Review Exercise 6

Q.1 Choose the correct answer.

(i) **H.C.F of** $p^3q - pq^3$ **and** $p^5q^2 - pq^5$ **is** ________

(a) $pq(p^2 - q^2)$ (b) $pq(p-q)$

(c) $p^2q^2(p-q)$ (d) $pq(p^3 - q^3)$

(ii) **H.C.F of** $5x^2y^2$ *and* $20x^3y^3$ is ________

(a) $5x^2y^2$ (b) $20x^3y^3$

(c) $100x^5y^5$ (d) $5xy$

(iii) **H.C.F of** $x-2$ **and** x^2+x-6 ______

(a) x^2+x-6 (b) $x+3$

(c) $x-2$ (d) $x+2$

(iv) **H.C.F of** a^3+b^3 **and** a^2-ab+b^2 ______

(a) $a+b$ (b) a^2-ab+b^2

(c) $(a-b)^2$ (d) a^2+b^2

(v) **H.C.F of** x^2-5x+6 **and** x^2-x-6 is______

(a) $x-3$ (b) $x+2$

(c) x^2-4 (d) $x-2$

(vi) **H.C.F of** a^2-b^2 **and** a^3-b^3 is______

(a) $a-b$ (b) $a+b$

(c) a^2+ab+b^2 (d) a^2-ab+b^2

(vii) **H.C.F of** x^2+3x+2, x^2+4x+3 **and** x^2+5x+4 is______

(a) $x+1$ (b) $(x+1)(x+2)$

(c) $x+3$ (d) $(x+4)(x+1)$

(viii) **L.C.M of** $15x^2, 45xy$ **and** $30xyz$ is______

(a) $90xyz$ (b) $90x^2yz$

(c) $15xyz$ (d) $15x^2yz$

(ix) **L.C.M of** a^2+b^2 **and** a^4-b^4 is______

(a) a^2+b^2 (b) a^2-b^2

(c) a^4-b^4 (d) $a-b$

(x) **The product of two algebraic expression is equal to the ________ of their H.C.F and L.C.M**

(a) Sum (b) Difference

(c) Product (d) Quotitent

(xi) **Simplify** $\dfrac{a}{9a^2-b^2}+\dfrac{1}{3a-b}$ is______

(a) $\frac{4a}{9a^2-b^2}$ (b) $\frac{4a-b}{9a^2-b^2}$

(c) $\frac{4a+b}{9a^2-b^2}$ (d) $\frac{b}{9a^2-b^2}$

(xii) **Simplify** $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=$ ______

(a) $\frac{a+7}{a-6}$ (b) $\frac{a+7}{a-2}$

(c) $\frac{a+3}{a-6}$ (d) $\frac{a-2}{a+3}$

(xiii) **Simplify the** $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}=$ ______

(a) $\frac{1}{a+b}$ (b) $\frac{1}{a-b}$

(c) $\frac{a-b}{a^2+b^2}$ (d) $\frac{a+b}{a^2+b^2}$

(xiv) **Simplify** $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{x}{x+y}\right)=$ ______

(a) $\frac{x}{x+y}$ (b) $\frac{y}{x+y}$

(c) $\frac{y}{x}$ (d) $\frac{x}{y}$

(xv) **The square root of** a^2-2a+1 is ______

(a) $\pm(a+1)$ (b) $\pm(a-1)$

(c) $a-1$ (d) $a+1$

(xvi) **What should be added to complete the square of** x^4+64 ?______

(a) $8x^2$ (b) $-8x^2$

(c) $16x^2$ (d) $4x^2$

(xvii) **The square root to** $x^4+\frac{1}{x^4}+2$ **is**______

(a) $\pm\left(x+\frac{1}{x}\right)$ (b) $\pm\left(x^2+\frac{1}{x^2}\right)$

(c) $\pm\left(x-\frac{1}{x}\right)$ (d) $\pm\left(x^2-\frac{1}{x^2}\right)$

ANSWER KEYS

1	b	5	a	9	c	13	a	17	b
2	a	6	a	10	c	14	d		
3	c	7	a	11	c	15	b		
4	b	8	b	12	a	16	c		

Q.2 Find the H.C.F of the following by factorization.

$8x^4-128, 12x^3-96$

Solution:

$8x^4-128 \quad =8\left(x^4-16\right) \quad =8\left[(x)^2-(4)^2\right]$

$=2\times2\times2\left(x^2+4\right)\left(x^2-4\right)$

$=2\times2\times2\left(x^2+4\right)(x+2)(x-2)$

$12x^3-96 \quad =12(x^3-8)$

$=\left(12(x^3-2^3\right)$

$=12(x-2)(x^2+2x+4)$

$2\times2\times3(x-2)\left(x^2+2x+4\right)$

H.C.F = $2\times2(x-2)$

$=4(x-2)$

Q.3 Find the H.C.F of the following by division method $y^3+3y^2-3y-9, 3y^2-8y-2y$**.**

Solution: $y^3+3y^2-3y-9,$

$=y^3+3y^2-3y-9$

$$
\begin{array}{r|l}
 & \hfill 1 \\
y^3+3y^2-8y-24 & y^3+3y^2-3y-9 \\
 & \pm y^3 \pm 3y^2 \pm 8y \pm 24 \\
\hline
 & \hfill 5y+15 \\
 & 5(y+3)
\end{array}
$$

$$
\begin{array}{r|l}
 & \hfill y^2-8 \\
y+3 & y3+3y-8y-24 \\
 & \pm y3 \pm 3y \\
\hline
 & -8y-24 \\
 & \pm 8y \pm 24 \\
\hline
 & \times
\end{array}
$$

H.C.F $=(y+3)$

Q.4 Find the L.C.M of the following by factorization.

$12x^2-75,\ 6x^2-13x-5,\ 4x^2-20x+25$

Solution:

$$12x^2-75 = 3\left(4x^2-25\right)$$
$$= 3\left[(2x)^2-(5)^2\right]$$
$$= 3(2x-5)(2x+5)$$

$$6x^2-15x+2x-5 = 3x(2x-5)+1(2x-5)$$
$$= (2x-5)(3x+1)$$

$$4x^2-20x+25 = 4x^2-10x-10x+25$$
$$= 2x(2x-5)-5(2x-5)$$
$$= (2x-5)(2x-5)$$

Common factor = (2x-5)

Non common factor $= 3(3x+1)(2x-5)2x+5)$

L.C.M = common factor $\times$ non common factor

$$\text{L.C.M} = (2x-5)3(3x+1)(2x+5)(2x-5)$$

$$\text{L.C.M} = 3(2x+5)(2x-5)^2(3x+1)$$

Q.5 If H.C.F of $x^4+3x^3+5x^2+26x+56$ and $x^4+2x^3-4x^2-x+28$ is x^2+5x+7 find their L.C.M.

Solution: $p(x)=x^4+3x^3+5x^2+26x+56$ and $q(x)=x^4+2x^3-4x^2-x+28$

$HCF = x^2+5x+7$, LCM=?

$$L.C.M = \frac{P(x)\times q(n)}{H.C.F}$$

$$\text{L.C.M} = \frac{\left(x^4+3x^2+5x^2+26x+56\right)\times\left(x^4+2x^3-4x^2-x+28\right)}{\left(x^2+5x+7\right)}$$

$$\begin{array}{r|l}
 & x^2-3x+4 \\
x^2+5x+7 & x^4+2x^3-4x^2-x+28 \\
 & \pm x^4 \pm 5x^3 \pm 7x^2 \\
 & -3x^3-11x^2-x+28 \\
 & \mp 3x^3 \mp 15x^2 \mp 21x \\
 & +4x^2+20x+28 \\
 & \pm 4x^2 \pm 20x \pm 28 \\
 & 0
\end{array}$$

$$x^2-3x+4$$

$$L.C.M=\frac{(x^4+3x^3+5x^2+26x+56)\,(x^4+2x^3-4x^2-x+28)}{(x2+5x+7)}$$

$$L.C.M=(x^4+3x^3+5x^2+26x+56)(x^2-3x+4)$$

Q.6 Simplify:

Solution:

(i) $\frac{3}{x^3+x^2+x+1}-\frac{3}{x^3-x^2+x-1}$

Solution: $\frac{3}{x^3+x^2+x+1}-\frac{3}{x^3-x^2+x-1}$

$$=\frac{3}{x^2(x+1)+1(x+1)}-\frac{3}{x^2(x-1)+1(x-1)}$$

$$=\frac{3}{(x^2+1)(x+1)}-\frac{3}{(x-1)(x^2+1)}$$

$$=\frac{3(x-1)-3(x+1)}{(x+1)(x-1)(x^2+1)}$$

$$=\frac{3x-3-3x-3}{(x-1)(x-1)(x^2+1)}$$

$$=\frac{-6}{(x+1)(x-1)(x^2+1)}=\frac{-6}{(x^2-1)(x^2+1)}$$

$$=\frac{-6}{(x^4-1)}$$

$$=\frac{6}{1-x^4}$$

(ii) $\frac{a+b}{a^2-b^2}\div\frac{a^2-ab}{a^2-2ab+b^2}$

Solution: $\frac{a+b}{a^2-b^2}\div\frac{a^2-ab}{a^2-2ab+b^2}$

$$=\frac{a+b}{a^2-b^2}\times\frac{a^2-2ab+b^2}{a^2-ab}$$

$$=\frac{a+b}{(a-b)(a+b)}\times\frac{(a-b)^2}{a(a-b)}$$

$$=\frac{(a-b)^2}{a(a-b)^2}$$

$$=\frac{1}{a}$$

Q.7 Find the square root by using factorization. $\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \quad (x\neq 0)$.

Solution: $(x^2+\frac{1}{x^2}+10\left(x+\frac{1}{x}\right)+27$

$$=(x^2+\frac{1}{x^2}+10\left(x+\frac{1}{x}\right)+25+2$$

$$=x^2+\frac{1}{x^2}+2+10\left(x+\frac{1}{x}\right)+25$$

$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)\times 5+(5)^2$$

$$=\left[x+\frac{1}{x}+5\right]^2$$

Taking the square root

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left[x+\frac{1}{x}+5\right]^2}$$

$$=\pm\left(x+\frac{1}{x}+5\right)$$

Q.8 Find the square roots by using division method. $\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x}$

Solution:

$$\frac{2x}{y}+5-\frac{3y}{x}$$

$$\frac{2x}{y}\,\Big)\;\frac{4x^2}{y^2}+\frac{20x}{4}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\frac{4x^2}{y^2}$$

$$\frac{4x}{y}+5\,\Big)\;\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\frac{20x}{y}\pm 25$$

$$\frac{4x}{y}+10-\frac{3y}{x}\Bigg)\,-\cancel{12}-\cancel{\frac{30y}{x}}+\cancel{\frac{9y^2}{x^2}}$$

$$\mp\cancel{12}\mp\cancel{\frac{30y}{x}}\pm\cancel{\frac{9y^2}{x^2}}$$

$$\textbf{Square root}=\pm\left[\frac{2x}{y}+5-\frac{3y}{x}\right]$$

Unit 6: Algebraic Manipulation

Overview

Highest Common Factor:

If two or more algebraic expressions are given then their common factor of highest power is called the H.C.F. of the expression.

Least Common Multiple(L.C.M):

The Least common Multiple (L.C.M) is the product of common factors together with non-common factors of the given expressions.