Exercise 5.4

Q.1 x^3-2x^2-x+2

Solution: Given that

$P(x)=x^3-2x^2-x+2$

P=2 and possible factor of 2 are $\pm1,\pm2$.

Here q=1 and possible factor of 1 are ±1.

So possible factor of P(x) will be form $\frac{P}{q}=\pm1,\pm2$

$P(x)=x^3-2x^2-x+2$

Put x=1

$P(1)=(1)^3-2(1)^2-1+2 \quad =1-2-1+2=0$ Remainder is equal to zero so (x-1) is factor

Put x=-1

$P(-1)=(-1)^3-2(-1)^2-(-1)+2=-1-2+1+2=0$ Remainder is equal to zero so (x+1) is factor

Put x=2

$P(2)=(2)^3-2(2)^2-(2)+2=8-8-2+2=0$ Remainder is equal to zero so (x-2) is factor

$x^3-2x^2-x+2=(x-1)(x+1)(x-2)$

Q.2 $x^3-x^2-22x+40$

Solution: Given that

$P(x)=x^3-x^2-22x+40$

P=40

possible factor of 40= $\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

Here q=1 and possible factor of 1 are ±1

So possible factor of P(x) will be from

$\frac{P}{q}=\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

$P(x)=x^3-x^2-22x+40$

Put $x=2$

$P(2)=(2)^3-(2)^2-22(2)+40$

$=8-4-44+40=0$

Remainder is equal to zero so (x-2) is a factor

Put x=4

$P(4)=(4)^3-(4)^2-22(4)+40$

$=64-16-88+40=0$

Remainder is not equal to zero so (x-4) is a factor

Put x=-5

$$P(-5) = (-5)^3 - (-5)^2 - 22(-5) + 40$$
$$= -125 - 25 + 110 + 40$$
$$= -150 + 150$$
$$= 0$$
Remainder is equal to zero so (x+5) is a factor

Hence $x^3 - x^2 - 22x + 40 = (x-2)(x-4)(x+5)$

Q.3 $x^3-6x^2+3x+10$

Solution: Given that

$P(x) = x^3 - 6x^2 + 3x + 10$

P=10

So possible factor of 10 are $\pm1, \pm2, \pm5, \pm10$

Here q=1 So, possible factor of 1are ±1.

So possible of factor of P(x) can be found from $\frac{P}{q} = \pm1, \pm2, \pm5, \pm10$

P(x)=x^3-6x^2+3x+10

Put x=-1

P(-1)=$(-1)^3$-6$(-1)^2$+3(-1)+10=-1-6-3+10=0

Remainder is equal to zero so (x+1) is a factor

Put x=2

P(2)=$(2)^3$-6$(2)^2$+3(2)+10=8-24+6+10=0

Remainder is equal to zero so (x-2) is a factor

Put x=5

$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0$Remainder is equal to zero so (x-5) is a factor

Hence $x^3 - 6x^2 + 3x + 10 = (x+1)(x-2)(x-5)$

Q.4 $x^3 + x^2 - 10x + 8$

Solution: Given that

P(x)=x^3+x^2-10x+8

P=8 So possible factor of 8

are $\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm7, \pm8$.

Here q=1 So possible factor can be found from $\frac{P}{q} = \pm1, \pm2, \pm4, \pm8$

P(x)=x^3+x^2-10x+8

Put x=1

$P(1) = (1)^3 + (1)^2 - 10(1) + 8 = 1 + 1 - 10 + 8 = 0$

Remainder is equal to zero so (x-1) is a factor

Put x=2

$P(2) = 2^3 + 2^2 - 10(2) + 8$

$= 8 + 4 - 20 + 8$

$= 20 - 20$

$= 0$

Remainder is equal to zero so (x-2) is a factor

Put x=-4

P(-4) $=(-4)^3+(-4)^2-10(-4)+8$

=-64+16+40+8

=-64+64

=0

Remainder is equal to zero so (x+4) is a factor

Hence $x^3+x^2-10x+8=(x-1)(x-2)(x+4)$

Q.5 x^3-2x^2-5x+6

Solution: Given that

$P(x)=x^3-2x^2-5x+6$

$P=6$ So factors of 3 are $\pm1, \pm2, \pm3, \pm6$

Here q=1 So factors of 1 are $\pm$ 1.

So possible factors of P(x) can be found from $\frac{P}{q}=\pm1, \pm2, \pm3, \pm6$

P(x)=x^3-2x^2-5x+6

Put x=1

P(1) $=(1)^3-2(1)^2-5(1)+6$

=1-2-5+6

=-7+7

=0

Remainder is equal to zero so (x-1) is a factor

Put x=-2

$P(-2)=(-2)^3-2(-2)^2-5(-2)+6$

$=-8-8+10+6$

$=-16+16$

$=0$

Remainder is equal to zero so (x+2) is a factor

Put x=3

$P(3)=(3)^3-2(3)^2-5(3)+6$

$=27-6-15+6$

27-27

=0

Remainder is equal to zero so (x-3) is a factor

Hence $x^3-2x^2-5x+6=(x-1)(x+2)(x-3)$

Q.6 $x^3+5x^2-2x-24$

Solution: Given that

$P(x)=x^3+5x^2-2x-24$

P= -24 So possible factors of 24 are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Here q=1. So possible factors of 1 are $\pm$ 1.

So possible factors of P(x) will be found from

$\frac{P}{q}=\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$P(x)=x^3+5x^2-2x-24$
Put $x=2$
$P(2)=(2)^3+5(2)^2-2(2)-24$
$=8+20-4-24$
$=28-28$
$=0$
Remainder is equal to zero so $(x-2)$ is a factor
Put $x=-3$
$P(-3) \quad =(-3)^3+5(-3)^2-2(-3)-24$
$=-27+45+6-24$
$=-51+51$
$=0$
Remainder is equal to zero so $(x+3)$ is a factor
Put $x=-4$
$P(-4) \quad =(-4)^3+5(-4)^2-2(-4)-24$
$=-64+80+8-24$
$=-88+88$
$=0$
Remainder is equal to zero so $(x+4)$ is a factor
Hence $x^3+5x^2-2x-24=(x-2)(x+3)(x+4)$

Q.7 $3x^3-x^2-12x+4$
Solution: Given that
$P(x)=3x^3-x^2-12x+4$
P=4 So possible factors of 4 are $\pm1,\pm2,\pm4$.
Here q=3 So possible factors of 3 are $\pm1,\pm3$.
So possible factors of P(x) can be found from
$$\frac{P}{q}=\pm1,\pm2,\pm4,\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$$
Put $x=2$
$P(2)=3(2)^3-(2)^2-12(2)+4$
$=-24-4+24+4$
$=28-28$
$=0$
Remainder is equal to zero so $(x-2)$ is a factor
Put $x=-2$
$P(-2)=3(-2)^3-(-2)^2-12(-2)+4$
$=-24-4+24+4$
$=28-28$
$=0$
Remainder is equal to zero so $(x+2)$ is a factor
Put $x=\frac{1}{3}$
$$P\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3-\left(\frac{1}{3}\right)^2-\cancel{12}^{4}\left(\frac{1}{3}\right)+4$$

$$= \cancel{3}\left(\frac{1}{\cancel{27}\,9}\right) - \frac{1}{9} - \cancel{4} + \cancel{4}$$

$$P\left(\frac{1}{3}\right) = \frac{\cancel{1}}{\cancel{9}} - \frac{\cancel{1}}{\cancel{9}}$$

$$= 0$$

$$x = \frac{1}{3} \Rightarrow \begin{matrix} 3x = 1 \\ 3x - 1 \end{matrix}$$

Remainder is equal to zero so (3x-1) is a factor

Hence $3x^3 - x^2 - 10x + 4 = (x-2)(x+2)(3x-1)$

Q.8 $2x^3 + x^2 - 2x - 1$

Solution: Given that

$P(x)=2x^3+x^2-2x-1$

P= -1So possible factors of -1 are $\pm 1, \pm 2$.

Here q=1. So possible factors of P(x) will be found from $\frac{P}{q}$

$$\frac{P}{q} = \pm 1, \pm 2, \pm \frac{1}{2}$$

$P(x)=2x^3+x^2-2x-1$

Put x=1

P(1) $=2(1)^3+(1)^2-2(1)-1$

=2+1-2-1

=3-3

=0

Remainder is equal to zero (x-1) is a factor

Put x=-1

P(-1) $=2(-1)^3+(-1)^2-2(-1)-1$

=2+1-2-1

=3-3

=0

Remainder is equal to zero (x+1) is a factor

Put $x = \frac{-1}{2}$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{2}\right]^3 + \left[\frac{-1}{2}\right]^2 - \cancel{2}\left[\frac{-1}{\cancel{2}}\right] - 1$$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{\cancel{8}4}\right] + \frac{1}{4} + \cancel{1} - \cancel{1}$$

$$P\left(\frac{-1}{2}\right) = -\cancel{\frac{1}{4}} + \cancel{\frac{1}{4}}$$

$$= 0$$

$$x = -\frac{1}{2}$$

$$2x = -1$$

$$2x + 1 = 0$$

Remainder is equal to zero so $(2x+1)$ is a factor

Hence $2x^3 + x^2 - 2x - 1 = (x-1)(x+1)(2x+1)$