(2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$

$=(2x+5)^3$

(iii) $x^3-18x^2+108x-216$

Solution: $x^3-18x^2+108x-216$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$

$=(x-6)^3$

(iv) $8x^3-125y^3-60x^2y+150xy^2$

Solution: $8x^3-125y^3-60x^2y+150xy^2$

$=8x^3-60x^2y+150xy^2-125y^3$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3$

$=(2x-5y)^3$

Q.6

(i) $27+8x^3$

Solution: $27+8x^3$

$=(3)^3+(2x)^3$

$=(3+2x)[(3)^2-(3)(2x)+(2x)^2]$

$=(3+2x)(9-6x+4x^2)$

(ii) $125x^3-216y^3$

Solution: $125x^3-216y^3$

$=(5x)^3-(6y)^3$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$=(5x-6y)[(5x)^2+(5x)(6y)+(6y)^2]$

$= (5x - 6y)(25x^2 + 30xy + 36y^2)$

(iii) $64x^3 + 27y^3$

Solution: $64x^3 + 27y^3$

$= (4x)^3 + (3y)^3$

$(a + b)(a^2 + ab + b^2) = a^3 + b^3$

$= (4x + 3y)\left[(4x)^2 - (4x)(3y) + (3y)^2\right]$

$= (4x + 3y)(16x^2 - 12xy + 9y^2)$

(iv) $(2x)^3 + (5y)^3$

Solution: $(2x)^3 + (5y)^3$

$(a - b)(a^2 + ab + b^2) = a^3 - b^3$

$= (2x + 5y)\left[(2x)^2 - (2x)(5y) + (5y)^2\right]$

$= (2x + 5y)(4x^2 - 10xy + 25y^2)$

Exercise 5.3

Q.1 Use the remainder theorem to find the remainder when

(i) $3x^3-10x^2+13x-6$ **is divided by** $(x-2)$.

Solution:

$P(x)=3x^3-10x^2+13x-6$

Since $P(x)$ is divided by $(x-2)$.

$\therefore P(2)=R$

$R=3(2)^3-10(2)^2+13(2)-6$

$=3(2)^3-10(2)^2+13(2)-6$

$=24-40+26-6$

R=4

Hence 4 is the remainder

(ii) $4x^3-4x+3$ **is divided by** $(2x-1)$

Solution:

$P(x)=4x^3-4x+3$

Since $P(x)$ is divided by (2x-1)

$\therefore R=P\left(\frac{1}{2}\right)$

$=4\left[\frac{1}{2}\right]^3-\not{4}^2\times\frac{1}{\not{2}}+3$

$=\not{4}\times\frac{1}{\not{8}^2}-2+3$

$=\frac{1}{2}-2+3$

$=\frac{1-4+6}{2}=\frac{3}{2}$

$R=\frac{3}{2}$

Hence $\frac{3}{2}$ is the remainder

(iii) $6x^4+2x^3-x+2$ **is divided by** $(x+2)$ **from** $x+2=0$

Solution: Given that

$P(x)=6x^4+2x^3-x+2$

Since $P(x)$ is divided by $(x+2)$

$\therefore R=P(-2)$

$=6(-2)^4+2(-2)^3-(-2)+2$

=96-16+2+2

R=84

Hence 84 is the remainder

(iv) $(2x-1)^3+6(3+4x)^2-10$ **is divided by** $2x+1$ **from** $2x+1=0$

$x=-\frac{1}{2}$

Solution: Given that

$P(x)=(2x-1)^3+6(3+4x)^2-10$

Since $P(x)$ is divided by $2x+1$

$\therefore R=P\left(-\frac{1}{2}\right)$

$=\left[\not{2}\left(-\frac{1}{\not{2}}\right)-1\right]^3+6\left[3+\not{4}^2\left(\frac{-1}{\not{2}}\right)\right]^2-10$

$=[-1-1]^3+6[3-2]^2-10$

$=[-2]^3+6-10=-8+6-10$

$R=-12$

Hence -12 is the remainder

(v) $x^3-3x^2+4x-14$ **is divided by** $(x+2)$ **from** $x+2=0, x=-2$

Solution: Given that

$P(x)=x^3-3x^2+4x-14$

Since $P(x)$ is divided by $(x+2)$

$\therefore R=P(-2)$

$=(-2)^3-3(-2)^2+4(-2)-14$

$=-8-12-8-14$

$R=-42$

Hence -42 is the remainder

Q.2

(i) If (x+2) is a factor of $3x^2-4kx-4k^2$ then find the values of k $x+2=0$ $x=-2$

Solution: Given that

$P(x)=3x^2-4kx-4k^2$

P(-2)=$3(-2)^2-4k(-2)-4k^2$

P(-2)=12+8k-4k^2

If (x+2) is the factor then remainder is equal to zero

P(-2)=0

$12+8k-4k^2=0$

$4(3+2k-k^2)=0$

$-k^2+2k+3=\frac{0}{4}$

$-k^2+3k-k+3=0$

$-k(k-3)-1(k-3)=0$

$(k-3)(-k-1)=0$

$k-3=0$ $\quad$ $-k-1=0$

$k=3$ $\quad$ $-1=k$

$k=-1$

(ii) If (x-1) is a factor of $x^3-kx^2+11x-6$ the find the value of k from $x-1=0$ $x=1$

Solution: Given that

$P(x)=x^3-kx^2+11x-6$

P(1)=$(1)^3-k(1)^2+11(1)-6$

P(1)=$1-k+11-6$

P(1)=$6-k$

If (x-1) is the factor then remainder is equal to zero

$P(1)=0$

6-k=0

k=6

Q.3 Without long division determine whether

(i) $(x-2)$ **and** $(x-3)$ **are factor of** $P(x)=x^3-12x^2+44x-48$ **from** $x-2=0$ $x=2$

Solution: Given that

$P(x)=x^3-12x^2+44x-48$

If (x-2) is the factor then remainder is equal to zero

P(2)=$(2)^3-12(2)^2+44(2)-48=8-48+88-48=0$

Hence $x-2$ is a factor of $P(x)$

For $x-3$

$R=P(3)$

=$(3)^3$-12$(3)^2$+44(3)-48

=$(3)^3$-12$(3)^2$+44(3)-48

=27-108+132-48

=159-156

R=3

3 is remainder hence $x-3$ is not factor of $P(x)$

P(3) is not equal to zero then x-3 is not factor of $P(x)=x3-12x^2+44x-48$

(ii) $(x-2)$, $(x+3)$ **and** $(x-4)$ **are factor of** $q(x)=x^3+2x^2-5x-6$ **from** $x-2=0$, $x=2$

Solution: Given that

$q(x)=x^3+2x^2-5x-6$

For (x-2), putt x-2=0

$x=2$

$R=q(2)$

$=(2)^3+2(2)^2-5(2)-6$

$R=8+8-10-6$

$R=16-16$

$R=0$

Hence $x-2$ is factor of $q(x)=x^3+2x^2-5x-6$

For (x+3), putt x+3=0

x=-3

$R=q(-3)$

$=(-3)^3+2(-3)^2-5(-3)-6$

$=-27+18+15-6$

$R=0$

Hence x-2 is factor of $q(x)=x^3+2x^2-5x-6$

For x-4, x-4=0

x=4

$R=q(4)$

$=(4)^3+2(4)^2-5(4)-6$

$=64+32-20-6$

R=70

Hence x-4 is not a factor of $q(x)=x^3+2x^2-5x-6$

Q.4 For what value of m is the polynomial P(x) =4x^3-7x^2+6x-3m exactly divisible by x+2?

Solution:

$P(x)=4x^3-7x^2+6x-3m$

From x+2=0, x=-2

P(-2)=4(-2)^3-7(-2)^2 $+6(-2)-3m$

P(-2)=-32-28-12-3m=-72-3m

If (x+2) is the factor then remainder is equal to zero

$P(-2)=0$

$-72-3m=0$

$-72=3m$

$m=-\frac{72}{3}$

$m=-24$

Q.5 Determine the value of k if $P(x)=kx^3+4x^2+3x-4$ and $q(x)=x^3-4x+k$ leaves the same remainder when divided by $(x-3)$.

Solution:

$q(x)=x^3-4x+k$

from x-3=0 x=3

$R_1=q(3)$

$=(3)^3-4(3)+k$

$=27-12+k$

$=15+k$

$R_1 =15+k$ ....(i)

$R_2=P(3)$

$=k(3)^3+4(3)^2+3(3)-4$

$=27k+36+9-4$

$R_2 =27k+41$ ....(ii)

Since it leaves the same remainder.

Hence $R_1=R_2$

$15+k=27k+41$

$15-41=27k-k$

$-26=26k$

$k=\frac{-26}{26}$

$k=-1$

Q.6 The remainder after dividing the polynomial $P(x)=x^3+ax^2+7$ by $(x+1)$ is $2b$ calculate the value of a and b if this expression leaves a remainder of $(b+5)$ on being dividing by $(x-2)$

Solution:

Let

$P(x)=x^3+ax^2+7$

Since $P(x)$ is divided by (x+1)

Put $x+1=0$ $x=-1$

R=P(−1)

$=(-1)^3+a(-1)^2+7$

$=-1+a+7$

$R=a+6$

According to first condition remainder is 2b

$2b=a+6$ ...(i)

Since $P(x)$ is divided by (x-2)

Put $x-2=0$

$x=2$

P(2)=(2)^3+a(2)^2+7

=8+4a+7

R=15+4a

According to second condition remainder is (b+5)

15+4a=b+5

4a-b=5-15

4a-b=-10 ...(ii)

Solving equations (i) and (ii)
From equation (ii) b=10+4a putting the value of be in equation (i)
a+6=2(10+4a)
a=20+8a-6
-8a+a=14
-7a=14

$$a=\frac{14}{-7}$$

a=-2
Putting the value of a in equation (ii)

$$4a-b=-10$$
$$4(-2)-b=-10$$
$$-8-b=-10$$
$$-8+10=b$$
$$2=b$$
$$b=2$$

Q.7 The polynomial $x^3+lx^2+mx+24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$
Find the values of *l* and *m*.

Solution:
Let

$$P(x)=x^3+lx^2+mx+24$$

From $x+4=0$ $\quad x=-4$

$$P(-4)=(-4)^3+l(-4)^2+m(-4)+24$$
$$P(-4)=-64+16l-4m+24$$
$$P(-4)=16l-4m-40$$

According to condition (*x*+4) is the factor then

$$16l-4m-40=0$$
$$4[4l-m-10]=0$$
$$4l-m-10=0 \qquad \text{(i)}$$

from $x-2=0$ $\quad x=2$

Now $P(2)=(2)^3+l(2)^2+m(2)+24$

$$P(2)=8+4l+2m+24$$
$$P(2)=4l+2m+32$$

According the condition

$$4l+2m+32=36$$
$$4l+2m=36-32$$
$$4l+2m=4$$
$$4l+2m-4=0 \qquad \text{(ii)}$$

Subtracting (i) from (ii)

$$\begin{array}{r} \not{4}l+2m-4=0 \\ \underline{\pm\not{4}l\mp m\mp 10=0} \\ 3m+6=0 \end{array}$$

$$3m+6=0$$
$$3m=-6$$
$$m=\frac{-\not{6}\,2}{\not{3}}$$
$$m=-2$$

Putting the value of m is equation (i)

$$4l-(-2)-10=0$$
$$4l+2-10=0$$
$$4l-8=0$$
$$4l=8$$
$$l=\frac{2\not{8}}{\not{4}}$$
$$l=2$$

Q.8 The expression lx^3+mx^2-7 leaves remainder of -3 and 12 when divided by $(x-1)$ and $(x+2)$ respectively. Calculate the value of *l* and *m*.

Solution:

$$P(x)=lx^3+mx^2-7$$

from *x*-1=0 $\quad$ *x*=1

$$P(1)=l(1)^3+m(1)^2-4$$
$$P(1)=l+m-4$$

According to conditions *l*+*m*-4=-3

$$l+m=4-3$$
$$l=1-m \qquad \text{(i)}$$

From $x+2=0$ $\quad x=-2$

$$P(-2)=l(-2)^3+m(-2)^2-4$$
$$P(-2)=-8l+4m-4$$

According to condition

$$-8l+4m-4=12$$

Putting the value of *l* in the equation

$-8[1-m]+4m=16$

$-8+8m+4m=16$

$12m=16+8$

$12m=24$

$m=\frac{24^2}{12}$

$m=2$

Putting the value of m is equation (i)

$l=1-2$

$l=-1$

$m=2$

$l=-1$

Q.9 The expression $ax^3-9x^2+bx+3a$ is exactly divisible by x^2-5x+6. Find the value of a and b.

Solution: Given that

$P(x)=ax^3-9x^2+bx+3a$

$x^2-5x+6 \quad =x^2-2x-3x+6$

$=x[x-2]-3[x-2]$

$=[x-2][x-3]$

$(x-2)(x-3)$ is divides the expression $ax^3-9x^2+bx+3a$ from $x-2=0$, $x=2$

$P(2)=a(2)^3-9(2)^2+b(2)+3a$

$P(2)=8a-36+2b+3a$

$P(2)=11a+2b-36$

According to condition (x-2) is the factor so

11a+2b-36=0 (i)

From x-3=0,x=3

$P(3)=a(3)^3-9(3)^2+b(3)+3a$

$P(3)=27a-81+3b+3a$

$P(3)=30a+3b-81$

According to condition (x-3) is the factor so

$30a+3b-81=0$ (ii)

$3(10a+b-27)=0$

$10a+b-27=\frac{0}{3}$

$b=27-10a$ (iii)

Putting the value of b in equation (i)

$11a+2[27-10a)-36=0$

$11a+54-20a-36=0$

$-9a+18=0$

$+18=9a$

$a=\frac{+18\,2}{9}$

$a=+2$

Putting the value of a in equation (iii)

$b=27-10(+2)$

$b=27-20$

$b=7$

$a=2$

Exercise 5.4

Q.1 x^3-2x^2-x+2

Solution: Given that

$P(x)=x^3-2x^2-x+2$

P=2 and possible factor of 2 are $\pm1,\pm2$.

Here q=1 and possible factor of 1 are ±1.

So possible factor of P(x) will be form $\frac{P}{q}=\pm1,\pm2$

$P(x)=x^3-2x^2-x+2$

Put x=1

$P(1)=(1)^3-2(1)^2-1+2 \quad =1-2-1+2=0$ Remainder is equal to zero so (x-1) is factor

Put x=-1

$P(-1)=(-1)^3-2(-1)^2-(-1)+2=-1-2+1+2=0$ Remainder is equal to zero so (x+1) is factor

Put x=2

$P(2)=(2)^3-2(2)^2-(2)+2=8-8-2+2=0$ Remainder is equal to zero so (x-2) is factor

$x^3-2x^2-x+2=(x-1)(x+1)(x-2)$

Q.2 $x^3-x^2-22x+40$

Solution: Given that

$P(x)=x^3-x^2-22x+40$

P=40

possible factor of 40= $\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

Here q=1 and possible factor of 1are ±1

So possible factor of P(x) will be from

$\frac{P}{q}=\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

$P(x)=x^3-x^2-22x+40$

Put $x=2$

$P(2)=(2)^3-(2)^2-22(2)+40$

$=8-4-44+40=0$

Remainder is equal to zero so (x-2) is a factor

Put x=4

$P(4)=(4)^3-(4)^2-22(4)+40$

$=64-16-88+40=0$

Remainder is not equal to zero so (x-4) is a factor

Put x=-5

$P(-5)=(-5)^3-(-5)^2-22(-5)+40$

$=-125-25+110+40$

$=-150+150$

$=0$

Remainder is equal to zero so (x+5) is a factor

Hence $x^3-x^2-22x+40=(x-2)(x-4)(x+5)$

Q.3 $x^3-6x^2+3x+10$

Solution: Given that

$P(x)=x^3-6x^2+3x+10$

P=10

So possible factor of 10 are $\pm1,\pm2,\pm5,\pm10$

Here q=1 So, possible factor of 1are ±1.

So possible of factor of P(x) can be found from $\frac{P}{q}=\pm1,\pm2,\pm5,\pm10$

P(x)=x^3-6x^2+3x+10

Put x=-1

P(-1)=(-1)^3-6(-1)^2+3(-1)+10=-1-6-3+10=0

Remainder is equal to zero so (x+1) is a factor

Put x=2

P(2)=(2)^3-6(2)^2+3(2)+10=8-24+6+10=0

Remainder is equal to zero so (x-2) is a factor

Put x=5

$P(5)=(5)^3-6(5)^2+3(5)+10=125-150+15+10=0$Remainder is equal to zero so (x-5) is a factor

Hence $x^3-6x^2+3x+10=(x+1)(x-2)(x-5)$

Q.4 $x^3+x^2-10x+8$

Solution: Given that

P(x)=x^3+x^2-10x+8

P=8 So possible factor of 8

are $\pm1,\pm2,\pm3,\pm4,\pm5,\pm6,\pm7,\pm8$.

Here q=1 So possible factor can be found from $\frac{P}{q}=\pm1,\pm2,\pm4,\pm8$

P(x)=x^3+x^2-10x+8

Put x=1

$P(1)=(1)^3+(1)^2-10(1)+8=1+1-10+8=0$

Remainder is equal to zero so (x-1) is a factor

Put x=2

$P(2)=2^3+2^2-10(2)+8$

$=8+4-20+8$

$=20-20$

$=0$

Remainder is equal to zero so (x-2) is a factor

Put x=-4

P(-4) $=(-4)^3+(-4)^2-10(-4)+8$

$=-64+16+40+8$

$=-64+64$

$=0$

Remainder is equal to zero so (x+4) is a factor

Hence $x^3+x^2-10x+8=(x-1)(x-2)(x+4)$

Q.5 x^3-2x^2-5x+6

Solution: Given that

$P(x)=x^3-2x^2-5x+6$

$P=6$ So factors of 3 are $\pm1, \pm2, \pm3, \pm6$

Here q=1 So factors of 1 are $\pm$ 1.

So possible factors of P(x) can be found from $\frac{P}{q}=\pm1, \pm2, \pm3, \pm6$

P(x)=x^3-2x^2-5x+6

Put x=1

P(1) $=(1)^3-2(1)^2-5(1)+6$

$=1-2-5+6$

$=-7+7$

$=0$

Remainder is equal to zero so (x-1) is a factor

Put x=-2

$P(-2)=(-2)^3-2(-2)^2-5(-2)+6$

$=-8-8+10+6$

$=-16+16$

$=0$

Remainder is equal to zero so (x+2) is a factor

Put x=3

$P(3)=(3)^3-2(3)^2-5(3)+6$

$=27-6-15+6$

27-27

=0

Remainder is equal to zero so (x-3) is a factor

Hence $x^3-2x^2-5x+6=(x-1)(x+2)(x-3)$

Q.6 $x^3+5x^2-2x-24$

Solution: Given that

$P(x)=x^3+5x^2-2x-24$

P= -24 So possible factors of 24 are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Here q=1. So possible factors of 1 are $\pm$ 1.

So possible factors of P(x) will be found from

$\frac{P}{q}=\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$P(x)=x^3+5x^2-2x-24$
Put $x=2$
$P(2)=(2)^3+5(2)^2-2(2)-24$
$=8+20-4-24$
$=28-28$
$=0$
Remainder is equal to zero so $(x-2)$ is a factor
Put $x=-3$
$P(-3) \quad =(-3)^3+5(-3)^2-2(-3)-24$
$=-27+45+6-24$
$=-51+51$
$=0$
Remainder is equal to zero so $(x+3)$ is a factor
Put $x=-4$
$P(-4) \quad =(-4)^3+5(-4)^2-2(-4)-24$
$=-64+80+8-24$
$=-88+88$
$=0$
Remainder is equal to zero so $(x+4)$ is a factor
Hence $x^3+5x^2-2x-24=(x-2)(x+3)(x+4)$

Q.7 $\quad 3x^3-x^2-12x+4$
Solution: Given that
$P(x)=3x^3-x^2-12x+4$
P=4 So possible factors of 4 are $\pm1,\pm2,\pm4$.
Here q=3 So possible factors of 3 are $\pm1,\pm3$.
So possible factors of P(x) can be found from
$$\frac{P}{q}=\pm1,\pm2,\pm4,\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$$
Put $x=2$
$P(2)=3(2)^3-(2)^2-12(2)+4$
$=-24-4+24+4$
$=28-28$
$=0$
Remainder is equal to zero so $(x-2)$ is a factor
Put $x=-2$
$P(-2)=3(-2)^3-(-2)^2-12(-2)+4$
$=-24-4+24+4$
$=28-28$
$=0$
Remainder is equal to zero so $(x+2)$ is a factor
Put $x=\frac{1}{3}$
$$P\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3-\left(\frac{1}{3}\right)^2-\cancel{12}^{4}\left(\frac{1}{3}\right)+4$$

$$= \not{3}\left(\frac{1}{\not{27}9}\right) - \frac{1}{9} - \not{4} + \not{4}$$

$$P\left(\frac{1}{3}\right) = \frac{1}{\not{9}} - \frac{1}{\not{9}}$$

$$= 0$$

$$x = \frac{1}{3} \Rightarrow \begin{matrix} 3x = 1 \\ 3x - 1 \end{matrix}$$

Remainder is equal to zero so (3x-1) is a factor

Hence $3x^3 - x^2 - 10x + 4 = (x-2)(x+2)(3x-1)$

Q.8 $2x^3 + x^2 - 2x - 1$

Solution: Given that

$P(x)=2x^3+x^2-2x-1$

P= -1So possible factors of -1 are $\pm 1, \pm 2$.

Here q=1. So possible factors of P(x) will be found from $\frac{P}{q}$

$$\frac{P}{q} = \pm 1, \pm 2, \pm \frac{1}{2}$$

$P(x)=2x^3+x^2-2x-1$

Put x=1

P(1) $=2(1)^3+(1)^2-2(1)-1$

=2+1-2-1

=3-3

=0

Remainder is equal to zero (x-1) is a factor

Put x=-1

P(-1) $=2(-1)^3+(-1)^2-2(-1)-1$

=2+1-2-1

=3-3

=0

Remainder is equal to zero (x+1) is a factor

Put $x = \frac{-1}{2}$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{2}\right]^3 + \left[\frac{-1}{2}\right]^2 - \cancel{2}\left[\frac{-1}{\cancel{2}}\right] - 1$$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{\cancel{8}\,4}\right] + \frac{1}{4} + \cancel{1} - \cancel{1}$$

$$P\left(\frac{-1}{2}\right) = -\cancel{\frac{1}{4}} + \cancel{\frac{1}{4}}$$

$$= 0$$

$$x = -\frac{1}{2}$$

$$2x = -1$$

$$2x + 1 = 0$$

Remainder is equal to zero so $(2x+1)$ is a factor

Hence $2x^3 + x^2 - 2x - 1 = (x-1)(x+1)(2x+1)$

Review Exercise 5

Q.1 Filling the blanks:

1. **The factor of** x^2-5x+6 **are _________.**

(a) $x+1, x-6$ **(b)** $x-2, x-3$

(c) $x+6, x-1$ **(d)** $x+2, x+3$

2. **Factors of** $8x^3+27y^3$ **are ___________.**

(a) $(2x-3y),(4x^2+9y^2)$ **(b)** $(2x-3y),(4x^2-9y^2)$

(c) $(2x+3y),(4x^2-6xy+9y^2)$ **(d)** $(2x-3y),(4x^2+6xy+9y^2)$

3. **Factors of** $3x^2-x-2$ **are___________.**

(a) $(x+1),(3x-2)$ **(b)** $(x+1),(3x+2)$

(c) $(x-1),(3x-2)$ **(d)** $(x-1),(3x+2)$

4. **Factors of** a^4-4b^4 **are__________.**

(a) $(a-b),(a+b),(a^2+4b^2)$ **(b)**$(a^2-2b^2),(a^2+2b^2)$

(c) $(a-b),(a+b)(a^2+4b^2)$ **(d)** $(a-2b),(a^2+2b^2)$

5. **What will be added to complete the square of** $9a^2-12ab$ **?.....**

(a) $-16b^2$ **(b)** $16b^2$ **(c)** $4b^2$ **(d)** $-4b^2$

6. **Find m so that** x^2+4x+m **is a complete square**

(a) 8 **(b)** –8

(c) 4 **(d)** 16

7. **Factors of** $5x^2-17xy-12y^2$ **are__________.**

(a) $(x+4y),(5x+3y)$ **(b)** $(x-4y),(5x-3y)$

(c) $(x-4y),(5x+3y)$ **(d)** $(5x-4y),(x+3y)$

8. **Factors of** $27x^3-\frac{1}{x^3}$ **are**

(a) $\left(3x-\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$ **(b)** $\left(3x+\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$

(c) $\left(3x-\frac{1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$ **(d)** $\left(\frac{3x+1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$

ANSWERS KEYS

1	2	3	4	5	6	7	8
b	c	d	b	c	c	c	a

Q.2 Completion items

(i) $x^2+5x+6=$ ____________

(ii) $4a^2-16=$ ____________

(iii) $4a^2+4ab+($_____$)$ **is a complete square.**

(iv) $\frac{x^2}{y^2}-2+\frac{y^2}{x^2}=$ __________

(v) $(x+y)(x^2-xy+y^2)=$ ____________

(vi) **Factored form of** x^4-16 **is** ____________

(vii) **If x–2 is factor of** $P(x)=x^2+2kx+8$ **then** $=$ ____________

ANSWER KEYS

(i) $(x+3)(x+2)$

(ii) $(2a+4)(2a-4)=4(a+2)(a-2)$

(iii) $(b)^2$

(iv) $\left(\frac{x}{y}-\frac{y}{x}\right)^2$

(v) x^3+y^3

(vi) $(x+2)(x-2)(x^2+4)$

(vii) -3

Q.3 Factorize the following

(i) $x^2+8x+16-4y^2$

Solution: $x^2+8x+16-4y^2$

$=\left[x^2+8x+16\right]-4y^2$

$=\left[(x)^2+2(x)(4)+(4)^2\right]-(2y)^2$

$=(x+4)^2-(2y)^2$

Now arrange them

$=(x+4+2y)(x+4-2y)$

$=(x+2y+4)(x-2y+4)$

(ii) $4x^2-16y^2$

Solution: $4x^2-16y^2$

$=4\left[x^2-4y^2\right]$

$=4\left[(x)^2-(2y)^2\right]$

$=4(x-2y)(x+2y)$

(iii) $9x^2+24x+3x+8$

Solution: $=9x^2+24x+3x+8$

$=3x(3x+8)+1(3x+8)$

$=(3x+8)(3x+1)$

(iv) $1-64z^3$

Solution: $1-64z^3$

$=(1)^3-(4z)^3$

$=(1-4z)\left[(1)^2+(1)(4z)+(4z)^2\right]$

$=(1-4z)(1+4z+16z^2)$

(v) $8x^3-\left(\frac{1}{3y}\right)^3$

$=(2x)^3-\left(\frac{1}{3y}\right)^3$

$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)$

(vi) $2y^2+5y-3$

Solution: $=2y^2+6y-y-3$

$=2y(y+3)-1(y+3)$

$=(2y-1)(y+3)$

(vii) x^3+x^2-4x-4

Solution: x^3+x^2-4x-4

$=x^2(x+1)-4(x+1)$

$=(x+1)(x^2-4)$

$=(x+1)(x-2)(x+2)$

(viii) $25m^2n^2+10mn+1$

Solution: $25m^2n^2+10mn+1$

$=(5mn)^2+2(5mn)(1)+(1)^2$

$=(5mn+1)^2$

(ix) $1-12pq+36p^2q^2$

Solution: $1-12pq+36p^2q^2$

$\therefore (a)^2-2ab+(b)^2$

$=(1)^2-2(1)(6pq)+(6pq)^2$

$=(1-6pq)^2$

Unit 5: Factorization

Overview

Factorization:

The process of expressing an algebraic expression in terms of its factors is called factorization.

Remainder Theorem:

If a polynomial $p(x)$ is divided by a linear divisor $(x-a)$, then the remainder is $p(a)$.

Zero of a Polynomial:

If a specific number $x=a$ is substituted for the variable x in a polynomial $p(x)$ so that the value $p(a)$ is zero, then $x=a$ is called a zero of the polynomial $p(x)$.

Factor Theorem:

The polynomial $(x-a)$ is a factor of the polynomial $p(x)$ if and only if $p(a)=0$.

Rational Root Theorem:

Let $a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n=0,\ a_0\neq 0$ be a polynomial equation of degree n with integral coefficients. If $\frac{p}{q}$ is a rational root (expressed in lowest terms) of the equation, then p is a factor of the constant term a_n and q is a factor of the leading coefficient a_0.