Exercise 6.3

Q.1 **Use factorization to find the square root of the following expression.**

(i) $4x^2 - 12xy + 9y^2$

Solution: $4x^2 - 12xy + 9y^2$

$4x^2 - 12xy + 9y^2 = 4x^2 - 6xy - 6xy + 9y^2$

$= 2x(2x-3y) - 3y(3x-3y)$

$= (2x-3y)(2x-3y)$

$4x^2 - 12y + 9y^2 = (2x-3y)^2$

Taking square root on both side

$$\sqrt{4x^2 - 12xy + 9y^2} = \sqrt{[2x-3y]^2}$$

$$= \pm(2x-3y)$$

(ii) $x^2 - 1 + \dfrac{1}{4x^2}$

Solution: $x^2 - 1 + \dfrac{1}{4x^2}$

$$= (x)^2 - 2(x)\left[\frac{1}{2x}\right] + \left[\frac{1}{2x}\right]^2$$

$$= \left[x - \frac{1}{2x}\right]^2$$

Taking square root

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \sqrt{\left[x - \frac{1}{2x}\right]^2}$$

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \pm\left(x - \frac{1}{2x}\right)$$

(iii) $\dfrac{1}{16}x^2 - \dfrac{1}{12}xy + \dfrac{1}{36}y^2$

Solution: $\dfrac{1}{16}x^2 - \dfrac{1}{12}xy + \dfrac{1}{36}y^2$

$$= \left(\frac{1}{4}x\right)^2 - 2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right) + \left(\frac{1}{6}y\right)^2$$

$$= \left(\frac{x}{4} - \frac{y}{6}\right)^2$$

Taking the square root

$$\sqrt{\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2} = \sqrt{\left(\frac{1}{4}x - \frac{1}{6}y\right)^2}$$

$$= \pm\left(\frac{1}{4}x - \frac{1}{6}y\right)$$

$$= \pm\left(\frac{x}{4} - \frac{y}{6}\right)$$

(iv) $4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2$

Solution: $4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2$

$= \left[2(a+b)^2\right] - 2\left[2(a+b)\right]\left[3(a-b)\right] + \left[3(a-b)\right]^2$

$= \left[2(a+b) - 3(a-b)\right]^2$

Taking square root

$\sqrt{4(a+b)^2 - 12(a2+b^2) + 9(a-b)^2} = \sqrt{\left[2(a+b) - 3(a-b)\right]^2}$

$= \pm\left[2a + 2b - 3a + 3b\right]$

$= \pm(5b - a)$

(v) $\dfrac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$

Solution: $\dfrac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}$

$$= \frac{(2x^3)^2 - 2(2x^3)(3y^3) + (3y^3)^2}{(3x^3)^2 + 2(3x^2)(4y^2) + (4y^2)^2}$$

=

$$= \frac{\left[2x^3 - 3y^3\right]^2}{\left[3x^3 + 4y^2\right]^2}$$

Taking square root

$$= \sqrt{\frac{4x^6 - 12x^3y^3 + 9y^6}{9x^4 + 24x^2y^2 + 16y^4}}$$

$$= \pm\left(\frac{2x^3 - 3y^3}{3x^2 + 4y^2}\right)$$

(vi) $\left(x + \dfrac{1}{x}\right)^2 - 4\left(x - \dfrac{1}{x}\right), (x \neq 0)$

Solution: $\left(x + \dfrac{1}{x}\right)^2 - 4\left(x - \dfrac{1}{x}\right), (x \neq 0)$

By adding and substituting 4

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right)$$

$$= x^2 + \frac{1}{x^2} + 2 - 4\left(x - \frac{1}{x}\right) - 4 + 4$$

$$= x^2 + \frac{1}{x^2} - 2 - 4\left(x - \frac{1}{x}\right) + 4$$

$$= \left(x - \frac{1}{x}\right)^2 - 2\left(x - \frac{1}{x}\right)(2) + (2)^2$$

$$\left[\left(x - \frac{1}{x}\right) - 2\right]^2$$

Taking square root

$$\sqrt{\left(x + \frac{1}{x}\right)^2 - 4\left(x - \frac{1}{x}\right)} = \sqrt{\left[x - \frac{1}{x} - 2\right]^2}$$

$$= \pm\left(x - \frac{1}{x} - 2\right)$$

(vii) $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

Solution: $\left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x + \frac{1}{x}\right)^2 + 12$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4\left[x^2 + \frac{1}{x^2} + 2\right] + 12$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 4x^2 - \frac{4}{x^2} - 8 + 12$$

$$= \left(x^2 + \frac{1}{x^2}\right)^2 - 4\left(x^2 + \frac{1}{x^2}\right) + 4$$

$$= \left[x^2 + \frac{1}{x^2}\right]^2 - 2\left[x^2 + \frac{1}{x^2}\right](2) + (2)^2$$

$$= \left[x^2 + \frac{1}{x^2} - 2\right]^2$$

Taking square root

$$= \sqrt{\left[x^2 + \frac{1}{x^2}\right] - 4\left[x + \frac{1}{x}\right]2 + 12}$$

$$= \sqrt{\left[x^2 \frac{1}{x^2} - 2\right]^2}$$

$$= \pm\left(x^2 + \frac{1}{x^2} - 2\right)$$

(viii) $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

Solution: $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

$=\left[x^2+2x+x+2\right]\left[x^2+3x+x+3\right]\left[x^2+3x+2x+6\right]$

$=\left[x(x+2)+1(x+2)\right]\left[x(x+3)+1(x+3)\right]\left[x(x+3)+2(x+3)\right]$

$=(x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$

$=(x+2)^2(x+1)^2(x+3)^2$

Taking square root

$=\sqrt{(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)}$

$=\sqrt{(x+2)^2(x+1)^2(x+3)^2}$

$=\pm(x+1)(x+2)(x+3)$ **Ans**

(ix) $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

Solution: $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

$=(x^2+7x+1x+7)(2x^2-3x+2x-3)(2x^2+14x-3x-21)$

$=\left[(x(x+7)+1(x+7)\right]\left[x(2x-x)+1(2x-3)\right]\left[(2x(x+7)-3(x+7)\right]$

$=(x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$

$=(x+7)^2(x+1)^2(2x-3)^2$

Taking square root

$=\sqrt{(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)}$

$=\sqrt{(x+7)^2(x+1)^2(2x-3)^2}$

$=\pm(x+1)(x+7)(2x-3)$ **Ans**

Q.2 **Use division method to find the square root of the following expression.**

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution: $4x^2+12xy+9y^2+16x+24y+16$

$$2x+3y+4$$

$$2x\,\big)\,\cancel{4x^2}+12xy+9y^2+16x+24y+16$$

$$\pm\cancel{4x^2}$$

$$4x+3y\,\big)\,12\cancel{xy}+9\cancel{y^2}+16x+24y+16$$

$$\pm12\cancel{xy}\pm9\cancel{y^2}$$

$$4x+6y+4\,\big)\,\cancel{16x}\ \cancel{24y}+\cancel{16}$$

$$\cancel{16x}\pm\cancel{24y}\pm\cancel{16}$$

$$0$$

Square root $=\pm\left(2x+3y+4\right)$

(ii) $x^4-10x^3+37x^2-60x+36$

Solution: $x^4-10x^3+37x^2-60x+36$

$$
\begin{array}{rl}
 & \overline{\qquad\qquad\qquad\qquad x^2-5x+6} \\
x^2 \big) & \cancel{x^4}-10x^3+37x^2-60x+36 \\
 & \underline{\pm\cancel{x^4}\qquad\qquad\qquad} \\
2x^2-5x \big) & \overline{\cancel{10x^3}+37x^2-60x+36} \\
 & \underline{\pm\cancel{10x^3}\pm 25x^2} \\
2x^2-10x+6 \big) & \overline{\cancel{12x^2}+\cancel{60x}+\cancel{36}} \\
 & \underline{\pm\cancel{12x^2}\pm\cancel{60x}\pm\cancel{36}} \\
 & \times
\end{array}
$$

Square root $=\pm\left(x^2-5x+6\right)$

(iii) $9x^4-6x^3+7x^2-2x+1$

Solution: $9x^4-6x^3+7x^2-2x+1$

$$
\begin{array}{rl}
 & \overline{\qquad\qquad\qquad\qquad 3x^2-x+1} \\
3x^2 \big) & \cancel{9x^4}-6x^3+7x^2-2x+1 \\
 & \underline{\pm\cancel{9x^4}\qquad\qquad} \\
6x^2-x \big) & \overline{\cancel{6x^3}+\cancel{7x^2}-2x+1} \\
 & \underline{\pm\cancel{6x^3}\pm\cancel{7x^2}} \\
6x^2-2x+1 \big) & \overline{\cancel{6x^2}-\cancel{2x}+\cancel{1}} \\
 & \underline{\pm\cancel{6x^2}\pm\cancel{2x}\pm\cancel{1}} \\
 & \times
\end{array}
$$

Square root $\pm\left(=3x^2-x+1\right)$

(iv) $4+25x^2+7x^2-2x+1$

Solution: $4+25x^2-12x-24x^3+16x^4$

$$4x^2-3x+2$$

$$4x^2\Big)\ \cancel{16x^4}-24x^3+25x^2-12x+4$$

$$\underline{\pm\cancel{16x^4}}$$

$$8x^2-3x\Big)\ -\cancel{24x^3}+25x^2-12x+4$$

$$\underline{\pm\cancel{24x^3}\ \pm 9x^2}$$

$$8x^2-6x+2\Big)\ \cancel{16x^2}-\cancel{12x}+\cancel{4}$$

$$\underline{\pm\cancel{16x^2}\ \pm\cancel{12x}\ \pm\cancel{4}}$$

$$\times$$

Square root $=\pm\left(4x^2-3x+2\right)$

(v) $\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},(x\neq 0, y\neq 0)$

Solution: $\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},(x\neq 0, y\neq 0)$

$$\frac{x}{y}-5+\cancel{y}/\cancel{x}$$

$$\frac{x}{y}\Bigg)\ \cancel{\frac{x^2}{y^2}}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},$$

$$\underline{\pm\cancel{\frac{x^2}{y^2}}}$$

$$\frac{2x}{y}-5\Bigg)\ -\cancel{\frac{10x}{y}}+27-\frac{10y}{x}+\frac{y^2}{x^2}$$

$$\underline{\pm\cancel{\frac{x^2}{y^2}}\ \pm 25}$$

$$\frac{2x}{y}-10+\frac{y}{x}\Bigg)\ +\cancel{2}-\cancel{\frac{10y}{c}}+\cancel{\frac{y^2}{x^2}}$$

$$\underline{\pm\cancel{2}\mp\cancel{\frac{10y}{x}}\pm\cancel{\frac{y^2}{x^2}}}$$

$$\times$$

Square root $=\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$

Q.3 **Find the value of k for which the following expressions will become a perfect square.**

(i) $4x^4 - 12x^3 + 37x^2 - 42x + k$

Solution: $4x^4 - 12x^3 + 37x^2 - 42x + k$

$$\begin{array}{r|l} & \qquad\qquad\qquad\qquad 2x^2 - 3x + 7 \\ \hline 2x^2 & \cancel{4x^4} - 12x^3 + 37x^2 - 42x + k \\ & \pm\cancel{4x^4} \\ \hline 4x^2 - 3x & -\cancel{12x^3} + 37x^2 - 42x + k \\ & \pm\cancel{12x^3} \pm 9x^2 \\ \hline 4x^2 - 6x + 7 & \cancel{28x^2} - \cancel{42x} + k \\ & \pm\cancel{28x^2} \pm \cancel{42x} \pm 49 \\ \hline & k - 49 \end{array}$$

In the case of perfect square remainder is always is equal to zero so

$k - 49 = 0$

$k = 49$

(ii) $x^4 - 4x^3 + 10x^2 - kx + 9$

Solution: $x^4 - 4x^3 + 10x^2 - kx + 9$

$$\begin{array}{r|l} & \qquad\qquad x^2 - 2x + 3 \\ \hline = x^2 & \cancel{x^4} - 4x^3 + 10x^2 - kx + 9 \\ & \pm\cancel{x^4} \\ \hline 2x^2 - 2x & -\cancel{4x^3} + 10x^2 - kx + 9 \\ & \pm\cancel{4x^3} \pm 4x^2 \\ \hline 2x^2 - 4x + 3 & 6x^2 - kx + 9 \\ & -6x^2 \mp 12x \pm 9 \\ \hline \end{array}$$

$-kx + 12x = 0$

In the case of square root remainder is always equal to zero

$-x(k - 12) = 0$

$$k - 12 = \frac{0}{-x}$$

$k - 12 = 0$

$k = 12$

Q.4 **Find the value of l and m for which the following expression will be perfect square**

(i) $x^4+4x^3+16x^2+ln+m$

Solution: $x^4+4x^3+16x^2+ln+m$

$$x^2+2x+6$$

$$=x^2\Big)\,\cancel{x^4}+4x^3+16x^2+lx+m$$

$$\pm\cancel{x^4}$$

$$2x^2+2x\Big)\,\cancel{4x^3}+16x^2+lx+m$$

$$\pm\cancel{4x^3}\pm4x^2$$

$$2x^2+4x+6\Big)\,\cancel{12x^2}+lx+m$$

$$\pm\cancel{12x^2}\pm24x-36$$

In the case of square root remainder is always zero

$(lx-24x)$, $m-36=0$

$x(l-24)=0$, $m=36$ **Ans**

$l-24=\dfrac{0}{x}$

$l-24=0$

$l=24$ **Ans**

(ii) $49x^4-70x^3+109x^2+lx-m$

Solution: $49x^4-70x^3+109x^2+lx-m$

$$7x^2-5x+6$$

$$7x^2\Big)\,\cancel{49x^4}-70x^3+109x^2+lx-m$$

$$\pm\cancel{49x^4}$$

$$14x^2-5x\Big)\,-\cancel{70x^3}+109x^2+lx-m$$

$$\mp\cancel{70x^3}\pm25x^2$$

$$14x^2-10x+6\Big)\,\cancel{84x^2}+lx-m$$

$$\pm\cancel{84x^2}\mp60x\pm36$$

$$lx+60x-m-36$$

$$(l+60)x-m-36$$

In the case of square root remainder is always equal to zero

$-m-36=0$

$-m=36$

$l+60=0$ $m=-36$

$l=-60$ **Ans**

Q.5 **To make the expression** $9x^4-12x^3+22x^2-13x+12$ **a perfect square**

Solution: $9x^4-12x^3+22x^2-13x+12$

$$3x^2-2x+3$$

$$=3x^2\overline{)\cancel{9x^4}-12x^3-+22x^2-13x+12}$$

$$\underline{\pm\cancel{9x^4}}$$

$$6x^2-2x\overline{)-\cancel{12x^3}+22x^2-13x+12}$$

$$\underline{\pm\cancel{12x^3}\pm4x^2}$$

$$6x^2-4x+3\overline{)\cancel{18x^2}-13x+12}$$

$$\underline{\pm\cancel{18x^2}\mp12x\pm9}$$

$$-x+3$$

(i) $+x-3$ **is to be added**

(ii) $-x+3$ **is to be subtract from it**

(iii) $-x+3=0$

$x=3$