Exercise 5.2

Q.1   Factorize

(i) $x^4 + \dfrac{1}{x^4} - 3$

Solution: $x^4 + \dfrac{1}{x^4} - 3$

$= (x^2)^2 + \left(\dfrac{1}{x^2}\right)^2 - 3$

By adding and subtracting by 2

$= (x^2)^2 + \left(\dfrac{1}{x^2}\right)^2 + 2 - 2 - 3$

$= \left[(x^2)^2 + \left(\dfrac{1}{x^2}\right)^2 - 2\right] + 2 - 3$

$= \left[(x^2)^2 + \left(\dfrac{1}{x^2}\right)^2 - 2\right] - 1$

$= \left(x^2 - \dfrac{1}{x^2}\right)^2 - (1)^2$

$= \left(x^2 - \dfrac{1}{x^2} + 1\right)\left(x^2 - \dfrac{1}{x^2} - 1\right)$

(ii) $3x^4 + 12y^4$

Solution: $3x^4 + 12y^4$

$= 3(x^4 + 4y^4)$

By adding and subtracting by $2(x^2)(2y^2)$

$= 3\left[(x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2) - 2(x^2)(2y^2)\right]$

$= 3\left[(x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2) - 2(x^2)(2y^2)\right]$

$= 3\left[(x^2 + 2y^2)^2 - 4x^2y^2\right]$

$= 3\left[(x^2 + 2y^2)^2 - (2xy)^2\right]$

$= 3\left[(x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)\right]$

$= 3\left[(x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)\right]$

(iii) $a^4 + 3a^2b^2 + 4b^4$

Solution: $a^4 + 3a^2b^2 + 4b^4$

$= (a^4 + 4b^4) + 3a^2b^2$

$= (a^2)^2 + (2b^2)^2 + 3a^2b^2$

By adding and subtracting by $2(a^2)(2b^2)$

$= (a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2) - 2(a^2)(2b^2) + 3a^2b^2$

$= \left[(a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2)\right] - 2(a^2)(2b^2) + 3a^2b^2$

$= (a^2 + 2b^2)^2 - a^2b^2$

$= (a^2 + 2b^2)^2 - (ab)^2$

$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$

(iv) $4x^4 + 81$

Solution: $4x^4 + 81$

$= (2x^2)^2 + (9)^2$

By adding and subtracting by $2(2x^2)(9)$

$= \left[(2x^2)^2 + (9)^2 + 2(2x^2)(9) - 2(2x^2)(9)\right]$

$= \left[(2x^2)^2 + (9)^2 + 2(2x^2)(9)\right] - 2(2x^2)(9)$

$= (2x^2 + 9)^2 - 36x^2$

$= (2x^2 + 9)^2 - (6x)^2$

$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$

$= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$

(v) $x^4 + x^2 + 25$

Solution: $x^4 + x^2 + 25$

$= (x^4 + 25) + x^2$

$= \left[(x^2)^2 + (5)^2\right] + x^2$

By adding and subtracting by $2(x^2)(5)$

$= \left[(x^2)^2 + (5)^2 + 2(x^2)(5) - 2(x^2)(5)\right] + x^2$

$= \left[(x^2)^2 + (5)^2 + 2(x^2)(5)\right] - 2(x^2)(5) + x^2$

$= (x^2 + 5)^2 - 10x^2 + x^2$

$= (x^2 + 5)^2 - 9x^2$

$$= (x^2+5)^2 - (3x)^2$$
$$= \left(x^2+5+3x\right)\left(x^2+5-3x\right)$$
$$= (x^2+3x+5)(x^2-3x+5)$$

(vi) $\quad x^4+4x^2+16$

Solution: x^4+4x^2+16
$$= (x^2)^2+16+4x^2$$
$$= (x^2)^2+(4)^2+4x^2$$

By adding and subtracting by $2\left(x^2\right)(4)$
$$= \left(x^2\right)^2+(4)^2+2\left(x^2\right)(4)-2\left(x^2\right)(4)+4x^2$$
$$= (x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$$
$$= \left(x^2+4\right)^2-8x^2+4x^2$$
$$= (x^2+4)^2-4x^2$$
$$= (x^2+4)^2-(2x)^2$$
$$= \left(x^2+4+2x\right)\left(x^2+4-2x\right)$$
$$= \left(x^2+2x+4\right)\left(x^2-2x+4\right)$$

Q.2   Factorize

(i) $\quad x^2+14x+48$

Solution: $x^2+14x+48$
$$= x^2+8x+6x+48$$
$$= x(x+8)+6(x+8)$$
$$= (x+8)(x+6)$$

(ii) $\quad x^2-21x+108$

Solution: $x^2-21x+108$
$$= x^2-12x-9x+108$$
$$= x(x-12)-9(x-12)$$
$$= (x-9)(x-12)$$

(iii) $\quad x^2-11x-42$

Solution: $x^2-11x-42$
$$= x^2-14x+3x-42$$
$$= x(x-14)+3(x-14)$$
$$= (x+3)(x-14)$$

(iv) $\quad x^2+x-132$

Solution: $x^2+x-132$
$$= x^2+12x-11x-132$$

$$= x(x+12)-11(x+12)$$
$$= (x-11)(x+12)$$

Q.3   Factorize

(i) $\quad 4x^2+12x+5$

Solution: $4x^2+12x+5$
$$= 4x^2+2x+10x+5$$
$$= 2x(2x+1)+5(2x+1)$$
$$= (2x+5)(2x+1)$$

(ii) $\quad 30x^2+7x-15$

Solution: $30x^2+7x-15$
$$= 30x^2+25x-18x-15$$
$$= 5x(6x+5)-3(6x+5)$$
$$= (5x-3)(6x+5)$$

(iii) $\quad 24x^2-65x+21$

Solution: $24x^2-65x+21$
$$= 24x^2-56x-9x+21$$
$$= 8x(3x-7)-3(3x-7)$$
$$= (8x-3)(3x-7)$$

(iv) $\quad 5x^2-16x-21$

Solution: $5x^2-16x-21$
$$= 5x^2+5x-21x-21$$
$$= 5x(x+1)-21(x+1)$$
$$= (5x-21)(x+1)$$

(v) $\quad 4x^2-17xy+4y^2$

Solution: $4x^2-17xy+4y^2$
$$= 4x^2-16xy-xy+4y^2$$
$$= 4x(x-4y)-y(x-4y)$$
$$= (4x-y)(x-4y)$$

(vi) $\quad 3x^2-38xy-13y^2$

Solution: $3x^2-38xy-13y^2$
$$= 3x^2-39xy+xy-13y^2$$
$$= 3x(x-13y)+y(x-13y)$$
$$= (3x+y)(x-13y)$$

(vii) $\quad 5x^2+33xy-14y^2$

Solution: $5x^2+33xy-14y^2$

$$= 5x^2 + 35xy - 2xy - 14y^2$$
$$= 5x(x+7y) - 2y(x+7y)$$
$$= (5x - 2y)(x+7y)$$

(viii) $\left(5x - \dfrac{1}{x}\right)^2 + 4\left(5x - \dfrac{1}{x}\right) + 4, \; x \neq 0$

Solution: $\left(5x - \dfrac{1}{x}\right)^2 + 4\left(5x - \dfrac{1}{x}\right) + 4, \; x \neq 0$

$$= \left(5x - \dfrac{1}{x}\right)^2 + 2\left(5x - \dfrac{1}{x}\right)(2) + (2)^2$$

$$= \left(5x - \dfrac{1}{x} + 2\right)^2$$

$$= \left(5x - \dfrac{1}{x} + 2\right)\left(5x - \dfrac{1}{x} + 2\right)$$

Q.4

(i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Solution: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Suppose that

$x^2 + 5x = y$

So,

$(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

$= (y+4)(y+6) - 3$

$= [y(y+6) + 4(y+6) - 3$

$= (y^2 + 6y + 4y + 24) - 3$

$= (y^2 + 10y + 24) - 3$

$= y^2 + 10y + 24 - 3$

$= y^2 + 10y + 21$

$= y^2 + 7y + 3y + 21$

$= y(y+7) + 3(y+7)$

$= (y+3)(y+7)$

We know that $y = x^2 + 5x$

$= (x^2 + 5x + 3)(x^2 + 5x + 7)$

(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Solution: $(x^2 - 4x)(x^2 - 4x - 1) - 20$

Suppose that

$x^2 - 4x = y$

So,

$= (y)(y-1) - 20$

$= (y^2 - y) - 20$

$= y^2 - y - 20$

$= y^2 - 5y + 4y - 20$

$= y(y-5) + 4(y-5)$

$= (y+4)(y-5)$

We know that $a = x^2 - 4x$

$= (x^2 - 4x + 4)(x^2 - 4x - 5)$

$= \left[(x)^2 - 2(x)(2) + (2)^2\right]\left[x^2 - 5x + x - 5\right]$

$= (x-2)^2 \left[x(x-5) + 1(x-5)\right]$

$= (x-2)^2 (x-5)(x+1)$

$= (x-5)(x+1)(x-2)^2$

(iii) $(x+2)(x+3)(x+4)(x+5) - 15$

Solution: $(x+2)(x+3)(x+4)(x+5) - 15$

$= [(x+2)(x+5)][(x+3)(x+4)] - 15$

$= [x(x+5) + 2(x+5)][x(x+4) + 3(x+4)] - 15$

$= [x^2 + 5x + 2x + 10][x^2 + 4x + 3x + 12] - 15$

$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$

Suppose that

$x^2 + 7x = y$

So,

$(x^2 + 7x + 10)(x^2 + 7x + 12) - 15$

$= (y+10)(y+12) - 15$

$= [y(y+12) + 10(y+12)] - 15$

$= (y^2 + 12y + 10y + 120) - 15$

$= (y^2 + 22y + 120) - 15$

$= y^2 + 22y + 120 - 15$

$= y^2 + 22y + 105$

$= y^2 + 15y + 7y + 105$

$= y(y+15) + 7(y+15)$

$= y(y+15) + 7(y+15)$

$= (y+7)(y+15)$

We know that $y = x^2 + 7x$

$= (x^2 + 7x + 7)(x^2 + 7x + 15)$

(iv) $(x+4)(x-5)(x+6)(x-7) - 504$

Solution: $(x+4)(x-5)(x+6)(x-7)-504$

$=[(x+4)(x-5)][(x+6)(x-7)]-504$

$=[x(x-5)+4(x-5)][x(x-7)+6(x-7)]-504$

$=(x^2-5x+4x-20)(x^2-7x+6x-42)-504$

$=(x^2-x-20)(x^2-x-42)-504$

Suppose that

$x^2-x=y$

So,

$=(y-20)(y-42)-504$

$=[y(y-42)-20(y-42)]-504$

$=(y^2-42y-20y+840)-504$

$=y^2-62y+840-504$

$=y^2-62y+336$

$=y^2-56y-6y+336$

$=y(y-56)-6(y-56)$

$=(y-6)(y-56)$

We know that $a=x^2-x$

$=(x^2-x-6)(x^2-x-56)$

$=(x^2-3x+2x-6)(x^2-8x+7x-56)$

$=[x(x-3)+2(x-3)][x(x-8)+7(x-8)]$

$=(x+2)(x-3)(x+7)(x-8)$

(v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

Solution: $(x+1)(x+2)(x+3)(x+6)-3x^2$

$=[(x+1)(x+6)][(x+2)(x+3)]-3x^2$

$=[x(x+6)+1(x+6)][x(x+3)+2(x+3)]-3x^2$

$=(x^2+6x+x+6)(x^2+3x+2x+6)-3x^2$

$=(x^2+6+7x)(x^2+6+5x)-3x^2$

Suppose that

$x^2+6=y$

So,

$=(y+7x)(y+5x)-3x^2$

$=[y(y+5x)+7x(y+5x)]-3x^2$

$=(y^2+5xy+7xy+35x^2-3x^2)$

$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$

$=y(y+8x)+4x(y+8x)$

$=(y+4x)(y+8)$

We know that $y=x^2+6$

$=(x^2+6+4x)(x^2+6+8x)$

$=(x^2+4x+6)(x^2+8x+6)$

Q.5

(i) $x^3+48x-12x^2-64$

Solution: $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(4)+3(x)(4)^2-(4)^3$

$=(x-4)^3$

(ii) $8x^3+60x^2+150x+125$

Solution: $8x^3+60x^2+150x+125$

$a^3+3a^2b+3ab^2+b^3=(a+b)^3$

$=(2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$

$=(2x+5)^3$

(iii) $x^3-18x^2+108x-216$

Solution: $x^3-18x^2+108x-216$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$

$=(x-6)^3$

(iv) $8x^3-125y^3-60x^2y+150xy^2$

Solution: $8x^3-125y^3-60x^2y+150xy^2$

$=8x^3-60x^2y+150xy^2-125y^3$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3$

$=(2x-5y)^3$

Q.6

(i) $27+8x^3$

Solution: $27+8x^3$

$=(3)^3+(2x)^3$

$=(3+2x)[(3)^2-(3)(2x)+(2x)^2]$

$=(3+2x)(9-6x+4x^2)$

(ii) $125x^3$-$216y^3$

Solution: $125x^3-216y^3$

$=(5x)^3-(6y)^3$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$=(5x-6y)[(5x)^2+(5x)(6y)+(6y)^2]$

$$= (5x - 6y)(25x^2 + 30xy + 36y^2)$$

(iii) $64x^3 + 27y^3$

Solution: $64x^3 + 27y^3$

$$= (4x)^3 + (3y)^3$$

$$(a + b)(a^2 + ab + b^2) = a^3 + b^3$$

$$= (4x + 3y)\left[(4x)^2 - (4x)(3y) + (3y)^2\right]$$

$$= (4x + 3y)(16x^2 - 12xy + 9y^2)$$

(iv) $(2x)^3 + (5y)^3$

Solution: $(2x)^3 + (5y)^3$

$$(a - b)(a^2 + ab + b^2) = a^3 - b^3$$

$$= (2x + 5y)\left[(2x)^2 - (2x)(5y) + (5y)^2\right]$$

$$= (2x + 5y)(4x^2 - 10xy + 25y^2)$$