Exercise 5.3

Q.1 Use the remainder theorem to find the remainder when

(i) $3x^3 - 10x^2 + 13x - 6$ **is divided by** $(x-2)$.

Solution:

$P(x) = 3x^3 - 10x^2 + 13x - 6$

Since $P(x)$ is divided by $(x-2)$.

$\therefore P(2) = R$

$R = 3(2)^3 - 10(2)^2 + 13(2) - 6$

$= 3(2)^3 - 10(2)^2 + 13(2) - 6$

$= 24 - 40 + 26 - 6$

R=4

Hence 4 is the remainder

(ii) $4x^3 - 4x + 3$ **is divided by** $(2x-1)$

Solution:

$P(x) = 4x^3 - 4x + 3$

Since $P(x)$ is divided by (2x-1)

$\therefore R = P\left(\frac{1}{2}\right)$

$= 4\left[\frac{1}{2}\right]^3 - \cancel{4}^2 \times \frac{1}{\cancel{2}} + 3$

$= \cancel{4} \times \frac{1}{\cancel{8}^2} - 2 + 3$

$= \frac{1}{2} - 2 + 3$

$= \frac{1-4+6}{2} = \frac{3}{2}$

$R = \frac{3}{2}$

Hence $\frac{3}{2}$ is the remainder

(iii) $6x^4 + 2x^3 - x + 2$ **is divided by** $(x+2)$ **from** $x + 2 = 0$

Solution: Given that

$P(x) = 6x^4 + 2x^3 - x + 2$

Since $P(x)$ is divided by (x+2)

$\therefore R = P(-2)$

$=6(-2)^4 + 2(-2)^3 - (-2) + 2$

=96-16+2+2

R=84

Hence 84 is the remainder

(iv) $(2x-1)^3 + 6(3+4x)^2 - 10$ **is divided by** $2x+1$ **from** $2x+1=0$

$x = -\frac{1}{2}$

Solution: Given that

$P(x) = (2x-1)^3 + 6(3+4x)^2 - 10$

Since $P(x)$ is divided by 2x+1

$\therefore R = P\left(-\frac{1}{2}\right)$

$= \left[\cancel{2}\left(-\frac{1}{\cancel{2}}\right) - 1\right]^3 + 6\left[3 + \cancel{4}^2\left(\frac{-1}{\cancel{2}}\right)\right]^2 - 10$

$= [-1-1]^3 + 6[3-2]^2 - 10$

$= [-2]^3 + 6 - 10 = -8 + 6 - 10$

$R = -12$

Hence -12 is the remainder

(v) $x^3 - 3x^2 + 4x - 14$ **is divided by** $(x+2)$ **from** $x+2=0, x=-2$

Solution: Given that

$P(x) = x^3 - 3x^2 + 4x - 14$

Since $P(x)$ is divided by (x+2)

$\therefore R = P(-2)$

$=(-2)^3 - 3(-2)^2 + 4(-2) - 14$

$= -8 - 12 - 8 - 14$

$R = -42$

Hence -42 is the remainder

Q.2

(i) If $(x+2)$ is a factor of $3x^2-4kx-4k^2$ then find the values of k $x+2=0$ $x=-2$

Solution: Given that

$P(x)=3x^2-4kx-4k^2$

$P(-2)=3(-2)^2-4k(-2)-4k^2$

$P(-2)=12+8k-4k^2$

If $(x+2)$ is the factor then remainder is equal to zero

$P(-2)=0$

$12+8k-4k^2=0$

$4(3+2k-k^2)=0$

$-k^2+2k+3=\frac{0}{4}$

$-k^2+3k-k+3=0$

$-k(k-3)-1(k-3)=0$

$(k-3)(-k-1)=0$

$k-3=0 \qquad -k-1=0$

$k=3 \qquad -1=k$

$k=-1$

(ii) If $(x-1)$ is a factor of $x^3-kx^2+11x-6$ the find the value of k from $x-1=0$ $x=1$

Solution: Given that

$P(x)=x^3-kx^2+11x-6$

$P(1)=(1)^3-k(1)^2+11(1)-6$

$P(1)=1-k+11-6$

$P(1)=6-k$

If $(x-1)$ is the factor then remainder is equal to zero

$P(1)=0$

$6-k=0$

$k=6$

Q.3 Without long division determine whether

(i) $(x-2)$ and $(x-3)$ are factor of $P(x)=x^3-12x^2+44x-48$ from $x-2=0$ $x=2$

Solution: Given that

$P(x)=x^3-12x^2+44x-48$

If $(x-2)$ is the factor then remainder is equal to zero

$P(2)=(2)^3-12(2)^2+44(2)-48=8-48+88-48=0$

Hence $x-2$ is a factor of $P(x)$

For $x-3$

$R=P(3)$

$=(3)^3-12(3)^2+44(3)-48$

$=(3)^3-12(3)^2+44(3)-48$

$=27-108+132-48$

$=159-156$

$R=3$

3 is remainder hence $x-3$ is not factor of $P(x)$

$P(3)$ is not equal to zero then x-3 is not factor of $P(x)=x3-12x^2+44x-48$

(ii) $(x-2),(x+3)$ and $(x-4)$ are factor of $q(x)=x^3+2x^2-5x-6$ from $x-2=0, x=2$

Solution: Given that

$q(x)=x^3+2x^2-5x-6$

For $(x-2)$, putt $x-2=0$

$x=2$

$R=q(2)$

$=(2)^3+2(2)^2-5(2)-6$

$R=8+8-10-6$

$R=16-16$

$R=0$

Hence $x-2$ is factor of $q(x)=x^3+2x^2-5x-6$

For $(x+3)$, putt $x+3=0$

$x=-3$

$R=q(-3)$

$=(-3)^3+2(-3)^2-5(-3)-6$

$=-27+18+15-6$

$R=0$

Hence x-2 is factor of

$q(x)=x^3+2x^2-5x-6$

For x-4, x-4=0

x=4

$R=q(4)$

$=(4)^3+2(4)^2-5(4)-6$

$=64+32-20-6$

R=70

Hence x-4 is not a factor of

$q(x)=x^3+2x^2-5x-6$

Q.4 For what value of m is the polynomial P(x) =4x^3-7x^2+6x-3m exactly divisible by x+2?

Solution:

$P(x)=4x^3-7x^2+6x-3m$

From x+2=0, x=-2

P(-2)=4(-2)^3-7(-2)^2 $+6(-2)-3m$

P(-2)=-32-28-12-3m=-72-3m

If (x+2) is the factor then remainder is equal to zero

$P(-2)=0$

$-72-3m=0$

$-72=3m$

$m=-\dfrac{72}{3}$

$m=-24$

Q.5 Determine the value of k if $P(x)=kx^3+4x^2+3x-4$ and $q(x)=x^3-4x+k$ leaves the same remainder when divided by $(x-3)$.

Solution:

$q(x)=x^3-4x+k$

from x-3=0 x=3

$R_1=q(3)$

$=(3)^3-4(3)+k$

$=27-12+k$

$=15+k$

$R_1 \quad =15+k$ ....(i)

$R_2=P(3)$

$=k(3)^3+4(3)^2+3(3)-4$

$=27k+36+9-4$

$R_2 \quad =27k+41$ ....(ii)

Since it leaves the same remainder.

Hence $R_1=R_2$

$15+k=27k+41$

$15-41=27k-k$

$-26=26k$

$k=\dfrac{-26}{26}$

$k=-1$

Q.6 The remainder after dividing the polynomial $P(x)=x^3+ax^2+7$ by $(x+1)$ is $2b$ calculate the value of a and b if this expression leaves a remainder of $(b+5)$ on being dividing by $(x-2)$

Solution:

Let

$P(x)=x^3+ax^2+7$

Since $P(x)$ is divided by (x+1)

Put $x+1=0$ $x=-1$

R=P(−1)

$=(-1)^3+a(-1)^2+7$

$=-1+a+7$

$R=a+6$

According to first condition remainder is 2b

$2b=a+6$ ...(i)

Since $P(x)$ is divided by (x-2)

Put $x-2=0$

$x=2$

P(2)=(2)^3+a(2)^2+7

=8+4a+7

R=15+4a

According to second condition remainder is (b+5)

15+4a=b+5

4a-b=5-15

4a-b=-10 ...(ii)

Solving equations (i) and (ii)
From equation (ii) b=10+4a putting the value of be in equation (i)
a+6=2(10+4a)
a=20+8a-6
-8a+a=14
-7a=14

$$a=\frac{14}{-7}$$

a=-2
Putting the value of a in equation (ii)

$$4a-b=-10$$
$$4(-2)-b=-10$$
$$-8-b=-10$$
$$-8+10=b$$
$$2=b$$
$$b=2$$

Q.7 The polynomial $x^3+lx^2+mx+24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$

Find the values of *l* and *m*.

Solution:
Let

$$P(x)=x^3+lx^2+mx+24$$

From $x+4=0$ $\quad x=-4$

$$P(-4)=(-4)^3+l(-4)^2+m(-4)+24$$
$$P(-4)=-64+16l-4m+24$$
$$P(-4)=16l-4m-40$$

According to condition (x+4) is the factor then

$$16l-4m-40=0$$
$$4[4l-m-10]=0$$
$$4l-m-10=0 \qquad \text{(i)}$$

from $x-2=0$ $\quad x=2$

Now $P(2)=(2)^3+l(2)^2+m(2)+24$

$$P(2)=8+4l+2m+24$$
$$P(2)=4l+2m+32$$

According the condition

$$4l+2m+32=36$$
$$4l+2m=36-32$$
$$4l+2m=4$$
$$4l+2m-4=0 \qquad \text{(ii)}$$

Subtracting (i) from (ii)

$$\cancel{4l}+2m-4=0$$
$$\underline{\pm\cancel{4l}\mp m\mp 10=0}$$
$$3m+6=0$$
$$3m+6=0$$
$$3m=-6$$
$$m=\frac{-\cancel{6}\,2}{\cancel{3}}$$
$$m=-2$$

Putting the value of m is equation (i)

$$4l-(-2)-10=0$$
$$4l+2-10=0$$
$$4l-8=0$$
$$4l=8$$
$$l=\frac{2\cancel{8}}{\cancel{4}}$$
$$l=2$$

Q.8 The expression lx^3+mx^2-7 leaves remainder of -3 and 12 when divided by $(x-1)$ and $(x+2)$ respectively. Calculate the value of *l* and *m*.

Solution:

$$P(x)=lx^3+mx^2-7$$

from x-1=0 $\quad x$=1

$$P(1)=l(1)^3+m(1)^2-4$$
$$P(1)=l+m-4$$

According to conditions l+m-4=-3

$$l+m=4-3$$
$$l=1-m \qquad \text{(i)}$$

From $x+2=0$ $\quad x=-2$

$$P(-2)=l(-2)^3+m(-2)^2-4$$
$$P(-2)=-8l+4m-4$$

According to condition

$$-8l+4m-4=12$$

Putting the value of l in the equation

$-8[1-m]+4m=16$

$-8+8m+4m=16$

$12m=16+8$

$12m=24$

$m=\frac{\not{24}^{2}}{\not{12}}$

$m=2$

Putting the value of m is equation (i)

$l=1-2$

$l=-1$

$m=2$

$l=-1$

Q.9 **The expression $ax^3-9x^2+bx+3a$ is exactly divisible by x^2-5x+6. Find the value of a and b.**

Solution: Given that

$P(x)=ax^3-9x^2+bx+3a$

$x^2-5x+6 \quad =x^2-2x-3x+6$

$=x[x-2]-3[x-2]$

$=[x-2][x-3]$

$(x-2)(x-3)$ is divides the expression ax^3-$9x^2+bx+3a$ from $x-2=0$, $x=2$

$P(2)=a(2)^3-9(2)^2+b(2)+3a$

$P(2)=8a-36+2b+3a$

$P(2)=11a+2b-36$

According to condition (x-2) is the factor so

11a+2b-36=0 (i)

From x-3=0,x=3

$P(3)=a(3)^3-9(3)^2+b(3)+3a$

$P(3)=27a-81+3b+3a$

$P(3)=30a+3b-81$

According to condition (x-3) is the factor so

$30a+3b-81=0$ (ii)

$3(10a+b-27)=0$

$10a+b-27=\frac{0}{3}$

$b=27-10a$ (iii)

Putting the value of b in equation (i)

$11a+2[27-10a)-36=0$

$11a+54-20a-36=0$

$-9a+18=0$

$+18=9a$

$a=\frac{+\not{18}\,2}{\not{9}}$

$a=+2$

Putting the value of a in equation (iii)

$b=27-10(+2)$

$b=27-20$

$b=7$

$a=2$