Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).

(i) $3x^2 + \frac{1}{x} - 5$

No (Because of $\frac{1}{x}$) Ans.

(ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$

No (Because $\sqrt{x}$ or $(x)^{\frac{1}{2}}$) Ans.

(iii) $x^2 - 3x + \sqrt{2}$

Yes (Because no variable has power in fraction). **Ans**

(iv) $\frac{3x}{2x-1} + 8$

No (Because of $\frac{1}{2x-1}$) **Ans**

Q.2 State whether each of the following expressions is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Irrational **Ans**

(ii) $\frac{x^3 - 2x^3 + \sqrt{3}}{2 + 3x - x^2}$

Rational **Ans**

(iii) $\frac{x^2 + 6x + 9}{x^2 - 9}$

Rational **Ans**

(iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Irrational **Ans**

Q.3 Reduce the following expression to the lowest form.

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$

Solution: $\frac{{}^{4}\cancel{120}x^2y^3z^5}{\cancel{30}x^3yz^2}$

$= \frac{120x^2y^3z^5}{30x^3yz^2}$

$= 4x^{2-3}y^{3-1}z^{5-2}$

$= 4x^{-1}y^2z^3$

$= \frac{4y^2z^3}{x}$ **Ans**

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

Solution: $\frac{8a(x+1)}{2(x^2-1)}$

$= \frac{{}^{4}\cancel{8}a(x+1)}{\cancel{2}(x^2-1)}$

$= \frac{4a\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$

$= \frac{4a}{x-1}$ **Ans**

(iii) $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

Solution: $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

$\because (x+y)^2 = x^2 + y^2 + 2xy$

$\because (x-y)^2 = x^2 + y^2 - 2xy$

$= \frac{x^2 + y^2 + 2xy - 4xy}{x^2 + y^2 - 2xy}$

$= \frac{x^2 + y^2 - 2xy}{x^2 + y^2 - 2xy}$

$$= \frac{(x-y)^2}{(x-y)^2}$$

$= 1$ **Ans**

(iv) $$\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$$

Solution: $$\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$$

$$(a^3+b^3)=(a-b)(a^2+ab+b^2)$$

$$= \frac{(x^3-y^3)(x^2-2xy+y^2)}{(x^3-y^3)}$$

$$= x^2-2xy+y^2$$

$$\therefore (x-y)^2 = x^2-2xy+y^2$$

$= (x-y)^2$ **Ans**

(v) $$\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$$

Solution: $$\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$$

$$= \frac{(x+2)\left[(x)^2-(1)^2\right]}{(x+1)\left[(x)^2-(2)^2\right]}$$

$$= \frac{(x+2)(x-1)(x+1)}{(x+1)(x-2)(x+2)}$$

$= \frac{(x-1)}{(x-2)}$ **Ans**

(vi) $$\frac{x^2-4x+4}{2x^2-8}$$

Solution: $$\frac{x^2-4x+4}{2x^2-8}$$

$$\therefore (a-b)^2 = a^2-2ab+b^2$$

$$\therefore a^2-b^2 = (a+b)(a-b)$$

$$= \frac{(x)^2-2(x)(2)+(2)^2}{2(x^2-4)}$$

$$= \frac{(x-2)^2}{2\left[(x)^2-(2)^2\right]}$$

$$= \frac{(x-2)^2}{2(x+2)(x-2)}$$

$$= \frac{(x-2)(x-2)}{2(x+2)(x-2)}$$

$= \frac{x-2}{2(x+2)}$ **Ans**

(vii) $$\frac{64x^5-64x}{(8x^2+8)(2x+2)}$$

Solution: $$\frac{64x^5-64x}{(8x^2+8)(2x+2)}$$

$$= \frac{64x(x^4-1)}{8(x^2+1).2(x+1)}$$

$$= \frac{64\left[(x^2)^2-(1)^2\right]}{16(x^2+1)(x+1)}$$

$$= \frac{{}^{4}64(x^2-1)(x^2+1)}{16(x^2+1)(x+1)}$$

$$= \frac{4x(x-1)(x+1)}{(x+1)}$$

$= 4x(x-1)$ **Ans**

(viii) $$\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$$

Solution: $$\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$$

$$= \frac{(3x)^2 - (x^2 - 4)^2}{4 + 3x - x^2}$$

$$= \frac{(3x + x^2 - 4)(3x - x^2 + 4)}{4 + 3x - x^2}$$

$$= \frac{(x^2 + 3x - 4)(-x^2 + 3x + 4)}{(-x^2 + 3x + 4)}$$

$= x^2 + 3x - 4$ **Ans**

Q.4 Evaluate

(a) $\frac{x^3y - 2z}{xz}$ **for**

(i) $x = 3, y = -1, z = -2$

(ii) $x = -1, y = -9, z = 4$

Solution for 1st part

When $x = 3,\ y = -1,\ z = -2$

$$\frac{x^3y - 2z}{xz} =$$

$$= \frac{(3)^3(-1) - 2(-2)}{(3)(-2)}$$

$$= \frac{27(-1) + 4}{-6}$$

$$= \frac{-27 + 4}{-6}$$

$$= \frac{-23}{-6}$$

$= \frac{23}{6}$ **Ans**

Solution for 2nd Part.

When $x = -1,\ y = -9,\ z = 4$

$$\frac{x^3y - 2z}{xz} =$$

$$= \frac{(-1)^3(-9) - 2(4)}{(-1)(4)}$$

$$= \frac{-1(-9) - 8}{-4}$$

$$= \frac{9 - 8}{-4}$$

$$= \frac{1}{-4}$$

$= -\frac{1}{4}$ **Ans**

(b) $\frac{x^2y^2 - 5z^4}{xyz}$ for $x = 4, y = -2$ and $z = -1$

Solution: $\frac{x^2y^3 - 5z^4}{xyz}$

$$= \frac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)}$$

$$= \frac{16(-8) - 5(1)}{8}$$

$$= \frac{16(-8) - 5(1)}{8}$$

$$= \frac{-128 - 5}{8}$$

$$= -\frac{133}{8}$$

$= -16\frac{5}{8}$ **Ans**

Q.5 Perform the indicated operation and simplify.

(i) $\frac{15}{2x - 3y} - \frac{4}{3y - 2x}$

Solution: $\frac{15}{2x - 3y} - \frac{4}{3y - 2x}$

$$= \frac{15}{2x - 3y} - \frac{4}{-2x + 3y}$$

$$= \frac{15}{2x - 3y} - \frac{4}{-(2x - 3y)}$$

$$= \frac{15}{2x - 3y} + \frac{4}{2x - 3y}$$

$= \frac{19}{2x - 3y}$ **Ans**

(ii) $\frac{1+2x}{1-2x}-\frac{1-2x}{1+2x}$

Solution: $\frac{1+2x}{1-2x}-\frac{1-2x}{1+2x}$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$

$$=\frac{(1)^2+(2x)^2+2(2x)(1)-\left[(1)^2+(2x)^2-2(2x)(1)\right]}{(1)^2-(2x)^2}$$

$$=\frac{1+4x^2+4x-\left[1+4x^2-4x\right]}{1-4x^2}$$

$$=\frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$

$$=\frac{4x+4x}{1-4x^2}$$

$$=\frac{8x}{1-4x^2}$$ **Ans**

(iii) $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

Solution: $\frac{x^2-25}{x^2-36}-\frac{x+5}{x+6}$

$$=\frac{(x)^2-(5)^2}{(x)^2-(6)^2}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)}{(x+6)(x-6)}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)-(x-6)(x+5)}{(x+6)(x-6)}$$

$$=\frac{(x+5)\left[(x-5)-(x-6)\right]}{x^2-6^2}$$

$$=\frac{(x+5)(x-5-x+6)}{x^2-36}$$

$$=\frac{(x+5)(1)}{x^2-36}$$

$$=\frac{x+5}{x^2-36}$$ **Ans**

(iv) $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

Solution: $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

$$=\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{(x)^2-(y)^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2-2xy}{x^2-y^2}$$

$$=\frac{(x-y)^2}{x^2-y^2}$$

$$=\frac{(x-y)\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}$$

$$=\frac{x-y}{x+y}$$ **Ans**

(v) $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$

Solution: $\frac{x-2}{x^2+6x+9}-\frac{x+2}{2x^2-18}$

$$=\frac{x-2}{(x)^2+2(3)(x)+3^2}-\frac{x+2}{2(x^2-9)}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2\left[(x)^2-(3)^2\right]}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{x-2}{(x+3)(x+3)}-\frac{x+2}{2(x+3)(x-3)}$$

$$=\frac{2(x-3)(x-2)-(x+3)(x+2)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-2x-3x+6)-(x^2+2x+3x+6)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x+3)^2(x-3)}$$

$$= \frac{x^2 - 15x + 6}{2(x+3)^2(x-3)}$$ **Ans**

(vi) $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$

Solution: $\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$

$$= \frac{(x+1)-(x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{x + 1 - x + 1}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2(x^2+1) - 2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1}$$

$$= \frac{2x^2 + 2 - 2x^2 + 2}{(x^2)^2 - (1)^2} - \frac{4}{x^4-1}$$

$$= \frac{4}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4-4}{x^4-1}$$

$$= \frac{0}{x^4-1}$$

$= 0$ **Ans**

Q.6 Perform the indicated operation and simplify.

(i) $(x^2 - 49).\frac{5x+2}{x+7}$

Solution: $(x^2 - 49).\frac{5x+2}{x+7}$

$$= \left[(x)^2 - (7)^2\right].\frac{5x+2}{x+7}$$

$$= (x+7)(x-7)\frac{(5x+2)}{(x+7)}$$

$= (x-7)(5x+2)$ **Ans**

(ii) $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9}$

Solution: $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

$$= \frac{4(x-3)}{(x^2)-(3)^2} \div \frac{2(9-x^2)}{(x+3)^2}$$

$$= \frac{4(x-3)}{(x-3)(x+3)} \times \frac{(x+3)^2}{2(9-x^2)}$$

$$= \frac{4}{x+3} \times \frac{(x+3)^2}{2(3+x)(3-x)}$$

$$= \frac{{}^2 4x(x+3)^2}{2(x+3)^2(3-x)}$$

$$= \frac{2}{3-x}$$ **Ans**

(iii) $\frac{x^6 - y^6}{x^2 - y^2} \div (x^4 + x^2y^2 + y^4)$

Solution: $\frac{x^6 - y^6}{x^2 - y^2} \div (x^4 + x^2y^2 + y^4)$

$$= \frac{(x^2)^3 - (y^2)^3}{x^2 - y^2} \div (x^4 + x^2y^2 + y^4)$$

$$= \frac{(x^2 - y^2)\left[(x^2)^2 + x^2y^2 + (y^2)^2\right]}{(x^2 - y^2)} \div (x^4 + x^2y^2 + y^4)$$

$$= (x^4 + x^2y^2 + y^4) \times \frac{1}{(x^4 + x^2y^2 + y^4)}$$

$= 1$ **Ans**

(iv) $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$

Solution: $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$

$$= \frac{(x+1)(x-1)}{(x^2+2(x)(1)+(1)^2)} \times \frac{x+5}{-(x-1)}$$

$$= \frac{(x+1)(x-1)}{(x+1)^2} \times \frac{(x+5)}{-(x-1)}$$

$$= -\frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x+1)}$$

$$= -\frac{(x+5)}{x+1}$$ **Ans**

(v) $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

Solution: $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

$$= \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \cdot \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \div \frac{x(x-1)}{y(x-2)}$$

$$= \frac{x.\cancel{x}}{y.\cancel{y}} \times \frac{\cancel{y}(x-2)}{\cancel{x}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)}$$ **Ans**

Exercise 4.2

Q.1 Solve

(i) **If** $a+b=10$ **and** $a-b=6,$ **then find the value of** $\left(a^2+b^2\right)$

Solution:

$$2\left(a^2+b^2\right)=(a+b)^2+(a-b)^2$$
$$2\left(a^2+b^2\right)=(10)^2+(6)^2$$
$$2\left(a^2+b^2\right)=100+36$$
$$2\left(a^2+b^2\right)=136$$
$$\left(a^2+b^2\right)=\frac{\cancel{136}^{68}}{\cancel{2}}$$
$$\left(a^2+b^2\right)=68 \textbf{ Ans}$$

(ii) **If** $a+b=5, a-b=\sqrt{17},$ **then find the value of** ab**.**

Solution:

$$4ab=(a+b)^2-(a-b)^2$$
$$4ab=(5)^2-\left(\sqrt{17}\right)^2$$
$$4ab=25-17$$
$$4ab=8$$
$$ab=\frac{8}{4}$$
$$ab=2$$
$$ab=2 \textbf{ Ans}$$

Q.2 **If** $a^2+b^2+c^2=45$ **and** $a+b+c=-1$**, then find the value of** $ab+bc+ca$**.**

Solution: $a^2+b^2+c^2=45$

$a+b+c=-1$

$ab+bc+ca=$**?**

We know that

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$
$$(-1)^2=45+2(ab+bc+ca)$$
$$1=45+2(ab+bc+ca)$$
$$1-45=2(ab+bc+ca)$$
$$-44=2(ab+bc+ca)$$
$$\frac{\cancel{-44}^{22}}{\cancel{2}}=(ab+bc+ca)$$
$$(ab+bc+ca)=-22 \textbf{ Ans}$$

Q.3 **If** $m+n+p=10$ **and** $mn+np+np=27,$ **find the value of** $m^2+n^2+p^2$

Solution: $m+n+p=10$

$mn+np+np=27,$

$m^2+n^2+p^2=$**?**

We know that

$$(m+n+p)^2=m^2+n^2+p^2+2mn+2np+2mp$$
$$(10)^2=m^2+n^2+p^2+2(mn+np+mp)$$
$$100=m^2+n^2+p^2+2(27)$$
$$100=m^2+n^2+p^2+54$$
$$100-54=m^2+n^2+p^2$$
$$m^2+n^2+p^2=46 \textbf{ Ans}$$

Q.4 **If** $x^2+y^2+z^2=78$ **and** $xy+yz+zx=59,$ **find the value of** $x+y+z$**.**

Solution: $x^2+y^2+z^2=78$

$xy+yz+zx=59,$

$x+y+z=$**?**

We know that

$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$$

$$(x+y+z)^2=78+2(xy+yz+zx)$$
$$(x+y+z)^2=78+2(59)$$
$$(x+y+z)^2=78+118$$
$$(x+y+z)^2=196$$

Taking square root at both sides

$$\sqrt{(x+y+z)^2} = \pm\sqrt{196}$$

$$x+y+z = \pm 14 \textbf{ Ans}$$

Q.5 If $x+y+z=12$ **and** $x^2+y^2=64$, **find the value of** $xy+yz+zx$.

Solution: $x+y+z=12$

$x^2+y^2=64$

$xy+yz+zx=$**?**

We know that

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$$

$$(12)^2 = 64+2(xy+yz+zx)$$

$$144-64 = 2(xy+yz+zx)$$

$$80 = 2(xy+yz+zx)$$

$$\frac{\overset{40}{\cancel{80}}}{\cancel{2}} = (xy+yz+zx)$$

$$40 = xy+yz+zx$$

$$xy+yz+zx = 40 \textbf{ Ans}$$

Q.6 If $x+y=7$ **and** $xy=12$, **then find the value of** x^3+y^3

Solution: $x+y=7$

$xy=12$

$x^3+y^3=$**?**

We know that

$$(x+y)^3 = x^3+y^3+3xy(x+y)$$

$$(7)^3 = x^3+y^3+3(12)(7)$$

$$343 = x^3+y^3+252$$

$$343-252 = x^3+y^3$$

$$91 = x^3+y^3$$

$$x^3+y^3 = 91 \textbf{ Ans}$$

Q.7 If $3x+4y=11$ **and** $xy=12$, **then find the value of** $27x^3+64y^3$.

Solution: $3x+4y=11$

$xy=12$

$27x^3+64y^3=$**?**

$$(x+y)^3 = x^3+y^3+3xy(x+y)$$

$$(3x+4y)^3 = (3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$$

$$(3x+4y)^3 = 27x^3+64y^3+36xy(3x+4y)$$

$$(11)^3 = 27x^3+64y^3+36(12)(11)$$

$$1331 = 27x^3+64y^3+4752$$

$$1331-4752 = 27x^3+64y^3$$

$$-3421 = 27x^3+64y^3$$

$$27x^3+64y^3 = -3421 \text{ Ans}$$

Q.8 If $x-y=4$ **and** $xy=21$, **then find the value of** x^3-y^3

Solution: $x-y=4$

$xy=21$

$x^3-y^3=$**?**

We know that

$$(x-y)^3 = x^3-y^3-3xy(x-y)$$

$$(4)^3 = x^3-y^3-3(21)(4)$$

$$64 = x^3-y^3-252$$

$$64+252 = x^3-y^3$$

$$316 = x^3-y^3$$

$$x^3-y^3 = 316 \textbf{ Ans}$$

Q.9 If $5x-6y=13$ **and** $xy=6$, **then find the value of b**$125x^3-216y^3$

Solution: $5x-6y=13$

$xy=6$

$125x^3-216y^3=$**?**

We know that

$$(x-y)^3 = x^3-y^3-3xy(x-y)$$

$$(5x-6y)^3 = (5x)^3-(6y)^3-3(5x)(6y)(5x-6y)$$

$$(5x-6y)^3 = 125x^3-216y^3-90xy(5x-6y)$$

$$(13)^3 = 125x^3-216y^3-90(6)(13)$$

$$2197 = 125x^3-216y^3-7020$$

$$2197+7020 = 125x^3-216y^3$$

$$9217 = 125x^3 - 216y^3$$

$$125x^3 - 216y^3 = 9217 \textbf{ Ans}$$

Q.10 If $x+\frac{1}{x}=3$ then find the value of $x^3+\frac{1}{x^3}$

Solution: $x+\frac{1}{x}=3$

$x^3+\frac{1}{x^3}$ **=?**

We know that

$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$(3)^3 = x^3+\frac{1}{x^3}+3(3)$$

$$27 = x^3+\frac{1}{x^3}+9$$

$$27-9 = x^3+\frac{1}{x^3}$$

$$18 = x^3+\frac{1}{x^3}$$

$$x^3+\frac{1}{x^3} = 18 \textbf{ Ans}$$

Q.11 If $x-\frac{1}{x}=7$, then find the value of $x^3-\frac{1}{x^3}$

Solution: $x-\frac{1}{x}=7$

$x^3-\frac{1}{x^3}$ **=?**

We know that

$$\left(x-\frac{1}{x}\right)^3 = x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$$

$$(7)^3 = x^3-\frac{1}{x^3}-3(7)$$

$$343 = x^3-\frac{1}{x^3}-21$$

$$343+21 = x^3-\frac{1}{x^3}$$

$$364 = x^3-\frac{1}{x^3}$$

$$x^3-\frac{1}{x^3} = 364 \textbf{ Ans}$$

Q.12 If $\left[3x+\frac{1}{3x}\right]=5$, then find the value of $\left[27x^3+\frac{1}{27x^3}\right]$

Solution: $\left[3x+\frac{1}{3x}\right]=5$

$\left[27x^3+\frac{1}{27x^3}\right]$ **=?**

We know that

$$\left(3x+\frac{1}{3x}\right)^3 = (3x)^3+\left(\frac{1}{3x}\right)^3+3(3x)\left(\frac{1}{3x}\right)\left(3x+\frac{1}{3x}\right)$$

$$(5)^3 = 27x^3+\frac{1}{27x^3}+3\left(3x+\frac{1}{3x}\right)$$

$$125 = 27x^3+\frac{1}{27x^3}+3(5)$$

$$125 = 27x^3+\frac{1}{27x^3}+15$$

$$125-15 = 27x^3+\frac{1}{27x^3}$$

$$110 = 27x^3+\frac{1}{27x^3}$$

$$27x^3+\frac{1}{27x^3} = 110 \textbf{ Ans}$$

Q.13 If $\left(5x-\frac{1}{5x}\right)=6$, then find the value of $\left(125x^3-\frac{1}{125x^3}\right)$

Solution: $\left(5x-\frac{1}{5x}\right)=6$

$\left(125x^3-\frac{1}{125x^3}\right)$ **=?**

We know that

$$\left(5x-\frac{1}{5x}\right)^3=(5x)^3-\left(\frac{1}{5x}\right)^3-3(5x)\left(\frac{1}{5x}\right)\left(5x-\frac{1}{5x}\right)$$

$$(6)^3=125x^3-\frac{1}{125x^3}-3(6)$$

$$216=125x^3-\frac{1}{125x^3}-18$$

$$216+18=125x^3-\frac{1}{125x^3}$$

$$234=125x^3-\frac{1}{125x^3}$$

$$125x^3-\frac{1}{125x^3}=234 \textbf{ Ans}$$

Q.14 Factorize

(i) x^3-y^3-x+y

Solution: x^3-y^3-x+y

$$=(x)^3-(y)^3-1(x-y)$$

$$=(x-y)(x^2+xy+y^2)-1(x-y)$$

$$=(x-y)(x^2+xy+y^2-1) \textbf{ Ans}$$

(ii) $8x^3-\frac{1}{27y^3}$

Solution: $8x^3-\frac{1}{27y^3}$

$$=(2x)^3-\left(\frac{1}{3y}\right)^3$$

$$=\left[2x-\frac{1}{3y}\right]\left[(2x)^2+(2x)\left(\frac{1}{3y}\right)+\left(\frac{1}{3y}\right)^2\right]$$

$$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right) \textbf{ Ans}$$

Q.15 Find the products, using formula.

(i) $(x^2+y^2)(x^4-x^2y^2+y^4)$

Solution: $(x^2+y^2)(x^4-x^2y^2+y^4)$

$$=(x^2+y^2)\left[(x^2)^2-(x^2)(y^2)+(y^2)^2\right]$$

$$\left[(x^2)^3+(y^2)^3\right]$$

$$=x^6+y^6 \textbf{ Ans}$$

(ii) $(x^3-y^3)(x^6+x^3y^3+y^6)$

Solution: $(x^3-y^3)(x^6+x^3y^3+y^6)$

$$(x^3-y^3)\left[(x^3)^2+(x^3)(y^3)+(y^3)^2\right]$$

$$=(x^3)^3-(y^3)^3$$

$$=x^9-y^9 \textbf{ Ans}$$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2+xy+y^2)(x^4-x^2y^2+y^4)$

Solution:

$$(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$$

$$(x^2+xy+y^2)(x^4-x^2y^2+y^4)$$

$$=\left[(x-y)(x^2+xy+y^2)\right]\left[(x+y)(x^2-xy+y^2)\right]$$

$$\left[(x^2+y^2)(x^4-x^2y^2+y^4)\right]$$

$$=\left[(x^3-y^3)(x^3+y^3)\right]\left[(x^2)^3+(y^2)^3\right]$$

$$=\left[(x^3)^2-(y^3)^2\right]\left[(x^6+y^6)\right]$$

$$=\left[(x^6-y^6)(x^6+y^6)\right]$$

$$=\left[(x^6)^2-(y^6)^2\right]$$

$$=x^{12}-y^{12} \textbf{ Ans}$$

(iv) $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$

Solution:

$$(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$$

$$=\left[(2x^2-1)(4x^4+2x^2+1)\right]\left[(2x^2+1)(4x^4-2x^2+1)\right]$$

$$=\left[(2x^2)^3-(1)^3\right]\left[(2x^2)^3+(1)^3\right]$$

$$=(8x^6-1)(8x^6+1)$$

$$=(8x^6)^2-(1)^2$$

$$=64x^{12}-1$$ **Ans**

Exercise 4.3

Q.1 Express each of the following surd in the simplest form:

(i) $\sqrt{180}$

Solution: $\sqrt{180}$

$$=(180)^{\frac{1}{2}}$$

$$=(2\times2\times3\times3\times5)^{\frac{1}{2}}$$

$$=(2^2\times3^2\times5)^{\frac{1}{2}}$$

$$=2^{2\times\frac{1}{2}}\times3^{2\times\frac{1}{2}}\times5^{\frac{1}{2}}$$

$$=2\times3\times\sqrt{5}$$

$$=6\sqrt{5}$$ **Ans**

(ii) $3\sqrt{162}$

Solution: $3\sqrt{162}$

$$3\left(\sqrt{81\times2}\right)$$

$$=3\left(\sqrt{9^2}\times\sqrt{2}\right)$$

$$=3\times9\left(\sqrt{2}\right)$$

$$=27\sqrt{2}$$ **Ans**

(iii) $\frac{3}{4}\sqrt[3]{128}$

Solution: $\frac{3}{4}\sqrt[3]{128}$

$$=\frac{3}{4}\left(\sqrt[3]{64\times2}\right)$$

$$=\frac{3}{4}\left(\sqrt[3]{4^3\times2}\right)$$

$$=\frac{3}{4}\left[\sqrt[3]{4^3}\times\sqrt[3]{2}\right]$$

$$=\frac{3}{4}\times4\times\sqrt[3]{2}$$

$$=3\times\sqrt[3]{2}$$

$$=3\sqrt[3]{2}$$ **Ans**

(iv) $\sqrt[5]{96x^6y^7z^8}$

Solution: $\sqrt[5]{96x^6y^7z^8}$

$$=\sqrt{32\times3\times x^5y^5z^5\times x^1y^2z^3}$$

$$=\sqrt[5]{2^5\times3\times x^5y^5z^5\times xy^2z^3}$$

$$=\sqrt[5]{2^5x^5y^5z^5}\times\sqrt[5]{3xy^2z^3}$$

$$=\sqrt[5]{2^5}\times\sqrt[5]{x^5}\times\sqrt[5]{y^5}\times\sqrt[5]{z^5}\times\sqrt[5]{3xy^2z^3}$$

$$=2xyz\sqrt[5]{3xy^2z^3}$$ **Ans**

Q.2 Simplify

(i) $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

Solution: $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$$=\frac{\sqrt{9\times2}}{\sqrt{3}\times\sqrt{2}}$$

$$=\frac{\sqrt{3^2}\times\cancel{\sqrt{2}}}{\sqrt{3}\times\cancel{\sqrt{2}}}$$

$$=\frac{3}{\sqrt{3}}$$

$$=\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{3\sqrt{3}}{\left(\sqrt{3}\right)^{\cancel{2}}}$$

$$=\frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$

$$=\sqrt{3}$$ **Ans**

(ii) $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

Solution: $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

$=\frac{\sqrt{21}\sqrt{3^2}}{\sqrt{9\times 7}}$

$=\frac{\sqrt{21}\times 3}{\sqrt{3^2}\times\sqrt{7}}$

$=\frac{\sqrt{21}\times 3}{3\sqrt{7}}$

$=\frac{\not{3}\sqrt{21}}{\not{3}\sqrt{7}}$

$=\frac{\sqrt{21}}{\sqrt{7}}$

$=\frac{\sqrt{7\times 3}}{\sqrt{7}}$

$=\frac{\not{\sqrt{7}}\times\sqrt{3}}{\not{\sqrt{7}}}$

$=\sqrt{3}$ **Ans**

(iii) $=\sqrt[5]{243x^5y^{10}z^{15}}$

Solution: $=\sqrt[5]{243x^5y^{10}z^{15}}$

$=\sqrt[5]{3^5x^5\left(y^2\right)^5\left(z^3\right)^5}$

$=\sqrt[3]{3^5}\times\sqrt[5]{x^5}\times\sqrt[5]{\left(y^2\right)^5}\times\sqrt[3]{\left(z^3\right)^5}$

$=3\times x\times y^2\times z^3$

$=3xy^2z^3$ **Ans**

(iv) $\frac{4}{5}\sqrt[3]{125}$

Solution: $\frac{4}{5}\sqrt[3]{125}$

$=\frac{4}{5}\sqrt[3]{5\times 5\times 5}$

$=\frac{4}{5}\sqrt[3]{5^3}$

$=\frac{4}{\not{5}}\times\not{5}$

$=4$ **Ans**

(v) $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

Solution: $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{7\times 3}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{7\times 3\times 7\times 3}$

$=\sqrt{7\times 7\times 3\times 3}$

$=\sqrt{7^2}\times\sqrt{3^2}$

$=7\times 3$

$=21$ **Ans**

Q.3 Simplify by combining similar terms.

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

Solution: $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

$=\sqrt{9\times 5}-3\sqrt{5\times 4}+4\sqrt{5}$

$=\sqrt{3^2}\times\sqrt{5}-3\sqrt{2^2}\times\sqrt{5}+4\sqrt{5}$

$=3\sqrt{5}-3\times 2\sqrt{5}+4\sqrt{5}$

$=3\sqrt{5}-6\sqrt{5}+4\sqrt{5}$

$=\sqrt{5}\left(3-6+4\right)$

$=\sqrt{5}\left(3-2\right)$

$=\sqrt{5}\left(1\right)$

$=\sqrt{5}$ **Ans**

(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

Solution: $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

$=4\sqrt{4\times 3}+5\sqrt{9\times 3}-3\sqrt{25\times 3}+\sqrt{100\times 3}$

$=4\times 2\sqrt{3}+5\times 3\sqrt{3}-3\times 5\sqrt{3}+10\sqrt{3}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$= \sqrt{3}\left(8+\cancel{15}-\cancel{15}+10\right)$
$= \sqrt{3}\left(8+10\right)$
$= \sqrt{3}\left(18\right)$
$= 18\sqrt{3}$ **Ans**

(iii) $\sqrt{3}\left(2\sqrt{3}+3\sqrt{3}\right)$

Solution: $\sqrt{3}\left(2\sqrt{3}+3\sqrt{3}\right)$
$= \sqrt{3}\times\sqrt{3}\left(2+3\right)$
$= \left(\sqrt{3}\right)^2\times\left(5\right)$
$= 3\left(5\right)$
$= 15$ Ans

(iv) $2\left(6\sqrt{5}-3\sqrt{5}\right)$

Solution: $2\left(6\sqrt{5}-3\sqrt{5}\right)$
$= 2\times\sqrt{5}\left(6-3\right)$
$= 2\times\sqrt{5}\left(3\right)$
$= 6\sqrt{5}$ Ans

Q.4 Simplify

(i) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

Solution: $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$
$= \left(3\right)^2-\left(\sqrt{3}\right)^2$
$= 9-3$
$= 6$ **Ans**

(ii) $\left(\sqrt{5}+\sqrt{3}\right)^2$

Solution: $\left(\sqrt{5}+\sqrt{3}\right)^2$
$= \left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2$
$= 5+2\sqrt{5\times3}+3$
$= 8+2\sqrt{15}$ **Ans**

(iii) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

Solution: $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$
$= \left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$
$= 5-3$
$= 2$ **Ans**

(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

Solution: $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$
$= \left(\sqrt{2}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2$
$= 2-\frac{(1)^2}{\left(\sqrt{3}\right)^2}$
$= 2-\frac{1}{3}$
$= \frac{6-1}{3}$
$= \frac{5}{3}$ **Ans**

(v) $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)\left(x+y\right)\left(x^2+y^2\right)$

Solution:
$\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)\left(x+y\right)\left(x^2+y^2\right)$

$= \left[\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right]\left(x+y\right)\left(x^2+y^2\right)$
$= \left(x-y\right)\left(x+y\right)\left(x^2+y^2\right)$
$= \left[\left(x\right)^2-\left(y\right)^2\right]\left(x^2+y^2\right)$
$= \left(x^2-y^2\right)\left(x^2+y^2\right)$

$$= \left[\left(x^2\right)^2 - \left(y^2\right)^2 \right]$$
$$= x^4 - y^4$$ **Ans**

Exercise 4.4

Q.1 Rationalize the denominator of the following

(i) $\frac{3}{4\sqrt{3}}$

Solution: $\frac{3}{4\sqrt{3}}$

$$=\frac{3}{4\sqrt{3}}$$

$$=\frac{3}{4\sqrt{3}}\times\frac{4\sqrt{3}}{4\sqrt{3}}$$

$$=\frac{3\left(4\sqrt{3}\right)}{\left(4\sqrt{3}\right)^2}$$

$$=\frac{12\sqrt{3}}{16\left(\sqrt{3}\right)^2}$$

$$=\frac{12\sqrt{3}}{16\times3}$$

$$=\frac{\cancel{12}\sqrt{3}}{\cancel{48}}$$

$$=\frac{\sqrt{3}}{4}\ \textbf{Ans}$$

(ii) $\frac{14}{\sqrt{98}}$

Solution: $\frac{14}{\sqrt{98}}$

$$=\frac{14}{\sqrt{98}}$$

$$=\frac{14}{\sqrt{98}}\times\frac{\sqrt{98}}{\sqrt{98}}$$

$$=\frac{14\left(\sqrt{98}\right)}{\left(\sqrt{98}\right)^2}$$

$$=\frac{14\left(\sqrt{7\times7\times2}\right)}{98}$$

$$=\frac{14\times7\times\sqrt{2}}{98}$$

$$=\frac{\cancel{98}\times\sqrt{2}}{\cancel{98}}$$

$$=\sqrt{2}\ \textbf{Ans}$$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

Solution: $\frac{6}{\sqrt{8}\sqrt{27}}$

$$=\frac{6}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6}{\sqrt{8}\sqrt{27}}\times\frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6\left(\sqrt{8}\sqrt{27}\right)}{\left(\sqrt{8}\right)^2\left(\sqrt{27}\right)^2}$$

$$=\frac{6\left(\sqrt{4\times2}\right)\left(\sqrt{9\times3}\right)}{8\times27}$$

$$=\frac{6\times2\sqrt{2}\times3\sqrt{3}}{216}$$

$$=\frac{6\times3\times2\left(\sqrt{2\times3}\right)}{216}$$

$$=\frac{\cancel{36}\sqrt{6}}{\cancel{216}\,6}$$

$$=\frac{\sqrt{6}}{6}\ \textbf{Ans}$$

(iv) $\dfrac{1}{3+2\sqrt{5}}$

Solution: $\dfrac{1}{3+2\sqrt{5}}$

$$=\frac{1}{3+2\sqrt{5}}$$

$$=\frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$=\frac{3-2\sqrt{5}}{(3)^2-\left(2\sqrt{5}^2\right)}$$

$$=\frac{3-2\sqrt{5}}{9-4.5}$$

$$=\frac{3-2\sqrt{5}}{9-20}$$

$$=\frac{3-2\sqrt{5}}{-11}$$ **Ans**

(v) $\dfrac{15}{\sqrt{31}-4}$

Solution: $\dfrac{15}{\sqrt{31}-4}$

$$=\frac{15}{\sqrt{31}-4}$$

$$=\frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$=\frac{\cancel{15}\left(\sqrt{31}+4\right)}{\cancel{15}}$$

$$=\sqrt{31}+4$$ **Ans**

(vi) $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

Solution: $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

$$\frac{2}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\cancel{\sqrt{5}}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}+\sqrt{3}\right)}{\cancel{2}}$$

$$=\sqrt{5}+\sqrt{3}$$ **Ans**

(vii) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution: $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$=\frac{\left(\sqrt{3}\right)^2-2\left(\sqrt{3}\right)(1)+(1)^2}{2}$$

$$=\frac{3-2\sqrt{3}+1}{2}$$

$$=\frac{4-2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{2}}$$

$= 2-\sqrt{3}$ **Ans**

(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2}{5-3}$$

$$= \frac{5+2\sqrt{15}+3}{2}$$

$$= \frac{8+2\sqrt{15}}{2}$$

$$= \frac{\cancel{2}\left(4+\sqrt{15}\right)}{\cancel{2}}$$

$= 4+\sqrt{15}$ **Ans**

Q.2 **find the conjugate of** $x+\sqrt{y}$

(i) $3+\sqrt{7}$

Solution

Conjugate $3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Solution

Conjugate $4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Solution

Conjugate $2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Solution

Conjugate $2-\sqrt{5}$

(v) $5+\sqrt{7}$

Solution

Conjugate $5-\sqrt{7}$

(vi) $4-\sqrt{15}$

Solution

Conjugate $4+\sqrt{15}$

(vii) $7-\sqrt{6}$

Solution

Conjugate $7+\sqrt{6}$

(viii) $9+\sqrt{2}$

Solution

Conjugate $9-\sqrt{2}$

Q.3

(i) **If** $x=2-\sqrt{3}$, **find** $\frac{1}{x}$

Solution: Given that $x=2-\sqrt{3}$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}}$$

$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2+\sqrt{3}}{4-3}$$

$$= \frac{2+\sqrt{3}}{1}$$

$\frac{1}{x} = 2+\sqrt{3}$ **Ans**

(ii) **If** $x=4-\sqrt{17}$, **find** $\frac{1}{x}$

Solution: Given that $x=4-\sqrt{17}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}}$$

$$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$=\frac{4+\sqrt{17}}{(4)^2-\left(\sqrt{17}\right)^2}$$

$$=\frac{4+\sqrt{17}}{16-17}$$

$$=\frac{4+17}{-1}$$

$$=-1\left(4+\sqrt{17}\right)$$

$$\frac{1}{x}=-4-\sqrt{17}\ \textbf{Ans}$$

(iii) **If** $x=\sqrt{3}+2$, **find** $x+\frac{1}{x}$

Solution: Given that $x=\sqrt{3}+2$

$$\frac{1}{x}=\frac{1}{\sqrt{3}+2}$$

$$=\frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$=\frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$=\frac{\sqrt{3}-2}{3-4}$$

$$=\frac{\sqrt{3}-2}{-1}$$

$$=-\left(\sqrt{3}-2\right)$$

$$=-\sqrt{3}+2$$

$$x+\frac{1}{x}=\left(\sqrt{3}+2\right)+\left(-\sqrt{3}+2\right)$$

$$=\sqrt{3}+2-\sqrt{3}+2$$

$$=2+2$$

$$x+\frac{1}{x}=4\ \textbf{Ans}$$

Q.4 Simplify

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$=\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{\left(\sqrt{5}-\sqrt{3}\right)+\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{5-3}$$

$$+\frac{1\left(\sqrt{5}+\sqrt{3}\right)-\sqrt{2}\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$=\frac{\sqrt{5}}{2}-\frac{\sqrt{3}}{2}+\frac{\sqrt{10}}{2}-\frac{\sqrt{6}}{2}+\frac{\sqrt{5}}{2}+\frac{\sqrt{3}}{2}-\frac{\sqrt{10}}{2}-\frac{\sqrt{6}}{2}$$

$$=\frac{2\sqrt{5}}{2}-\frac{2\sqrt{6}}{2}$$

$$=\sqrt{5}-\sqrt{6}\ \textbf{Ans}$$

(ii) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

Solution: $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

$$=\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$$

$$=\left(\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)+\left(\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$$

$$+\left(\frac{1}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$

$$=\left(\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{2\times\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}\right)$$

$$=\left(\frac{2-\sqrt{3}}{4-3}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}\right)+\left(\frac{2-\sqrt{5}}{4-5}\right)$$

$$=\left(\frac{2-\sqrt{3}}{1}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}\right)+\left(\frac{2-\sqrt{5}}{-1}\right)$$

$$=2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$

$$=\cancel{2}-\cancel{2}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}+\sqrt{5}+\sqrt{5}$$

$$=\sqrt{5}+\sqrt{5}$$

$=2\sqrt{5}$ **Ans**

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}-\dfrac{3}{\sqrt{5}+\sqrt{2}}$

Solution: $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}-\dfrac{3}{\sqrt{5}+\sqrt{2}}$

$$=\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

$$=\left(\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)-\left(\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{7}}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{1}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}\right)$$

$$=\cancel{\sqrt{5}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}-\cancel{\sqrt{5}}+\cancel{\sqrt{2}}$$

$=0$ **Ans**

Q.5 **If** $x=2+\sqrt{3}$, **then find the value of** $x-\dfrac{1}{x}$ **and** $\left(x-\dfrac{1}{x}\right)^2$

(i)

Solution: Given that $x=2+\sqrt{3}$

$$\frac{1}{x}=\frac{1}{2+\sqrt{3}}$$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2-\sqrt{3}}{4-3}$$

$$=\frac{2-\sqrt{3}}{1}$$

$$=2-\sqrt{3}$$

To find the value of $x-\dfrac{1}{x}$

$$x-\frac{1}{x}=\left(2+\sqrt{3}\right)-\left(2-\sqrt{3}\right)$$

$$=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$=\sqrt{3}+\sqrt{3}$$

$$=2\sqrt{3}$$

To find the value of $\left(x-\dfrac{1}{x}\right)^2$

We know that

$$x-\frac{1}{x}=2\sqrt{3}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{3}\right)^2$$

$$=4\left(\sqrt{3}\right)^2$$

$$=4(3)$$

$=12$ **Ans**

(ii) **If** $x=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$**, find the value of** $x+\dfrac{1}{x}$, $x^2+\dfrac{1}{x^2}$ **and** $x^3+\dfrac{1}{x^3}$

Solution: Given that $x=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$$\frac{1}{x}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$x+\frac{1}{x}=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{\left(\sqrt{5}-\sqrt{2}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

$$=\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2-2\sqrt{5}\times\sqrt{2}+\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{5}\times\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{5+2-\cancel{2\sqrt{10}}+5+2+\cancel{2\sqrt{10}}}{5-2}$$

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking square on both sides

$$\left(x+\frac{1}{x}\right)^2=\left(\frac{14}{3}\right)^2$$

$$x^2+\frac{1}{x^2}+2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)=\frac{196}{9}$$

$$x^2+\frac{1}{x^2}=\frac{196}{9}-2$$

$$x^2+\frac{1}{x^2}=\frac{196-18}{9}$$

$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

To find $x^3+\dfrac{1}{x^3}$

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking cube on both sides

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$$x^3+\frac{1}{x^3}+3(\cancel{x})\left(\frac{1}{\cancel{x}}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+\cancel{3}\left(\frac{14}{\cancel{3}}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+14=\frac{2744}{24}$$

$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$

$$x^3+\frac{1}{x^3}=\frac{2744-378}{27}$$

$$x^3+\frac{1}{x^3}=\frac{2366}{27}\quad \textbf{Ans}$$

Q.6 **Determine the rational numbers** a **and** b **if** $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}+\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$

Solution: Given that

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

$$a+b\sqrt{3}=\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$$=\frac{\left(\sqrt{3}\right)^2+(1)^2-\cancel{2\sqrt{3}}+\left(\sqrt{3}\right)^2+(1)^2+\cancel{2\sqrt{3}}}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2\left(\sqrt{3}\right)^2+2}{\left(\sqrt{3}\right)^2-1}$$

$$=\frac{2\left[\left(\sqrt{3}\right)^2+(1)^2\right]}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2(3+1)}{3-1}$$

$$=\frac{2(4)}{2}$$

$a+b\sqrt{3}=4$

$a+b\sqrt{3}=4+0\sqrt{3}$

Comparing both sides

$a=4$ $\qquad$ $b\sqrt{3}=0\sqrt{3}$

$$b=\frac{0\sqrt{3}}{\sqrt{3}}$$

$b=0$ **Ans**

Review Exercise 4

Q.1 Multiple type questions?

(i) **is an algebraic …**

(a) Expression **(b)** Sentence
(c) Equation **(d)** In-equation

(ii) **The degree of polynomial** $4x^4 + 3x^2y$ **is**

(a) 1 **(b)** 2
(c) 3 **(d)** 4

(iii) $a^3 + b^3$ **is equal to**

(a) $(a-b)(a^2+ab+b^2)$ **(b)** $(a+b)(a^2-ab+b^2)$
(c) $(a-b)(a^2-ab+b^2)$ **(d)** $(a-b)(a^2+ab+b^2)$

(iv) $(3+\sqrt{2})(3-\sqrt{2})$ **is equal to**

(a) 7 **(b)** -7
(c) -1 **(d)** 1

(v) **Conjugate of surd** $a+\sqrt{b}$ **is;**

(a) $-a+\sqrt{b}$ **(b)** $a-\sqrt{b}$
(c) $\sqrt{a}+\sqrt{b}$ **(d)** $\sqrt{a}-\sqrt{b}$

(vi) $\dfrac{1}{a-b} - \dfrac{1}{a+b}$ **is equal to**

(a) $\dfrac{2a}{a^2-b^2}$ **(b)** $\dfrac{2b}{a^2-b^2}$
(c) $\dfrac{-2a}{a^2-b^2}$ **(d)** $\dfrac{-2b}{a^2-b^2}$

(vii) $\dfrac{a^2-b^2}{a+b}$ **is equal to**

(a) $(a-b)^2$ **(b)** $(a+b)^2$
(c) $a+b$ **(d)** $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ **is equal to**

(a) a^2+b^2 **(b)** a^2-b^2
(c) $a-b$ **(d)** $a+b$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
a	d	b	a	b	b	d	c

Q.2 Fill in the blanks

(i) The degree of polynomial $x^2y^2 + 3xy + y^3$ is ________

(ii) $x^2 - 4$ ________

(iii) $x^3 + \frac{1}{x^3} = \left[x + \frac{1}{x}\right]$(________)

(iv) $2(a^2 + b^2) = (a+b)^2 + ($________$)^2$

(v) $\left[x - \frac{1}{x}\right]^2 =$ ________

(vi) Order of surd $\sqrt[3]{x}$ is ________

(vii) $\frac{1}{2-\sqrt{3}} =$ ________

ANSWER KEY

(i) 4

(ii) $(x-2)(x+2)$

(iii) $x^2 - 1 + \frac{1}{x^2}$

(iv) $a - b$

(v) $x^2 + \frac{1}{x^2} - 2$

(vi) 3

(vii) $2 + \sqrt{3}$

Q.3 If $x + \frac{1}{x} = 3$, find

(i) $x^2 + \frac{1}{x^2}$

Solution: Given that $x + \frac{1}{x} = 3$

$\therefore (a+b)^2 = a^2 + b^2 + 2ab$

Putting the values

$$\left[x + \frac{1}{x}\right]^2 = (x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right)$$

$$(3)^2 = x^2 + \frac{1}{x^2} + 2$$

$$9 = x^2 + \frac{1}{x^2} + 2$$

$$9 - 2 = x^2 + \frac{1}{x^2}$$

$x^2 + \frac{1}{x^2} = 7$ **Ans**

(ii) $x^4+\dfrac{1}{x^4}$

Solution: Given that $x^2+\dfrac{1}{x^2}=7$

$$\left(x^2+\frac{1}{x^2}\right)^2=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(7)^2=x^4+\frac{1}{x^4}+2$$

$$49=x^4+\frac{1}{x^4}+2$$

$$x^4+\frac{1}{x^4}=49-2$$

$$x^4+\frac{1}{x^4}=47\ \textbf{Ans}$$

Q.4 **If** $x-\dfrac{1}{x}=2$ **find**

(i) $x^2+\dfrac{1}{x^2}$ $\quad$ $(ii)\,x^4+\dfrac{1}{x^4}$

Solution (i)

Given that $x-\dfrac{1}{x}=2$

$$\therefore (a+b)^2=a^2+b^2+2ab$$

Putting the values

$$\left(x-\frac{1}{x}\right)^2=(x)^2+\left(\frac{1}{x}\right)^2-2(x)\left(\frac{1}{x}\right)$$

$$(2)^2=x^2+\frac{1}{x^2}-2$$

$$4+2=x^2+\frac{1}{x^2}$$

$$x^2+\frac{1}{x^2}=6\ \textbf{Ans}$$

Solution (ii)

Given that $x^2+\dfrac{1}{x^2}=6$

$$\left(x^2+\frac{1}{x}\right)=x^4+\frac{1}{x^4}+2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(6)^2=x^4+\frac{1}{x^4}+2$$

$$x^4+\frac{1}{x^4}=36-2$$

$$x^4+\frac{1}{x^4}=34\ \textbf{Ans}$$

Q.5 **Find the value of** x^3+y^3 **and** xy **if** $x+y=5$ **and** $x-y=3$**.**

Solution: Given that $x+y=5$

$x-y=3$

As we know that

$$\therefore (x+y)^2-(x-y)^2=4xy$$

Putting the values

$$4xy=(5)^2-(3)^3$$

$$4xy=25-9$$

$$4xy=16$$

$$xy=\frac{16^{4}}{4}$$

$$xy=4\ \textbf{Ans}$$

As we know that

$$(x+y)^3=x^3+y^3+3xy(x+y)$$

Putting the values

$$(5)^3=x^3+y^3.3\times4\times5$$

$$125=x^3+y^3+60$$

$$x^3+y^3=125-60$$

$$x^3+y^3=65$$

$$x^3+y^3=65\ \textbf{Ans}$$

Q.6 If $P = 2+\sqrt{3}$, find

(i) $P+\dfrac{1}{P}$

Solution: Given that $P = 2+\sqrt{3}$

$$\frac{1}{P} = \frac{1}{2+\sqrt{3}}$$

$$\frac{1}{P} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{P} = \frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$\frac{1}{P} = \frac{2-\sqrt{3}}{4-3}$$

$$\frac{1}{P} = \frac{2-\sqrt{3}}{1}$$

$$\frac{1}{P} = 2-\sqrt{3}$$

$$P+\frac{1}{P} = 2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}$$

$$P+\frac{1}{P} = 4 \textbf{ Ans}$$

(ii) $P-\dfrac{1}{P}$

As we know that

$\dfrac{1}{P} = 2-\sqrt{3}$ and

$P = 2+\sqrt{3}$

$$P-\frac{1}{P} = 2+\sqrt{3}-\left(2-\sqrt{3}\right)$$

$$= \cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$= 2\sqrt{3} \textbf{ Ans}$$

(iii) $P^2+\dfrac{1}{P^2}$

Solution: Given that $P+\dfrac{1}{P} = 4$

$$\therefore (a+b)^2 = a^2+b^2+2ab$$

$$\left(P+\frac{1}{P}\right)^2 = (P)^2+\left(\frac{1}{P}\right)^2+2\left(\cancel{P}\right)\left(\frac{1}{\cancel{P}}\right)$$

$$(4)^2 = P^2+\frac{1}{P^2}+2$$

$$16-2 = P^2+\frac{1}{P^2}$$

$$P^2+\frac{1}{P^2} = 14 \textbf{ Ans}$$

(iv) $P^2-\dfrac{1}{P^2}$

Solution:

$$P^2-\frac{1}{P^2} = \left(P+\frac{1}{P}\right)\left(P-\frac{1}{P}\right)$$

$$P^2-\frac{1}{P^2} = (4)\left(2\sqrt{3}\right)$$

$$= 8\sqrt{3} \textbf{ Ans}$$

Q.7 If $q = \sqrt{5}+2$ find.

(i) $q+\dfrac{1}{q}$

Solution: Given that $q = \sqrt{5}+2$

$$\frac{1}{q} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$= \frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$= \frac{\sqrt{5}-2}{5-4}$$

$$= \sqrt{5}-2$$

$$q+\frac{1}{q} = \sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2}$$

$$q+\frac{1}{q} = 2\sqrt{5} \textbf{ Ans}$$

(ii) $q-\frac{1}{q}$

Solution: Given that $q=\sqrt{5}+2$

$$\frac{1}{q}=\sqrt{5}-2$$

$$q-\frac{1}{q}=\sqrt{5}+2-\left(\sqrt{5}-2\right)$$

$$=\cancel{\sqrt{5}}+2-\cancel{\sqrt{5}}+2$$

$$q-\frac{1}{q}=4 \textbf{ Ans}$$

(iii) $q^2+\frac{1}{q^2}$

Solution: Given that $q-\frac{1}{q}=4$

Squaring both sides

$$\left(q-\frac{1}{q}\right)^2=(4)^2$$

$$q^2+\frac{1}{q^2}-2=16$$

$$q^2+\frac{1}{q^2}=16+2$$

$$q^2+\frac{1}{q^2}=18 \textbf{ Ans}$$

(iv) $q^2-\frac{1}{q^2}$

Solution: Given that $q+\frac{1}{q}=2\sqrt{5}$

$$q-\frac{1}{q}=4$$

By using formula

$$q^2-\frac{1}{q^2}=\left(q+\frac{1}{q}\right)\left(q-\frac{1}{q}\right)$$

$$=\left(2\sqrt{5}\right)(4)$$

$$=8\sqrt{5} \textbf{ Ans}$$

Q.8 Simplify

(i) $\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$

Solution:

$$=\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^2-\left(\sqrt{a^2-2}\right)^2}$$

$$=\frac{\left(\sqrt{a^2+2}\right)^2+\left(\sqrt{a^2-2}\right)^2+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{a^2+2-a^2+2}$$

$$=\frac{a^2+\cancel{2}+a^2-\cancel{2}+2\left(\sqrt{a^4\cancel{-2a^2}+\cancel{2a^2}-4}\right)}{4}$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{\cancel{2}\left(a^2+\sqrt{a^4-4}\right)}{\cancel{4}\,2}$$

$$=\frac{a^2+\sqrt{a^4-4}}{2} \textbf{ Ans}$$

(ii) $\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$

$$=\left(\frac{1}{a-\sqrt{a^2-x^2}}\times\frac{a+\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}\right)$$

$$-\left(\frac{(1)}{\left(a+\sqrt{a^2-x^2}\right)}\frac{\left(a-\sqrt{a^2-x^2}\right)}{\left(a-\sqrt{a^2-x^2}\right)}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{a-\left(a^2-x^2\right)}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{a-\left(a^2-x^2\right)}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{a^2-a^2+x^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{\cancel{a^2}-\cancel{a^2}+x^2}\right)$$

$$= \left(\frac{a+\sqrt{a^2-x^2}}{x^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{x^2}\right)$$

$$= \frac{\not{a}+\sqrt{a^2-x^2}-\not{a}+\sqrt{a^2-x^2}}{x^2}$$

$$= \frac{2\sqrt{a^2-x^2}}{x^2} \quad \textbf{Ans}$$

Unit 4: Algebraic Expressions and Algebraic Formulas Overview

Algebraic expression:

An algebraic expression is that in which constants or variables or both are combined by basic operations.

Polynomial:

Polynomial means an expression with many terms.

Degree of Polynomial:

Degree of Polynomial means highest power of variable.

Rational expression:

Expression in the form $\frac{p(x)}{q(x)}, (q(x) \neq 0)$ is called rational expression.

Surd:

An irrational radical with rational radicand is called a surd.

e.g., $\sqrt{3}, \sqrt{\frac{2}{5}},$

Monomial Surd:

A surd which contains a single term is called monomial surd.

Binomial Surd:

A surd which contains sum or difference of two surds is called binomial surd.