Exercise 6.2

Q.1 Simplify each of the following as a rational expression.

(i) $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

Solution: $\dfrac{x^2-x-6}{x^2-9}+\dfrac{x^2+2x-24}{x^2-x-12}$

$$=\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$$

$$=\frac{x^2-3x-2x-6}{(x)^2-(3)^2}+\frac{x^2+6x-4x-24}{x^2-x-12}$$

$$=\frac{x(x-3)+2(x-3)}{(x-3)(x+3)}+\frac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$$

$$=\frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+3)}+\frac{(x+6)\cancel{(x-4)}}{\cancel{(x-4)}(x+3)}$$

$$=\frac{(x+2)}{(x+3)}+\frac{(x+6)}{(x+3)}$$

$$=\frac{x+2+x+6}{x+3}$$

$$=\frac{8+2x}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q.2 $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

Solution: $\left[\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}-\dfrac{4x}{x^2+1}\right]+\dfrac{4x}{x^4-1}$

$$=\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{x^2+2x+1-(x^2+1-2x)}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{\cancel{x^2}+2x\cancel{+1}-\cancel{x^2}\cancel{-1}+2x}{x^2-1}-\frac{4x}{x^2+1}\right]+\left[\frac{4x}{x^4-1}\right]$$

$$=\left[\frac{4x}{x^2-1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x(x^2+1)-4x(x-1)}{(x^2-1)(x^2+1)}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x^3+4x-4x^3+4x}{x^4-1}\right]+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}$$

$$=\frac{12x}{x^4-1}$$

Q.3 $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$

Solution: $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$

$$=\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$$

$$=\frac{1}{x^2-3x-5x+15}+\frac{1}{x^2-3x-1x+3}-\frac{2}{x^2-5x-x+5}$$

$$=\frac{1}{x(x-3)-5(x-3)}+\frac{1}{x(x-3)-1(x-3)}-\frac{2}{x(x-5)-1(x-5)}$$

$$=\frac{1}{(x-3)(x-5)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}$$

$$=\frac{(x-1)+(x-5)-2(x-3)}{(x-3)(x-5)(x-1)}$$

$$=\frac{\cancel{x}-\cancel{1}+\cancel{x}-\cancel{5}-\cancel{2x}+\cancel{6}}{(x-3)(x-)(x-1)}$$

$$=\frac{0}{(x-3)(x-5)(x-1)}$$

$$=0$$

Q.4 $\dfrac{(x+2)(x+3)}{x^2-9}+\dfrac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution: $\dfrac{(x+2)(x+3)}{x^2-9}+\dfrac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

$$=\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$

$$=\frac{(x+2)(x+3)}{(x)^2-(3)^2}+\frac{(x+2)\left[2(x^2-16)\right]}{(x-4)(x^2-3x+2x-6)}$$

$$=\frac{(x+2)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}}+\frac{(x+2)\left[2(x)^2-(4)^2\right]}{(x-4)\left[x(x-3)+2(x-3)\right]}$$

$$=\frac{(x+2)}{(x-3)}+\frac{\cancel{(x+2)}\left[2(x+4)\cancel{(x-4)}\right]}{\cancel{(x-4)}(x-3)\cancel{(x+2)}}$$

$$=\frac{(x+2)}{(x-3)}+\frac{2(x+4)}{(x-3)}$$

$$=\frac{x+2}{x-3}+\frac{2x+8}{x-3}$$

$$=\frac{x+2+2x+8}{x-3}$$

$$=\frac{3x+10}{x-3}$$

Q.5 $=\dfrac{x+3}{2x^2+9x+9}+\dfrac{1}{2(2x-3)}-\dfrac{4x}{4x^2-9}$

Solution: $=\dfrac{x+3}{2x^2+9x+9}+\dfrac{1}{2(2x-3)}-\dfrac{4x}{4x^2-9}$

$$=\frac{x+3}{2x^2+9x+9}+\frac{1}{2(2x-3)}-\frac{4x}{4x^2-9}$$

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-(3)^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{\cancel{(x+3)}}{\cancel{(x+3)}(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{1}{2x+3}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{2(2x-3)+(2x+3)-4x\times 2}{2(2x-3)(2x+3)}$$

$$=\frac{4x-6+2x+3-8x}{2(2x-3)(2x+3)}$$

$$=\frac{-2x-3}{2(2x-3)(2x+3)}$$

$$=\frac{-1(\cancel{2x+3})}{2(2x-3)(\cancel{2x+3})}$$

$$=\frac{-1}{2(2x-3)}$$

Q.6 $A-\frac{1}{A}$, **Where** A= $\frac{a+1}{a-1}$

Solution: $A-\frac{1}{A}$, Where A= $\frac{a+1}{a-1}$

$$=A-\frac{1}{A}=?$$

Put the value of A

$$=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$

$$=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$=\frac{a^2+2a+1-(a^2-2a+1)}{a^2-1}$$

$$=\frac{\cancel{a^2}+2a\cancel{+1}-\cancel{a^2}+2a\cancel{-1}}{a^2-1}$$

$$=\frac{4a}{a^2-1}$$

Q.7 $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

Solution: $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$

$$=\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$$

$$=\left[\frac{x-1}{x-2}+\frac{2}{-x+2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-x^2+4}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-\left(x^2-4\right)}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{x^2-4}\right]$$

$$=\left[\frac{x-1-2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{(x+2)(x-2)}\right]$$

$$=\frac{(x-3)}{(x-2)}-\frac{(x+1)(x-2)-4}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-2-4}{(x+2)(x-2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x(x-3)2(x-3)}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$=\frac{\cancel{x-3}}{\cancel{x-2}}-\frac{\cancel{x-3}}{\cancel{x-2}}$$

0 **Ans**

Q.8 What rational number should be subtracted from

$$=\frac{2x^2+2x-7}{x2+x-6}\text{ to get }\frac{x-1}{x-2}$$

Solution: let required rational number be *P*(*x*)

According to condition

$$\frac{2x^2+2x-7}{x^2+x-6}-P(x)=\frac{x-1}{x-2}$$

$$P(x)=\frac{2x^2+2x-7}{x^2+x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x^2+3x-2x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x(x+3)-2(x+3)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{(x+3)(x-2)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7-(x-1)(x+3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-(x^2+2x-3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$=\frac{x^2-4}{(x+3)(x-2)}$$

$$=\frac{x^2-2^2}{(x+3)(x-2)}$$

$$=\frac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$$

$$=\frac{x+2}{x+3}$$

Q.9 $=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$

Solution: $=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$

$$=\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$$

$$=\frac{x^2+3x-2x-6}{x^2-3x+2x-6}\times\frac{x^2-2^2}{x^2-3^2}$$

$$=\frac{x(x+3)-2(x+3)}{2(x-3)+2(x-3)}\times\frac{(x-2)(x+3)}{(x-3)(x+3)}$$

$$=\frac{(\cancel{x+3})(x-2)}{(x-3)(\cancel{x+2})}\times\frac{(x-2)(\cancel{x+2})}{(x-3)(\cancel{x+3})}$$

$$=\frac{(x-2)^2}{(x-3)^2}$$

Q.10 $\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$

Solution: $\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$

$$=\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

$$=\frac{(x)^3-(2)^3}{(x^2)-(2)^2}\times\frac{x^2+4x+2x+8}{x^2-x-x+1}$$

$$=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\times\frac{x(x+4)+2(x+4)}{x(x-1)-1(x-1)}$$

$$=\frac{x^2+2x+4}{(x+2)}\times\frac{(x+4)(x+2)}{(x-1)(x-1)}$$

$$=\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$$

Q.11 $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

Solution: $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

$$=\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$$

$$=\frac{x(x^3-8)}{2x^2+6x-x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{x\left[(x)^3-(2)^3\right]}{2x(x+3)-1(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{x(x-2)(x^2+2x+4)}{(2x-1)(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$=1$ **Ans**

Q.12 $\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$

Solution: $\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$

$$= \frac{2y^2+7y-4}{3y^2-13y+4} \div \frac{4y^2-1}{6y^2+y-1}$$

$$= \frac{2y^2+8y-1y-4}{3y^2-12y-y+4} \div \frac{(2y)^2-(1)^2}{6y^2+3y-2y-1}$$

$$= \frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)} \div \frac{(2y-1)(2y+1)}{3y(2y+1)-1(2y+1)}$$

$$= \frac{(y+4)(2y-1)}{(3y-1)(y-4)} \div \frac{(2y-1)\cancel{(2y+1)}}{(3y-1)\cancel{(2y+1)}}$$

$$= \frac{(y+4)\cancel{(2y-1)}}{\cancel{(3y-1)}(y-4)} \times \frac{\cancel{(3y-1)}}{\cancel{(2y-1)}}$$

$$= \frac{y+4}{y-4}$$

Q.13 $\left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$

Solution: $\left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$

$$= \left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$$

$$= \left[\frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)}\right] \div \left[\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}\right]$$

$$= \left[\frac{(x^4+2x^2y^2+y^4)-(x^4-2x^2y^2+y^4)}{(x^2-y^2)(x^2+y^2)}\right] \div \left[\frac{(x^2+2xy+y^2)-(x^2-2xy+y^2)}{x^2-y^2}\right]$$

$$= \left[\frac{\cancel{x^4}+2x^2y^2+\cancel{y^4}-\cancel{x^4}+2x^2y^2-\cancel{y^4}}{(x^2-y^2)(x^2+y^2)}\right] \div \left[\frac{\cancel{x^2}+2xy+\cancel{y^2}-\cancel{x^2}+2xy-\cancel{y^2}}{x^2-y^2}\right]$$

$$= \left[\frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)}\right] \div \left[\frac{4xy}{x^2-y^2}\right]$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{x^2-y^2}{4xy}$$

$$= \frac{\cancel{4xy}.xy}{\cancel{(x^2-y^2)}(x^2+y^2)} \times \frac{\cancel{x^2-y^2}}{\cancel{4xy}}$$

$$= \frac{xy}{x^2 + y^2}$$ **Ans**