Exercise 6.1

Q.1 Find the H.C.F of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3\times13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 7\times13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

H.C.F $=13\times x.x.x.x.x.y.y.y.z$

H.C.F = $13x^5y^2z$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times3\times17\times x.y.y.z$

$85x^2yz = 5\times17\times x.x.y.z$

$187xyz^2 = 11\times17\times x.y.z.z$

H.C.F $=17xyz$

Q.2 Find the H.C.F of the following expression by factorization.

(i) $x^2+5x+6, x^2-4x-12$

Solution: $x^2+5x+6, x^2-4x-12$

Factorizing x^2+5x+6

$=x^2+3x+2x+6$

$=x(x+3)+2(x+3)$

$=(x+3)(x+2)$

Factorizing $x^2-4x-12$

$=x^2-6x+2x-12$

$=x(x-6)+2(x-6)$

$=(x-6)(x+2)$

So,

H.C.F = $(x+2)$

(ii) $x^2-27, x^2+6x-27, 2x^2-18$

Solution: $x^2-27, x^2+6x-27, 2x^2-18$

Factorizing $x^3 - 27$

$= (x)^3 - (3)^3$

$= (x-3)\left[(x)^2 + (x)(3) + (3)^2\right]$

$= (x-3)(x^2 + 3x + 9)$

Factorizing $x^2 + 6x - 27$

$= x^2 + 9x - 3x - 27$

$= x(x+9) - 3(x+9)$

$= (x+9)(x-3)$

Factorizing $2x^2 - 18$

$= 2(x^2 - 9)$

$= 2\left[(x)^2 - (3)^2\right]$

$= 2(x-3)(x+3)$

So,

H.C.F = $(x-3)$

(iii) $x^3 - 2x^2 + x, x^2 + 2x - 3, x^2 + 3x - 4$

Factorizing $x^3 - 2x^2 + x$

$= x(x^2 - 2x + 1)$

$x(x^2 - x - x + 1)$

$= x\left[x(x-1) - 1(x-1)\right]$

$= x(x-1)(x-1)$

Factorizing $x^2 + 2x - 3$

$= x^2 + 3x - x - 3$

$= x(x+3) - 1(x+3)$

$= (x+3)(x-1)$

Factorizing $x^2 + 3x - 4$

$= x^2 + 4x - x - 4$

$= x(x+4) - 1(x+4)$

$= (x+4)(x-1)$

So,

H.C.F $= (x-1)$

(iv) $18(x^3 - 9x^2 + 8x), 24(x^2 + 3x + 2)$

Solution: $18(x^3 - 9x^2 + 8x), 24(x^2 + 3x + 2)$

Factorizing $18(x^3 - 9x^2 + 8x)$

$= 6 \times 3 \times x(x^2 - 9x + 8)$

$= 6\times 3\times x\left(x^2-8x-x+8\right)$

$= 6\times 3\times x\left[x\left(x-8\right)-1\left(x-8\right)\right]$

$= 6\times 3\times x\left(x-8\right)\left(x-1\right)$

Factorizing $24\left(x^2+3x+2\right)$

$= 6\times 4\left(x^2-3x+2\right)$

$= 6\times 4\left(x^2-2x-x+2\right)$

$= 6\times 4\left[x\left(x-2\right)-1\left(x-2\right)\right]$

$= 6\times 4\left(x-2\right)\left(x-1\right)$

So,

H.C.F = $6\left(x-1\right)$

(v) $36\left(3x^4+5x^2-2x^2\right), 54\left(27x^4-x\right)$

Factorizing $36\left(3x^4+6x^3-2x^2\right)$

$= 3\times 3\times 2\times 2\times x^2\left(3x^2+5x-2\right)$

$= 3\times 3\times 2\times 2\times x^2\left(3x^2+6x-x-2\right)$

$= 3\times 3\times 2\times 2\times x^2\left[3x\left(x+2\right)-1\left(x+2\right)\right]$

$= 3\times 3\times 2\times 2\times x^2\left(x+2\right)\left(3x-1\right)$

Factorizing $54\left(27x^4-x\right)$

$= 3\times 3\times 3\times 2\times x\left(27x^3-1\right)$

$= 3\times 3\times 3\times 2\times x\left[\left(3x\right)^3-\left(1\right)\right]$

$= 3\times 3\times 3\times 2\times x\left(3x-1\right)\left[\left(3x\right)^2+\left(3x\right)\left(1\right)+\left(1\right)^2\right]$

$= 3\times 3\times 3\times 2\times x\left(3x-1\right)\left(9x^2+3x+1\right)$

So,

H.C.F = $3\times 3\times 2\times x\left(3x-1\right)$

$= 18x\left(3x-1\right)$

Q.3 Find the H.C.F of the following by division method.

(i) $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

Solution: $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

$$
\begin{array}{r|l}
 & 1 \\
x^3+x^2-10x+8 & x^3+3x^2-16x+12 \\
 & \underline{\pm x^3\pm x^2\mp 10x\pm 8} \\
 & 2x^2-6x+4 \\
 & 2\left(x^2-3x+2\right)
\end{array}
$$

$$
\begin{array}{r|l}
 & x+4 \\
x^2-3x+2 & \cancel{x^3}+x^2-10x+8 \\
 & \underline{\cancel{-x^3}\mp 3x^2\pm 2x} \\
 & 4x^2-12x+8 \\
 & \underline{\pm 4x^2\mp 12x\pm 8} \\
 & \times
\end{array}
$$

H.C.F $=\left(x^2-3x+2\right)$

(ii) $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

Solution: $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

$$
\begin{array}{r|l}
 & x+2 \\
5x^3+3x^2-17x+6 & x^4+x^3-2x^2+x-3 \\
 & \underline{\times 5} \\
 & 5x^4+5x^3-10x^2+5x-15 \\
 & \underline{\pm 5x^4\pm 3x^3\mp 17x^2\pm 6x} \\
 & 2x^3+7x^2-x-15 \\
 & \underline{\times 5} \\
 & 10x^3+35x^2-5x-75 \\
 & \underline{\pm 10x^3\pm 6x^2\mp 34x\pm 12} \\
 & 29x^2+29x-87 \\
 & 29(x^2+x-3)
\end{array}
$$

$$
\begin{array}{r|l}
 & 5x-2 \\
x^2+x-3 & 5x^3+3x^2-17x+6 \\
 & \underline{\pm 5x^3\pm 5x^2\mp 15x} \\
 & -2x^2-2x+6 \\
 & \underline{\mp 2x^2\mp 2x\pm 6} \\
 & \times
\end{array}
$$

H.C.F $=\left(x^2+x-3\right)$

(iii) $2x^5-4x^4-6x, x^5+x^4-3x^3-3x^2$

$$\begin{array}{r} 1 \\ 2x^5-4x^4-6x\overline{)\,x^5+x^4-3x^3-3x^2} \end{array}$$

$$\times 2$$

$$\overline{2x^5+2x^4-6x^3-6x^2}$$

$$\underline{-2x^5 \mp 4x^4 \qquad \mp 6x}$$

$$6x^4-6x^3-6x^2+6x$$

$$6\left(x^4-x^3-x^2+x\right)$$

$$\begin{array}{r} 2x-2 \\ x^4-x^3-x^2+x\overline{)\,2x^5-4x^2-6x} \end{array}$$

$$\underline{\pm 2x^5 \pm 2x^2 \qquad \mp 2x^4 \mp 2x^3}$$

$$-2x^4+2x^3-2x^2-6x$$

$$\underline{\mp 2x^4 \pm 2x^3 \pm 2x^2 \mp 2x}$$

$$-4x^2-4x$$

$$-4\left(x^2+x\right)$$

$$\begin{array}{r} x^2-2x+1 \\ x^2+x\overline{)\,x^4-x^3-x^2+x} \end{array}$$

$$\underline{-x^4 \pm x^3}$$

$$-2x^3-x^2+x$$

$$\underline{\mp 2x^3 \mp 2x^2}$$

$$x^2+x$$

$$\underline{\pm x^2 \pm x}$$

$$\times$$

H.C.F $=x^2+x$

Q.4 Find the L.C.M of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3\times 13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 7\times 13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

Common$=13x^5y^8z$

Uncommon$=3\times 7\times x^2y^3z^6$

$= 21x^2y^3z^6$

L.C.M= common factors$\times$uncommon factors

$=13x^5y^3z\times 21x^2y^3z^6$

$273x^7y^6z^7$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102\,xy^2z = 2\times3\times17.x.y.y.z$

$85\,x^2yz = 5\times17\times x.x.y.z$

$187\,xyz^2 = 11\times17.x.y.z.z$

Common=$17xyz$

Uncommon=$2\times3\times5\times11.xyz$

=$330\;xyz$

L.C.M=common x uncommon

$=17xyz\times330xyz$

$=5610x^2y^2z^2$

Q.5 Find the L.C.M of the following by factorizing.

(i) $x^2-25x+100$ **and** x^2-x-20

Solution: $x^2-25x+100$ **and** x^2-x-20

Factorizing $x^2-25x+100$

$=x^2-20x-5x+100$

$=x(x-20)-5(x-20)$

$=(x-20)(x-5)$

Factorizing x^2-x-20

$=x^2-5x+4x-20$

$=x(x-5)+4(x-5)$

$=(x-5)(x+4)$

So,

L.C.M = $(x-5)(x+4)(x-20)$

(ii) $x^2+4x+4, x^2-4, 2x^2+x-6$

Solution: $x^2+4x+4, x^2-4, 2x^2+x-6$

Factorizing x^2+4x+4

$=x^2+2x+2x+4$

$=x(x+2)+2(x+2)$

$=(x+2)(x+2)$

Factorizing x^2-4

$=(x)^2-(2)^2$

$=(x-2)(x+2)$

Factorizing $2x^2+x-6$

$=2x^2+4x-3-6$

$=2x(x+2)-3(x+2)$

$=(x+2)(2x-3)$

So,

$$\text{L.C.M} =(x+2)(x+2)(x-2)(2x-3)$$

$$=(x+2)^2(x-2)(2x-3)$$

(iii) $2(x^4-y^4),3(x^3+2x^2y-xy^2-2y^3)$

Factorizing $2(x^4-y^4)$

$$=2\left[(x^2)^2-(y^2)^2\right]$$

$$=2(x^2+y^2)(x^2-y^2)$$

$$=2(x^2+y^2)(x+y)(x-y)$$

Factorizing $3(x^3+2x^2y-xy^2-2y^3)$

$$=3\left[x^2(x+2y)-y^2(x+2y)\right]$$

$$=3(x+2y)(x^2-y^2)$$

$$=3(x+2y)(x+y)(x-y)$$

So,

$$\text{L.C.M}=(x+y)(x-y)(x^2+y^2)(x+2y)\times 2\times 3$$

$$=6(x+y)(x-y)(x^2+y^2)(x+2y)$$

$$=6(x+2y)(x^4-y^4)$$

(iv) $4(x^4-1),6(x^3-x^2-x+1)$

Solution: $4(x^4-1),6(x^3-x^2-x+1)$

Factorizing $4(x^4-1)$

$$= \;=2\times 2\left[(x^2)^2-(1)^2\right]$$

$$=2\times 2(x^2+1)(x^2-1)$$

$$=2\times 2(x^2+1)(x+1)(x-1)$$

$$=6(x^3-x^2-x+1)$$

$$=2\times 3\left[x^2(x-1)-1(x-1)\right]$$

$$=2\times 3\left[(x-1)(x^2-1)\right]$$

$$=2\times 3(x-1)(x-1)(x+1)$$

$$\text{L.C.M}=2\times 2\times 3(x-1)(x+1)(x-1)(x^2+1)$$

$$=12(x-1)^2(x+1)(x^2+1)$$

$$=12(x-1)(x^4-1)$$

Q.6 For what value of k is $(x+4)$ the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?

Solution:

$P(x)=x^2+x-(2k+2)$

Since $x+4$ is H.C.F of $P(x)$ and $q(x)$

$\therefore x+4$ is a factor of $P(x)$

Hence $P(-4)=0$

$x^2+x-(2k+2)=0$

By putting the value of x

$(-4)^2+(-4)-(2k+2)=0$

$16-4-2k-2=0$

$-2k+10=0$

$2k=10$

$k=\frac{\cancel{10}^{5}}{\cancel{2}}$

$k=5$

Q.7 If $(x+3)(x-2)$ is the H.C.F of $P(x)=(x+3)(2x^2-3x+k)$ and $q(x)=(x-2)$ $(3x^2+7x-l)$ the find k and l

Solution: $(x-2)$ $(x+3)$ will divide $P(x)=(x+3)$ $(2x^2-3x+K)$

$(x-2)$ $(x+3)$ will divide $P(x)=(x+3)$ $(2x^2-3x+K)$

$x-2=0$

$x=2$

$P(2)=(2+3)(2(2)^2-3(2)+K)$

$P(2)=(5)(2\times 4-6+K)$

$P(2)=5(8-6+K)$

$P(2)=5(2+K)$

Remainder is equal to zero

$5(2+\text{K})=0$

$2+\text{K}=\frac{0}{5}$

$2+\text{K}=0$

$\text{K}=-2$

$q(x)=(x-2)(3x^2+7x-1)$

$(x-2)(x+3)$ will be divide $q\ (x)=(x-2)(3x^2+7x-l)$

$x+3=0$

$x=-3$

$q(-3)=(-3-2)\left[3(-3)^2+7(-3)-l\right]$

$q(-3)=(-5)\left[3(9)-21-l\right]$

$q(-3)=(-5)[27-21-l]$

$q(-3)=(-5)(6-l)$

Remainder is equal to zero

$-5(6-l)=0$

$6-l=0$

$l=6$

Q.8 **The L.C.M and H.C.F of two polynomials** $P(x)$ **and** $q(x)$ **are** $2(x^4-1)$ **and** $(x+1)(x^2+1)$ **respectively. If** $P(x)=x^3+x^2+x+1$, **find** $q(x)$

Solution: $\therefore P(x)\times q(x)=\text{L.C.M}\times\text{H.C.F}$

$\therefore P(x)\times q(x)=\text{L.C.M}\times\text{H.C.F}$

$$q(x)=\frac{L.C.M\times H.C.F}{P(x)}$$

By putting the values

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$$

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^2(x+1)+1(x+1)}$$

$$q(x)=\frac{2(x^4-1)(\cancel{x+1})(\cancel{x^2+1})}{(\cancel{x+1})(\cancel{x^2+1})}$$

$$q(x)=2(x^4-1)$$

Q.9 **Let** $p(x)=10(x^2-9)(x^2-3x+2)$ **and** $q(x)=10x(x+3)(x-1)^2$ **. if the H.C.F of** $p(x),\text{q}(x)$ **is** $10(x+3)(x-1)$**, Find their L.C.M**

Solutions: $p(x)\times q(x)=:L.C.\text{M}\times H.C.F$

$p(x)\times q(x)=:L.C.\text{M}\times H.C.F$

$$L.C.M=\frac{p(x)\times q(x)}{H.C.F}$$

By putting the values

$$L.C.M=\frac{\cancel{10}(x^2-9)(x^2-3x+2)\times 10x(\cancel{x+3})(x-1)^{\cancel{2}}}{\cancel{10}(\cancel{x+3})(\cancel{x-1})}$$

$$L.C.M=10x(x^2-9)(x^2-3x+2)(x-1)$$

Q.10 **Let the product of L.C.M and H.C.F of two polynomial be** $(x+3)^2(x-2)(x+5)$**.** **If one polynomial is** $(x+3)(x-2)$ **and the second polynomial is** $x^2+kx+15$**, find the value of k.**

Solution: $p(x)\times q(x)=L.C.M\times H.C.F$

$p(x)\times q(x)=L.C.M\times H.C.F$

By putting the values

$$(x+3)(x-2)\left(x^2+kx+15\right)=(x+3)^2(x-2)(x+5)$$

$$x^2+kx+15=\frac{(x+3)^{\cancel{2}}\left(\cancel{x-2}\right)(x+5)}{\left(\cancel{x+3}\right)\left(\cancel{x-2}\right)}$$

$$x^2+kx+15=(x+3)(x+5)$$

$$x^2+kx+15=x^2+8x+15$$

$$kx=\cancel{x^2}+8x+\cancel{15}-\cancel{x^2}-\cancel{15}$$

$$kx=8x$$

$$k=\frac{8\cancel{x}}{\cancel{x}}$$

$$k=8$$

Q.11 **Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get fruit in this way.**

Solution

$$\begin{array}{r|l} & 1 \\ 128 & 176 \\ & \underline{128} \end{array}$$

$$\begin{array}{r|l} & 2 \\ 48 & 128 \\ & \underline{-96} \end{array}$$

$$\begin{array}{r|l} 32 & 48 \\ & \underline{-32} \end{array}$$

$$\begin{array}{r|l} & 2 \\ 16 & 32 \\ & \underline{-32} \\ & 0 \end{array}$$

Highest no. of children =16

Exercise 6.2

Q.1 **Simplify each of the following as a rational expression.**

(i) $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

Solution: $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

$$=\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$$

$$=\frac{x^2-3x-2x-6}{(x)^2-(3)^2}+\frac{x^2+6x-4x-24}{x^2-x-12}$$

$$=\frac{x(x-3)+2(x-3)}{(x-3)(x+3)}+\frac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$$

$$=\frac{(\cancel{x-3})(x+2)}{(\cancel{x-3})(x+3)}+\frac{(x+6)(\cancel{x-4})}{(\cancel{x-4})(x+3)}$$

$$=\frac{(x+2)}{(x+3)}+\frac{(x+6)}{(x+3)}$$

$$=\frac{x+2+x+6}{x+3}$$

$$=\frac{8+2x}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q.2 $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

Solution: $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

$$=\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{x^2+2x+1-(x^2+1-2x)}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{\cancel{x^2}+2x\cancel{+1}-\cancel{x^2}\cancel{-1}+2x}{x^2-1}-\frac{4x}{x^2+1}\right]+\left[\frac{4x}{x^4-1}\right]$$

$$=\left[\frac{4x}{x^2-1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x(x^2+1)-4x(x-1)}{(x^2-1)(x^2+1)}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x^3+4x-4x^3+4x}{x^4-1}\right]+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}$$

$$=\frac{12x}{x^4-1}$$

Q.3 $\dfrac{1}{x^2-8x+15}+\dfrac{1}{x^2-4x+3}-\dfrac{2}{x^2-6x+5}$

Solution: $\dfrac{1}{x^2-8x+15}+\dfrac{1}{x^2-4x+3}-\dfrac{2}{x^2-6x+5}$

$$=\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$$

$$=\frac{1}{x^2-3x-5x+15}+\frac{1}{x^2-3x-1x+3}-\frac{2}{x^2-5x-x+5}$$

$$=\frac{1}{x(x-3)-5(x-3)}+\frac{1}{x(x-3)-1(x-3)}-\frac{2}{x(x-5)-1(x-5)}$$

$$=\frac{1}{(x-3)(x-5)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}$$

$$=\frac{(x-1)+(x-5)-2(x-3)}{(x-3)(x-5)(x-1)}$$

$$=\frac{\cancel{x}-\cancel{1}+\cancel{x}-\cancel{5}-\cancel{2x}+\cancel{6}}{(x-3)(x-)(x-1)}$$

$$=\frac{0}{(\mathrm{x}-3)(\mathrm{x}-5)(\mathrm{x}-1)}$$

$$=0$$

Q.4 $\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution: $\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

$$=\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$

$$=\frac{(x+2)(x+3)}{(x)^2-(3)^2}+\frac{(x+2)\left[2(x^2-16)\right]}{(x-4)(x^2-3x+2x-6)}$$

$$=\frac{(x+2)(\cancel{x+3})}{(x-3)(\cancel{x+3})}+\frac{(x+2)\left[2(x)^2-(4)^2\right]}{(x-4)\left[x(x-3)+2(x-3)\right]}$$

$$=\frac{(x+2)}{(x-3)}+\frac{(\cancel{x+2})\left[2(x+4)(\cancel{x-4})\right]}{(\cancel{x-4})(x-3)(\cancel{x+2})}$$

$$=\frac{(x+2)}{(x-3)}+\frac{2(x+4)}{(x-3)}$$

$$=\frac{x+2}{x-3}+\frac{2x+8}{x-3}$$

$$=\frac{x+2+2x+8}{x-3}$$

$$=\frac{3x+10}{x-3}$$

Q.5 $=\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$

Solution: $=\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$

$$=\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$$

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-(3)^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{(\cancel{x+3})}{(\cancel{x+3})(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{1}{2x+3}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{2(2x-3)+(2x+3)-4x\times 2}{2(2x-3)(2x+3)}$$

$$=\frac{4x-6+2x+3-8x}{2(2x-3)(2x+3)}$$

$$=\frac{-2x-3}{2(2x-3)(2x+3)}$$

$$=\frac{-1(\cancel{2x+3})}{2(2x-3)(\cancel{2x+3})}$$

$$=\frac{-1}{2(2x-3)}$$

Q.6 $A-\frac{1}{A}$, **Where** $A=\frac{a+1}{a-1}$

Solution: $A-\frac{1}{A}$, Where $A=\frac{a+1}{a-1}$

$$=A-\frac{1}{A}=?$$

Put the value of A

$$=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$

$$=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$=\frac{a^2+2a+1-(\mathrm{a}^2-2\mathrm{a}+1)}{a^2-1}$$

$$=\frac{\cancel{a^2}+2a\cancel{+1}-\cancel{a^2}+2a\cancel{-1}}{a^2-1}$$

$$=\frac{4a}{a^2-1}$$

Q.7 $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

Solution: $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

$$=\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$$

$$=\left[\frac{x-1}{x-2}+\frac{2}{-x+2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-x^2+4}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-\left(x^2-4\right)}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{x^2-4}\right]$$

$$=\left[\frac{x-1-2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{(x+2)(x-2)}\right]$$

$$=\frac{(x-3)}{(x-2)}-\frac{(x+1)(x-2)-4}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-2-4}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x(x-3)2(x-3)}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$=\frac{x-\cancel{3}}{\cancel{x}-2}-\frac{x-\cancel{3}}{\cancel{x}-2}$$

0 **Ans**

Q.8 **What rational number should be subtracted from**

$$=\frac{2x^2+2x-7}{x2+x-6}\text{ to get }\frac{x-1}{x-2}$$

Solution: let required rational number be *P*(*x*)

According to condition

$$\frac{2x^2+2x-7}{x^2+x-6}-P(x)=\frac{x-1}{x-2}$$

$$P(x)=\frac{2x^2+2x-7}{x^2+x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x^2+3x-2x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x(x+3)-2(x+3)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{(x+3)(x-2)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7-(x-1)(x+3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$=\frac{x^2-4}{(x+3)(x-2)}$$

$$=\frac{x^2-2^2}{(x+3)(x-2)}$$

$$=\frac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$$

$$=\frac{x+2}{x+3}$$

Q.9 $$=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$$

Solution: $$=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$$

$$=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$$

$$=\frac{x^2+3x-2x-6}{x^2-3x+2x-6}\times\frac{x^2-2^2}{x^2-3^2}$$

$$=\frac{x(x+3)-2(x+3)}{2(x-3)+2(x-3)}\times\frac{(x-2)(x+3)}{(x-3)(x+3)}$$

$$=\frac{\cancel{(x+3)}(x-2)}{(x-3)\cancel{(x+2)}}\times\frac{(x-2)\cancel{(x+2)}}{(x-3)\cancel{(x+3)}}$$

$$=\frac{(x-2)^2}{(x-3)^2}$$

Q.10 $$\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

Solution: $$\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

$$=\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

$$=\frac{(x)^3-(2)^3}{(x^2)-(2)^2}\times\frac{x^2+4x+2x+8}{x^2-x-x+1}$$

$$=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\times\frac{x(x+4)+2(x+4)}{x(x-1)-1(x-1)}$$

$$=\frac{x^2+2x+4}{(\cancel{x+2})}\times\frac{(x+4)(\cancel{x+2})}{(x-1)(x-1)}$$

$$=\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$$

Q.11 $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

Solution: $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

$$=\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$$

$$=\frac{x(x^3-8)}{2x^2+6x-x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{x\left[(x)^3-(2)^3\right]}{2x(x+3)-1(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{\cancel{x}(\cancel{x-2})(\cancel{x^2+2x+4})}{(\cancel{2x-1})(\cancel{x+3})}\times\frac{\cancel{2x-1}}{\cancel{x^2+2x+4}}\times\frac{\cancel{x+3}}{\cancel{x}(\cancel{x-2})}$$

=1 **Ans**

Q.12 $\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$

Solution: $\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$

$$=\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$$

$$=\frac{2y^2+8y-1y-4}{3y^2-12y-y+4}\div\frac{(2y)^2-(1)^2}{6y^2+3y-2y-1}$$

$$=\frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)}\div\frac{(2y-1)(2y+1)}{3y(2y+1)-1(2y+1)}$$

$$=\frac{(y+4)(2y-1)}{(3y-1)(y-4)}\div\frac{(2y-1)\cancel{(2y+1)}}{(3y-1)\cancel{(2y+1)}}$$

$$=\frac{(y+4)\cancel{(2y-1)}}{\cancel{(3y-1)}(y-4)}\times\frac{\cancel{(3y-1)}}{\cancel{(2y-1)}}$$

$$=\frac{y+4}{y-4}$$

Q.13 $\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$

Solution: $\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$

$$=\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$$

$$=\left[\frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}\right]$$

$$=\left[\frac{(x^4+2x^2y^2+y^4)-(x^4-2x^2y^2+y^4)}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{(x^2+2xy+y^2)-(x^2-2xy+y^2)}{x^2-y^2}\right]$$

$$=\left[\frac{\cancel{x^4}+2x^2y^2+\cancel{y^4}-\cancel{x^4}+2x^2y^2-\cancel{y^4}}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{\cancel{x^2}+2xy+\cancel{y^2}-\cancel{x^2}+2xy-\cancel{y^2}}{x^2-y^2}\right]$$

$$=\left[\frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)}\right]\div\left[\frac{4xy}{x^2-y^2}\right]$$

$$=\frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)}\times\frac{x^2-y^2}{4xy}$$

$$=\frac{\cancel{4xy}.xy}{\cancel{(x^2-y^2)}(x^2+y^2)}\times\frac{\cancel{x^2-y^2}}{\cancel{4xy}}$$

$$= \frac{xy}{x^2 + y^2}$$ **Ans**

Exercise 6.3

Q.1 **Use factorization to find the square root of the following expression.**

(i) $4x^2-12xy+9y^2$

Solution: $4x^2-12xy+9y^2$

$4x^2-12xy+9y^2=4x^2-6xy-6xy+9y^2$

$=2x(2x-3y)-3y(3x-3y)$

$=(2x-3y)(2x-3y)$

$4x^2-12y+9y^2=(2x-3y)^2$

Taking square root on both side

$\sqrt{4x^2-12xy+9y^2}=\sqrt{[2x-3y]^2}$

$=\pm(2x-3y)$

(ii) $x^2-1+\frac{1}{4x^2}$

Solution: $x^2-1+\frac{1}{4x^2}$

$=(x)^2-2(x)\left[\frac{1}{2x}\right]+\left[\frac{1}{2x}\right]^2$

$=\left[x-\frac{1}{2x}\right]^2$

Taking square root

$\sqrt{x^2-1+\frac{1}{4x^2}}=\sqrt{\left[x-\frac{1}{2x}\right]^2}$

$\sqrt{x^2-1+\frac{1}{4x^2}}=\pm\left(x-\frac{1}{2x}\right)$

(iii) $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

Solution: $\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2$

$=\left(\frac{1}{4}x\right)^2-2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right)+\left(\frac{1}{6}y\right)^2$

$=\left(\frac{x}{4}-\frac{y}{6}\right)^2$

Taking the square root

$$\sqrt{\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2} = \sqrt{\left(\frac{1}{4}x - \frac{1}{6}y\right)^2}$$
$$= \pm\left(\frac{1}{4}x - \frac{1}{6}y\right)$$
$$= \pm\left(\frac{x}{4} - \frac{y}{6}\right)$$

(iv) $4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2$

Solution: $4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2$

$$= \left[2(a+b)^2\right] - 2[2(a+b)][3(a-b)] + [3(a-b)]^2$$
$$= \left[2(a+b) - 3(a-b)\right]^2$$

Taking square root

$$\sqrt{4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2} = \sqrt{[2(a+b) - 3(a-b)]^2}$$
$$= \pm[2a + 2b - 3a + 3b]$$
$$= \pm(5b - a)$$

(v) $\dfrac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$

Solution: $\dfrac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$

$$= \frac{(2x^3)^2 - 2(2x^3)(3y^3) + (3y^3)^2}{(3x^3)^2 + 2(3x^2)(4y^2) + (4y^2)^2}$$

=

$$= \frac{\left[2x^3 - 3y^3\right]^2}{\left[3x^3 + 4y^2\right]^2}$$

Taking square root

$$= \sqrt{\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}}$$
$$= \pm\left(\frac{2x^3 - 3y^3}{3x^2 + 4y^2}\right)$$

(vi) $\left(x + \dfrac{1}{x}\right)^2 - 4\left(x - \dfrac{1}{x}\right), (x \neq 0)$

Solution: $\left(x + \dfrac{1}{x}\right)^2 - 4\left(x - \dfrac{1}{x}\right), (x \neq 0)$

By adding and substituting 4

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right)$$

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right) - 4 + 4$$

$$= x^2 + \frac{1}{x^2} - 2 - 4\left(x - \frac{1}{x}\right) + 4$$

$$= \left(x - \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right)(2) + (2)^2$$

$$\left[\left(x - \frac{1}{x}\right) - 2\right]^2$$

Taking square root

$$\sqrt{\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right)} = \sqrt{\left[x - \frac{1}{x} - 2\right]^2}$$

$$= \pm\left(x - \frac{1}{x} - 2\right)$$

(vii) $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

Solution: $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4\left[x^2 + \frac{1}{x^2} + 2\right] + 12$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4x^2 - \frac{4}{x^2} - 8 + 12$$

$$= \left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x^2 + \frac{1}{x^2}\right) + 4$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 2\left[x^2 + \frac{1}{x^2}\right](2) + (2)^2$$

$$= \left[x^2 + \frac{1}{x^2} - 2\right]^2$$

Taking square root

$$= \sqrt{\left[x^2 + \frac{1}{x^2}\right] - 4\left[x + \frac{1}{x}\right]2 + 12}$$

$$= \sqrt{\left[x^2 \frac{1}{x^2} - 2\right]^2}$$

$$= \pm\left(x^2 + \frac{1}{x^2} - 2\right)$$

(viii) $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

Solution: $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

$=\left[x^2+2x+x+2\right]\left[x^2+3x+x+3\right]\left[x^2+3x+2x+6\right]$

$=\left[x(x+2)+1(x+2)\right]\left[x(x+3)+1(x+3)\right]\left[x(x+3)+2(x+3)\right]$

$=(x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$

$=(x+2)^2(x+1)^2(x+3)^2$

Taking square root

$=\sqrt{(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)}$

$=\sqrt{(x+2)^2(x+1)^2(x+3)^2}$

$=\pm(x+1)(x+2)(x+3)$ **Ans**

(ix) $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

Solution: $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

$=(x^2+7x+1x+7)(2x^2-3x+2x-3)(2x^2+14x-3x-21)$

$=\left[(x(x+7)+1(x+7)\right]\left[x(2x-x)+1(2x-3)\right]\left[(2x(x+7)-3(x+7)\right]$

$=(x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$

$=(x+7)^2(x+1)^2(2x-3)^2$

Taking square root

$=\sqrt{(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)}$

$=\sqrt{(x+7)^2(x+1)^2(2x-3)^2}$

$=\pm(x+1)(x+7)(2x-3)$ **Ans**

Q.2 **Use division method to find the square root of the following expression.**

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution: $4x^2+12xy+9y^2+16x+24y+16$

$$\begin{array}{r|l} & 2x+3y+4 \\ \hline 2x & \cancel{4x^2}+12xy+9y^2+16x+24y+16 \\ & \underline{\pm\cancel{4x^2}} \\ 4x+3y & 12\cancel{xy}+9\cancel{y^2}+16x+24y+16 \\ & \underline{\pm 12\cancel{xy}\pm 9\cancel{y^2}} \\ 4x+6y+4 & \cancel{16x}\ \cancel{24y}+\cancel{16} \\ & \underline{\cancel{16x}\pm\cancel{24y}\pm\cancel{16}} \\ & 0 \end{array}$$

Square root = $\pm\left(2x+3y+4\right)$

(ii) $x^4-10x^3+37x^2-60x+36$

Solution: $x^4-10x^3+37x^2-60x+36$

$$\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad\qquad x^2-5x+6 \\
x^2\,\big)\,\cancel{x^4}-10x^3+37x^2-60x+36 \\
\quad\pm\cancel{x^4} \\
\hline
\quad 2x^2-5x\,\big)\,\cancel{10x^3}+37x^2-60x+36 \\
\qquad\qquad\quad \pm\cancel{10x^3}\pm25x^2 \\
\hline
\qquad\quad 2x^2-10x+6\,\big)\,\cancel{12x^2}+\cancel{60x}+\cancel{36} \\
\qquad\qquad\qquad\qquad\quad \pm\cancel{12x^2}\pm\cancel{60x}\pm\cancel{36} \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad \times
\end{array}$$

Square root $=\pm\left(x^2-5x+6\right)$

(iii) $9x^4-6x^3+7x^2-2x+1$

Solution: $9x^4-6x^3+7x^2-2x+1$

$$\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 3x^2-x+1 \\
3x^2\,\big)\,\cancel{9x^4}-6x^3+7x^2-2x+1 \\
\quad\pm\cancel{9x^4} \\
\hline
\qquad 6x^2-x\,\big)\,\cancel{6x^3}+\cancel{7x^2}-2x+1 \\
\qquad\qquad\quad \pm\cancel{6x^3}\pm\cancel{7x^2} \\
\hline
\qquad\qquad 6x^2-2x+1\,\big)\,\cancel{6x^2}-\cancel{2x}+\cancel{1} \\
\qquad\qquad\qquad\qquad\quad \pm\cancel{6x^2}\pm\cancel{2x}\pm\cancel{1} \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad \times
\end{array}$$

Square root $\pm\left(=3x^2-x+1\right)$

(iv) $4+25x^2+7x^2-2x+1$

Solution: $4+25x^2-12x-24x^3+16x^4$

$$4x^2-3x+2$$

$$4x^2\overline{)\cancel{16x^4}-24x^3+25x^2-12x+4}$$

$$\pm\cancel{16x^4}$$

$$8x^2-3x\overline{)-\cancel{24x^3}+25x^2-12x+4}$$

$$\pm\cancel{24x^3}\pm 9x^2$$

$$8x^2-6x+2\overline{)\cancel{16x^2}-\cancel{12x}+\cancel{4}}$$

$$\pm\cancel{16x^2}\pm\cancel{12x}\pm\cancel{4}$$

$$\times$$

Square root $=\pm\left(4x^2-3x+2\right)$

(v) $\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},(x\neq 0,y\neq 0)$

Solution: $\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},(x\neq 0,y\neq 0)$

$$\frac{x}{y}-5+\frac{y}{x}$$

$$\frac{x}{y}\overline{)\cancel{\frac{x^2}{y^2}}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},}$$

$$\pm\cancel{\frac{x^2}{y^2}}$$

$$\frac{2x}{y}-5\overline{)-\cancel{\frac{10x}{y}}+27-\frac{10y}{x}+\frac{y^2}{x^2}}$$

$$\pm\cancel{\frac{x^2}{y^2}}\pm 25$$

$$\frac{2x}{y}-10+\frac{y}{x}\overline{)+\cancel{2}-\cancel{\frac{10y}{c}}+\cancel{\frac{y^2}{x^2}}}$$

$$\pm\cancel{2}\mp\cancel{\frac{10y}{x}}\pm\cancel{\frac{y^2}{x^2}}$$

$$\times$$

Square root $=\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$

Q.3 **Find the value of k for which the following expressions will become a perfect square.**

(i) $4x^4-12x^3+37x^2-42x+k$

Solution: $4x^4-12x^3+37x^2-42x+k$

$$2x^2-3x+7$$

$$2x^2\overline{)\cancel{4x^4}-12x^3+37x^2-42x+k}$$

$$\underline{\pm\cancel{4x^4}}$$

$$4x^2-3x\overline{)-\cancel{12x^3}+37x^2-42x+k}$$

$$\underline{\pm\cancel{12x^3}\pm 9x^2}$$

$$4x^2-6x+7\overline{)\cancel{28x^2}-\cancel{42x}+k}$$

$$\underline{\pm\cancel{28x^2}\pm\cancel{42x}\pm 49}$$

$$k-49$$

In the case of perfect square remainder is always is equal to zero so

$k-49=0$

$k=49$

(ii) $x^4-4x^3+10x^2-kx+9$

Solution: $x^4-4x^3+10x^2-kx+9$

$$x^2-2x+3$$

$$=x^2\overline{)\cancel{x^4}-4x^3+10x^2-kx+9}$$

$$\underline{\pm\cancel{x^4}}$$

$$2x^2-2x\overline{)-\cancel{4x^3}+10x^2-kx+9}$$

$$\underline{\pm\cancel{4x^3}\pm 4x^2}$$

$$2x^2-4x+3\overline{)6x^2-kx+9}$$

$$\underline{-6x^2\mp 12x\pm 9}$$

$-kx+12x=0$

In the case of square root remainder is always equal to zero

$-x(k-12)=0$

$k-12=\dfrac{0}{-x}$

$k-12=0$

$k=12$

Q.4 **Find the value of l and m for which the following expression will be perfect square**

(i) $x^4+4x^3+16x^2+ln+m$

Solution: $x^4+4x^3+16x^2+ln+m$

$$x^2+2x+6$$

$$=x^2\overline{)\cancel{x^4}+4x^3+16x^2+lx+m}$$

$$\underline{\pm\cancel{x^4}}$$

$$2x^2+2x\overline{)\cancel{4x^3}+16x^2+lx+m}$$

$$\underline{\pm\cancel{4x^3}\pm4x^2}$$

$$2x^2+4x+6\overline{)\cancel{12x^2}+lx+m}$$

$$\underline{\pm\cancel{12x^2}\pm24x-36}$$

In the case of square root remainder is always zero

$(lx-24x)$, $m-36=0$

$x(l-24)=0$, $m=36$ **Ans**

$l-24=\frac{0}{x}$

$l-24=0$

$l=24$ **Ans**

(ii) $49x^4-70x^3+109x^2+lx-m$

Solution: $49x^4-70x^3+109x^2+lx-m$

$$7x^2-5x+6$$

$$7x^2\overline{)\cancel{49x^4}-70x^3+109x^2+lx-m}$$

$$\underline{\pm\cancel{49x^4}}$$

$$14x^2-5x\overline{)-\cancel{70x^3}+109x^2+lx-m}$$

$$\underline{\mp\cancel{70x^3}\pm25x^2}$$

$$14x^2-10x+6\overline{)\cancel{84x^2}+lx-m}$$

$$\underline{\pm\cancel{84x^2}\mp60x\pm36}$$

$$lx+60x-m-36$$

$$(l+60)x-m-36$$

In the case of square root remainder is always equal to zero

$-m-36=0$

$-m=36$

$l+60=0$ $m=-36$

$l=-60$ **Ans**

Q.5 **To make the expression** $9x^4 - 12x^3 + 22x^2 - 13x + 12$ **a perfect square**

Solution: $9x^4 - 12x^3 + 22x^2 - 13x + 12$

$$\begin{array}{r|l} & 3x^2 - 2x + 3 \\ \hline = 3x^2 & \cancel{9x^4} - 12x^3 -+22x^2 - 13x + 12 \\ & \underline{\pm\cancel{9x^4}} \\ 6x^2 - 2x & -\cancel{12x^3} + 22x^2 - 13x + 12 \\ & \underline{\pm\cancel{12x^3} \pm 4x^2} \\ 6x^2 - 4x + 3 & \cancel{18x^2} - 13x + 12 \\ & \underline{\pm\cancel{18x^2} \mp 12x \pm 9} \\ & -x + 3 \end{array}$$

(i) $+x - 3$ **is to be added**

(ii) $-x + 3$ **is to be subtract from it**

(iii) $-x + 3 = 0$

$x = 3$

Review Exercise 6

Q.1 **Choose the correct answer.**

(i) **H.C.F of** p^3q-pq^3 **and** $p^5q^2-pq^5$ **is** __________

(a) $pq(p^2-q^2)$ **(b)** $pq(p-q)$

(c) $p^2q^2(p-q)$ **(d)** $pq(p^3-q^3)$

(ii) **H.C.F of** $5x^2y^2$ *and* $20x^3y^3$ is __________

(a) $5x^2y^2$ **(b)** $20x^3y^3$

(c) $100x^5y^5$ **(d)** $5xy$

(iii) **H.C.F of** $x-2$ **and** x^2+x-6 _______

(a) x^2+x-6 **(b)** $x+3$

(c) $x-2$ **(d)** $x+2$

(iv) **H.C.F of** a^3+b^3 **and** a^2-ab+b^2 _______

(a) $a+b$ **(b)** a^2-ab+b^2

(c) $(a-b)^2$ **(d)** a^2+b^2

(v) **H.C.F of** x^2-5x+6 **and** x^2-x-6 is_______

(a) $x-3$ **(b)** $x+2$

(c) x^2-4 **(d)** $x-2$

(vi) **H.C.F of** a^2-b^2 **and** a^3-b^3 is_______

(a) $a-b$ **(b)** $a+b$

(c) a^2+ab+b^2 **(d)** a^2-ab+b^2

(vii) **H.C.F of** x^2+3x+2, x^2+4x+3 **and** x^2+5x+4 is_______

(a) $x+1$ **(b)** $(x+1)(x+2)$

(c) $x+3$ **(d)** $(x+4)(x+1)$

(viii) **L.C.M of** $15x^2, 45xy$ **and** $30xyz$ is_______

(a) $90xyz$ **(b)** $90x^2yz$

(c) $15xyz$ **(d)** $15x^2yz$

(ix) **L.C.M of** a^2+b^2 **and** a^4-b^4 is_______

(a) a^2+b^2 **(b)** a^2-b^2

(c) a^4-b^4 **(d)** $a-b$

(x) **The product of two algebraic expression is equal to the** _________ **of their H.C.F and L.C.M**

(a) Sum **(b)** Difference

(c) Product **(d)** Quotitent

(xi) **Simplify** $\frac{a}{9a^2-b^2}+\frac{1}{3a-b}$ is_______

(a) $\frac{4a}{9a^2-b^2}$ **(b)** $\frac{4a-b}{9a^2-b^2}$

(c) $\frac{4a+b}{9a^2-b^2}$ **(d)** $\frac{b}{9a^2-b^2}$

(xii) **Simplify** $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=$________

(a) $\frac{a+7}{a-6}$ **(b)** $\frac{a+7}{a-2}$

(c) $\frac{a+3}{a-6}$ **(d)** $\frac{a-2}{a+3}$

(xiii) **Simplify the** $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}=$________

(a) $\frac{1}{a+b}$ **(b)** $\frac{1}{a-b}$

(c) $\frac{a-b}{a^2+b^2}$ **(d)** $\frac{a+b}{a^2+b^2}$

(xiv) **Simplify** $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{x}{x+y}\right)=$__________

(a) $\frac{x}{x+y}$ **(b)** $\frac{y}{x+y}$

(c) $\frac{y}{x}$ **(d)** $\frac{x}{y}$

(xv) **The square root of** a^2-2a+1 is ________

(a) $\pm(a+1)$ **(b)** $\pm(a-1)$

(c) $a-1$ **(d)** $a+1$

(xvi) **What should be added to complete the square of** x^4+64 ?________

(a) $8x^2$ **(b)** $-8x^2$

(c) $16x^2$ **(d)** $4x^2$

(xvii) **The square root to** $x^4+\frac{1}{x^4}+2$ **is**________

(a) $\pm\left(x+\frac{1}{x}\right)$ **(b)** $\pm\left(x^2+\frac{1}{x^2}\right)$

(c) $\pm\left(x-\frac{1}{x}\right)$ **(d)** $\pm\left(x^2-\frac{1}{x^2}\right)$

ANSWER KEYS

1	b	5	a	9	c	13	a	17	b
2	a	6	a	10	c	14	d		
3	c	7	a	11	c	15	b		
4	b	8	b	12	a	16	c		

Q.2 Find the H.C.F of the following by factorization.

$8x^4 - 128, 12x^3 - 96$

Solution:

$$8x^4 - 128 \quad = 8\left(x^4 - 16\right) \quad = 8\left[\left(x\right)^2 - \left(4\right)^2\right]$$

$$= 2 \times 2 \times 2\left(x^2 + 4\right)\left(x^2 - 4\right)$$

$$= 2 \times 2 \times 2\left(x^2 + 4\right)\left(x + 2\right)\left(x - 2\right)$$

$$12x^3 - 96 \quad = 12(x^3 - 8)$$

$$= \left(12(x^3 - 2^3\right)$$

$$= 12(x - 2)(x^2 + 2x + 4)$$

$$2 \times 2 \times 3\left(x - 2\right)\left(x^2 + 2x + 4\right)$$

H.C.F = $2 \times 2\left(x - 2\right)$

$= 4\left(x - 2\right)$

Q.3 Find the H.C.F of the following by division method $y^3 + 3y^2 - 3y - 9, 3y^2 - 8y - 2y$**.**

Solution: $y^3 + 3y^2 - 3y - 9,$

$= y^3 + 3y^2 - 3y - 9$

$$\begin{array}{r|l} & \qquad\qquad\qquad\qquad 1 \\ y^3 + 3y^2 - 8y - 24 & \cancel{y^3} + \cancel{3y^2} - 3y - 9 \\ & \underline{\pm \cancel{y^3} \pm \cancel{3y^2} \pm 8y \pm 24} \\ & \qquad 5y + 15 \\ & 5(y + 3) \end{array}$$

$$\begin{array}{r|l} & \qquad\qquad y^2 - 8 \\ y + 3 & \cancel{y3} + \cancel{3y} - 8y - 24 \\ & \underline{\pm \cancel{y3} \pm \cancel{3y}} \\ & -\cancel{8y} - \cancel{24} \\ & \underline{\pm \cancel{8y} \pm \cancel{24}} \\ & \qquad \times \end{array}$$

H.C.F $= \left(y + 3\right)$

Q.4 Find the L.C.M of the following by factorization.

$12x^2-75,\ 6x^2-13x-5,\ 4x^2-20x+25$

Solution:

$$12x^2-75 \quad =3\left(4x^2-25\right)$$

$$=3\left[\left(2x\right)^2-\left(5\right)^2\right]$$

$$=3\left(2x-5\right)\left(2x+5\right)$$

$$6x^2-15x+2x-5 \quad =3x\left(2x-5\right)+1\left(2x-5\right)$$

$$=\left(2x-5\right)\left(3x+1\right)$$

$$4x^2-20x+25=4x^2-10x-10x+25$$

$$=2x\left(2x-5\right)-5\left(2x-5\right)$$

$$=\left(2x-5\right)\left(2x-5\right)$$

Common factor = (2x-5)

Non common factor $=3\left(3x+1\right)\left(2x-5\right)2x+5)$

L.C.M = common factor × non common factor

$$\text{L.C.M}=\left(2x-5\right)3\left(3x+1\right)\left(2x+5\right)\left(2x-5\right)$$

$$\text{L.C.M}=3\left(2x+5\right)\left(2x-5\right)^2\left(3x+1\right)$$

Q.5 If H.C.F of $x^4+3x^3+5x^2+26x+56$ and $x^4+2x^3-4x^2-x+28$ is x^2+5x+7 find their L.C.M.

Solution: $p\left(x\right)=x^4+3x^3+5x^2+26x+56$ and $q\left(x\right)=x^4+2x^3-4x^2-x+28$

$HCF=x^2+5x+7$, LCM=?

$$L.C.M=\frac{P(x)\times q(n)}{H.C.F}$$

$$\text{L.C.M}=\frac{\left(x^4+3x^2+5x^2+26x+56\right)\times\left(x^4+2x^3-4x^2-x+28\right)}{\left(x^2+5x+7\right)}$$

$$\begin{array}{r|l}
 & \qquad\qquad x^2-3x+4 \\ \hline
x^2+5x+7 & \cancel{x^4}+2x^3-4x^2-x+28 \\
 & \pm\cancel{x^4}\pm5x^3\pm7x^2 \\ \hline
 & -\cancel{3x^3}-11x^2-x+28 \\
 & \mp\cancel{3x^3}\mp15x^2\mp21x \\ \hline
 & +\cancel{4x^2}+\cancel{20x}+\cancel{28} \\
 & \pm\cancel{4x^2}\pm\cancel{20x}\pm\cancel{28} \\ \hline
 & \qquad 0
\end{array}$$

$$x^2-3x+4$$

$$L.C.M=\frac{(x^4+3x^3+5x^2+26x+56)\,\cancel{(x^4+2x^3-4x^2-x+28)}}{\cancel{(x2+5x+7)}}$$

$$L.C.M=(x^4+3x^3+5x^2+26x+56)(x^2-3x+4)$$

Q.6 Simplify:

Solution:

(i) $$\frac{3}{x^3+x^2+x+1}-\frac{3}{x^3-x^2+x-1}$$

Solution: $$\frac{3}{x^3+x^2+x+1}-\frac{3}{x^3-x^2+x-1}$$

$$=\frac{3}{x^2(x+1)+1(x+1)}-\frac{3}{x^2(x-1)+1(x-1)}$$

$$=\frac{3}{(x^2+1)(x+1)}-\frac{3}{(x-1)(x^2+1)}$$

$$=\frac{3(x-1)-3(x+1)}{(x+1)(x-1)(x^2+1)}$$

$$=\frac{\cancel{3x}-3-\cancel{3x}-3}{(x-1)(x-1)(x^2+1)}$$

$$=\frac{-6}{(x+1)(x-1)(x^2+1)}=\frac{-6}{(x^2-1)(x^2+1)}$$

$$=\frac{-6}{(x^4-1)}$$

$$=\frac{6}{1-x^4}$$

(ii) $$\frac{a+b}{a^2-b^2}\div\frac{a^2-ab}{a^2-2ab+b^2}$$

Solution: $$\frac{a+b}{a^2-b^2}\div\frac{a^2-ab}{a^2-2ab+b^2}$$

$$=\frac{a+b}{a^2-b^2}\times\frac{a^2-2ab+b^2}{a^2-ab}$$

$$=\frac{\cancel{a+b}}{(a-b)(\cancel{a+b})}\times\frac{(a-b)^2}{a(a-b)}$$

$$=\frac{(\cancel{a-b})^2}{a(\cancel{a-b})^2}$$

$$=\frac{1}{a}$$

Q.7 **Find the square root by using factorization.** $\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \quad (x\neq 0)$.

Solution: $(x^2+\frac{1}{x^2}+10\left(x+\frac{1}{x}\right)+27$

$$=(x^2+\frac{1}{x^2}+10\left(x+\frac{1}{x}\right)+25+2$$

$$=x^2+\frac{1}{x^2}+2+10\left(x+\frac{1}{x}\right)+25$$

$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)\times 5+(5)^2$$

$$=\left[x+\frac{1}{x}+5\right]^2$$

Taking the square root

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left[x+\frac{1}{x}+5\right]^2}$$

$$=\pm\left(x+\frac{1}{x}+5\right)$$

Q.8 **Find the square roots by using division method.** $\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x}$

Solution:

$$\frac{2x}{y}+5-\frac{3y}{x}$$

$$\frac{2x}{y}\,\Big)\,\frac{\cancel{4x^2}}{\cancel{y^2}}+\frac{20x}{4}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\frac{\cancel{4x^2}}{\cancel{y^2}}$$

$$\frac{4x}{y}+5\,\Big)\,\frac{\cancel{20x}}{\cancel{y}}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\frac{\cancel{20x}}{\cancel{y}}\pm 25$$

$$\frac{4x}{y}+10-\frac{3y}{x}\Big)\overline{-\cancel{12}-\cancel{\frac{30y}{x}}+\cancel{\frac{9y^2}{x^2}}}$$

$$\underline{\mp\cancel{12}\mp\cancel{\frac{30y}{x}}\pm\cancel{\frac{9y^2}{x^2}}}$$

Square root $=\pm\left[\frac{2x}{y}\overset{\times}{+}5-\frac{3y}{x}\right]$

Unit 6: Algebraic Manipulation

Overview

Highest Common Factor:

If two or more algebraic expressions are given then their common factor of highest power is called the H.C.F. of the expression.

Least Common Multiple(L.C.M):

The Least common Multiple (L.C.M) is the product of common factors together with non-common factors of the given expressions.