Exercise 5.1

Q.1 Factorize

(i) $2abc-4abx+2abd$

Solution: $2abc-4abx+2abd$

$=2ab(c-2x+d)$

(ii) $9xy-12x^2y+18y^2$

Solution: $9xy-12x^2y+18y^2$

$=3y(3x-4x^2+6y)$

(iii) $-3x^2y-3x+9xy^2$

Solution: $-3x^2y-3x+9xy^2$

$=-3x(xy+1-3y^2)$

(iv) $5ab^2c^3-10a^2b^3c-20a^3bc^2$

Solution: $5ab^2c^3-10a^2b^3c-20a^3bc^2$

$=5abc(bc^2-2ab^2-4a^2c)$

(v) $3x^3y(x-3y)-7x^2y^2(x-3y)$

Solution: $3x^3y(x-3y)-7x^2y^2(x-3y)$

$=(x-3y)(3x^3y-7x^2y^2)$

$=(x-3y)\ x^2y(3x-7y)$

$=x^2y(x-3y)(3x-7y)$

(vi) $2xy^3(x^2+5)+8xy^2(x^2+5)$

Solution: $2xy^3(x^2+5)+8xy^2(x^2+5)$

$=(x^2+5)(2xy^3+8xy^2)$

$=(x^2+5)2xy^2(y+4)$

$=2xy^2(x^2+5)(y+4)$

Q.2 Factorize

(i) $5ax-3ay-5bx+3by$

Solution: $5ax-3ay-5bx+3by$

$=5ax-5bx-3ay+3by$

$=5x(a-b)-3y(a-b)$

$=(a-b)(5x-3y)$

(ii) $3xy+2y-12x-8$

Solution: $3xy+2y-12x-8$

$=3xy-12x+2y-8$

$=3x(y-4)+2(y-4)$

$=(y-4)(3x+2)$

(iii) $x^3+3xy^2-2x^2y-6y^3$

Solution: $x^3+3xy^2-2x^2y-6y^3$

By cyclic order

$=x^3-2x^2y+3xy^2-6y^3$

$=x^2(x-2y)+3y^2(x-2y)$

$=(x-2y)(x^2+3y^2)$

(iv) $(x^2-y^2)z+(y^2-z^2)x$

Solution: $(x^2-y^2)z+(y^2-z^2)x$

$=x^2z-y^2z+xy^2-xz^2$

Arrange in cyclic order

$x^2z+xy^2-xz^2-y^2z$

$=x^2z+xy^2-y^2z-xz^2$

$=x(xz+y^2)-z(xz+y^2)$

$=(xz+y^2)(x-z)$

Q.3 Factorize

(i) $144a^2+24a+1$

Solution: $144a^2+24a+1$

By using formula

$(a+b)^2=a^2+2ab+b^2$

$=(12a)^2+2(12a)(1)+(1)^2$

$=(12a+1)^2$

(ii) $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

Solution: $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

Formula $a^2-2ab+b^2=(a-b)^2$

$=\left(\frac{a}{b}\right)^2-2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2$

$=\left(\frac{a}{b}-\frac{b}{a}\right)^2$

(iii) $(x+y)^2-14z(x+y)+49z^2$

Solution: $(x+y)^2-14z(x+y)+49z^2$

Formula $a^2-2ab+b^2=(a-b)^2$

$=(x+y)^2-2(x+y)(7z)+(7z)^2$

$=(x+y-7z)^2$

(iv) $12x^2-36x+27$

Solution: $12x^2-36x+27$

$=3\left(4x^2-12x+9\right)$

Formula $a^2-2ab+b^2=(a-b)^2$

$=3\left[(2x)^2-2(2x)(3)+(3)^2\right]$

$=3(2x-3)^2$

Q.4 Factorize

(i) $3x^2-75y^2$

Solution: $3x^2-75y^2$

$=3\left(x^2-25y^2\right)$

Formula $a^2-b^2=(a+b)(a-b)$

$=3\left[(x)^2-(5y)^2\right]$

$=3(x+5y)(x-5y)$

(ii) $x(x-1)-y(y-1)$

Solution: $x(x-1)-y(y-1)$

$=x^2-x-y^2+y$

Arranging in cyclic order

$=x^2-y^2-x+y$

Taking common

$=\left(x^2-y^2\right)-(x-y)$

$=\left[(x+y)(x-y)\right]-(x-y)$

$=(x-y)(x+y-1)$

(iii) $128am^2-242an^2$

Solution: $128am^2-242an^2$

$=2a\left(64m^2-121n^2\right)$

$=2a\left[(8m)^2-(11n)^2\right]$

$=2a(8m+11n)(8m-11n)$

(iv) $3x-243x^3$

Solution: $3x-243x^3$

$=3x\left(1-81x^2\right)$

$=3x\left[(1)^2-(9x)^2\right]$

$=3x(1+9x)(1-9x)$

Q.5 Factorize

(i) x^2-y^2-6y-9

Solution: x^2-y^2-6y-9

$=x^2-\left[y^2+6y+9\right]$

$=x^2-[(y)^2+2(y)(3)+(3)^2]$

$=x^2-(y+3)^2$

$=(x)^2-(y+3)^2$

$=(x+y+3)[x-(y+3)]$

$=(x+y+3)(x-y-3)$

(ii) x^2-a^2+2a-1

Solution: x^2-a^2+2a-1

$=x^2-\left[a^2-2a+1\right]$

$=x^2-(a-1)^2$

$=[x+(a-1)][x-(a-1)]$

$=(x+a-1)(x-a+1)$

(iii) $4x^2 - y^2 - 2y - 1$

Solution: $4x^2 - y^2 - 2y - 1$

$= 4x^2 - (y^2 + 2y + 1)$

$= 4x^2 - \left[(y)^2 + 2(y)(1) + (1)^2\right]$

$= 4x^2 - (y+1)^2$

$= (2x)^2 - (y+1)^2$

$= \left[2x + (y+1)\right]\left[2x - (y+1)\right]$

$= (2x+y+1)(2x-y-1)$

(iv) $x^2 - y^2 - 4x - 2y + 3$

Solution: $x^2 - y^2 - 4x - 2y + 3$

$= x^2 - 4x + 4 - y^2 - 2y - 1$

$= (x^2 - 4x + 4) - (y^2 + 2y + 1)$

$= \left[(x)^2 - 2(x)(2) + (2)^2\right]$

$- \left[(y)^2 + 2(y)(1) + (1)^2\right]$

$= (x-2)^2 - (y+1)^2$

$= (x - 2 + y + 1)\left[x - 2 - (y+1)\right]$

$= (x - 2 + y + 1)(x - 2 - y - 1)$

$= (x + y - 2 + 1)(x - y - 2 - 1)$

$= (x + y - 1)(x - y - 3)$

(v) $25x^2 - 10x + 1 - 36z^2$

Solution: $25x^2 - 10x + 1 - 36z^2$

$= (5x)^2 - 2(5x)(1) + (1)^2] - 36z^2$

$= (5x-1)^2 - (6z)^2$

$= \left[(5x-1) + 6Z\right]\left[(5x-1) - 6Z\right]$

$= (5x - 1 + 6z)(5x - 1 - 6z)$

(vi) $x^2 - y^2 - 4xz + 4z^2$

Solution: $x^2 - y^2 - 4xz + 4z^2$

$= x^2 - 4xz + 4z^2 - y^2$

$= \left[(x)^2 - 2(x)(2z) + (2z)^2\right] - y^2$

$= (x - 2z)^2 - (y)^2$

$= (x - 2z + y)(x - 2z - y)$

$= (x + y - 2z)(x - y - 2z)$