Exercise 9.3

Q.1 Find the midpoint of the line Segments joining each of the following pairs of points

Solution:

(a) $A(9,2), B(7,2)$

Let $M(x,y)$ the midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Midpoint formula

$$M(x,y)=M\left(\frac{9+7}{2},\frac{2+2}{2}\right)$$

$$=M\left(\frac{\cancel{16}^{8}}{\cancel{2}},\frac{\cancel{4}^{2}}{\cancel{2}}\right)$$

$$=M(8,2)$$

(b) $A(2,-6), B(3,-6)$

Let $M(x,y)$ the point of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Midpoint formula

$$M(x,y)=M\left(\frac{2+3}{2},\frac{-6-6}{2}\right)$$

$$M(x,y)=M\left(\frac{5}{2},\frac{-\cancel{12}^{6}}{\cancel{2}}\right)$$

$$M(x,y)=M(2.5,-6)$$

(c) $A(-8,1), B(6,1)$

Let $M(x,y)$ midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Formula

$$M(x,y)=M\left(\frac{-8+6}{2},\frac{1+1}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{2}}{\cancel{2}},\frac{\cancel{2}}{\cancel{2}}\right)$$

$$M(x,y)=M(-1,1)$$

(d) A (-4 , 9) , B (-4 , -3)

Let $M(x,y)$ midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$ Formula

$$M(x,y)=M\left(\frac{-4-4}{2},\frac{9-3}{2}\right)$$

$$M(x,y)=M\left(\frac{-\cancel{8}^{4}}{\cancel{2}},\frac{\cancel{6}^{3}}{\cancel{2}}\right)$$

$$M(x,y)=M(-4,3)$$

(e) $A(3,11), B(3,-4)$

Let $M(x,y)$ is the midpoint of AB

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M(x,y)=M\left(\frac{3+3}{2},\frac{-11-4}{2}\right)$$

$$M(x,y)=M\left(\frac{6}{2},\frac{-15}{2}\right)$$

$$M(x,y)=M(3,-7.5)$$

(f) A (0, 0) , B (0, -5)

Let $M(x,y)$ is the midpoint of AB

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

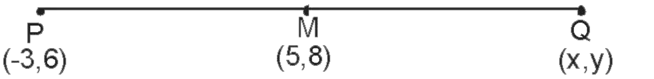
$$M(x,y)=M\left(\frac{0+0}{2},\frac{0-5}{2}\right)$$

$$M(x,y)=M\left(\frac{0}{2},\frac{-5}{2}\right)$$

$$=M(0,-2.5)$$

Q.2 The end point of line segment PQ is $(-3,6)$ and its midpoint is $(5,8)$ find the coordinates of the end point Q

Solution:



Let Q be the point (x,y), $M(5,8)$ is the midpoint of PQ

$$M(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$x=\frac{x_1+x_2}{2}$$
$$5=\frac{-3+x}{2}$$
$$5\times 2=-3+x$$
$$10+3=x$$
$$x=13$$
$$y=\frac{y_1+y_2}{2}$$
$$8=\frac{6+y}{2}$$
$$2\times 8=6+y$$
$$16-6=y$$
$$y=10$$

Hence point Q is $(13,10)$

Q.3 Prove that midpoint of the hypotenuse of a right triangle is equidistance from it three vertices $P(-2,5), Q(1,3)$ and $R(-1,0)$

Solution:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$d=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$
$$P(-2,5), Q(1,3)$$
$$|PQ|=\sqrt{|-2-1|^2+|5-3|^2}$$
$$|PQ|=\sqrt{(-3)^2+(2)^2}$$
$$|PQ|=\sqrt{9+4}$$
$$|PQ|=\sqrt{13}$$
$$Q(1,3), R(-1,0)$$
$$|QR|=\sqrt{|1-(-1)|^2+|3-0|^2}$$
$$|QR|=\sqrt{(1+1)^2+(3)^2}$$
$$|QR|=\sqrt{(2)^2+9}=\sqrt{4+9}$$
$$|QR|=\sqrt{13}$$
$$P(-2,5), R(-1,0)$$
$$|PR|=\sqrt{|-2-(-1)|^2+|5-0|^2}$$
$$|PR|=\sqrt{|-2+1|^2+|5|^2}$$
$$|PR|=\sqrt{(-1)^2+(5)^2}=\sqrt{1+25}$$
$$|PR|=\sqrt{26}$$

To find the length of hypotenuse and whether it is right angle triangle we use the Pythagoras theorem

$$(PR)^2=(PQ)^2+(QR)^2$$
$$\left(\sqrt{26}\right)^2=\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2$$
$$26 = 13 + 13$$
$$26 = 26$$

It is a right angle triangle and PR is hypotenuse

$$P(-2,5), R(-1,0)$$

Midpoint of PR

$$M(x,y)=\left(\frac{-2-1}{2},\frac{5+0}{2}\right)$$
$$M(x,y)=\left(\frac{-3}{2},\frac{5}{2}\right)$$
$$MP=MR$$
$$M\left(\frac{-3}{2},\frac{5}{2}\right), P(-2,5), R(-1,0)$$
$$|MP|=|MR|$$

(i)

$$|MP|=\sqrt{\left|\frac{-3}{2}-(-2)\right|^2+\left|\frac{5}{2}-5\right|^2}$$
$$=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5-10}{2}\right)^2}$$
$$|MP|=\sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$$
$$=\sqrt{\left(\frac{1}{2}\right)^2+\frac{25}{4}}$$
$$|MP|=\sqrt{\frac{1}{4}+\frac{25}{4}}=\sqrt{\frac{1+25}{4}}$$
$$|MP|=\sqrt{\frac{26}{4}}$$
$$|MP|=\frac{\sqrt{26}}{2}$$

(ii) $M\left(\frac{-3}{2},\frac{5}{2}\right), R(-1,0)$

$$|MR|=\sqrt{\left|\frac{-3}{2}-(-1)\right|^2+\left|\frac{5}{2}-0\right|^2}$$

$$|MR| = \sqrt{\left(\frac{-3}{2}+1\right)^2 + \left(\frac{5}{2}\right)^2}$$

$$|MR| = \sqrt{\left(\frac{-3+2}{2}\right)^2 + \frac{25}{4}}$$

$$= \sqrt{\left(\frac{-1}{2}\right)^2 + \frac{25}{4}}$$

$$|MR| = \sqrt{\frac{1}{4} + \frac{25}{4}}$$

$$|MR| = \sqrt{\frac{1+25}{4}} = \sqrt{\frac{26}{4}}$$

$$|MR| = \frac{\sqrt{26}}{2}$$

(iii) $M\left(\frac{-3}{2}, \frac{5}{2}\right)$

$Q(1,3)$

$$|MQ| = \sqrt{\left(\frac{-3}{2}-1\right)^2 + \left(\frac{5}{2}-3\right)^2}$$

$$= \sqrt{\left(\frac{-3-2}{2}\right)^2 + \left(\frac{5-6}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-5}{2}\right)^2 + \left(\frac{-1}{2}\right)^2}$$

$$= \sqrt{\frac{25}{4} + \frac{1}{4}} = \sqrt{\frac{26}{4}}$$

Hence proved $MP = MR = |MQ|$

Q.4 If $O(0,0)$, $A(3,0)$ and $B(3,5)$ are three points in the plane find M_1 and M_2 as the midpoint of the line segments AB and OB respectively find $|M_1M_2|$

Solution:

M_1 is the midpoint of AB

$$M_1(x,y) = M_1\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$A(3,0), B(3,5)$

$$M_1\left(\frac{3+3}{2}, \frac{0+5}{2}\right)$$

$$M_1\left(\frac{6}{2}, \frac{5}{2}\right)$$

$$M_1\left(3, \frac{5}{2}\right)$$

M_2 is the midpoint of OB

$$M_2\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$0(0,0), B(3,5)$

$$M_2\left(\frac{0+3}{2}, \frac{0+5}{2}\right)$$

$$M_2\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$M_1\left(3, \frac{5}{2}\right) M_2\left(\frac{3}{2}, \frac{5}{2}\right)$$

$$|M_1M_2| = \sqrt{\left|\frac{3}{2}-3\right|^2 + \left|\frac{5}{2}-\frac{5}{2}\right|^2}$$

$$|M_1M_2| = \sqrt{\left(\frac{3-6}{2}\right)^2 + (0)^2}$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2 + 0}$$

$$|M_1M_2| = \sqrt{\frac{9}{4}}$$

$$|M_1M_2| = \frac{3}{2}$$

Q.5 Show that the diagonals of the parallelogram having vertices $A(1,2), B(4,2), C(-1,-3)$ and $D(-4,-3)$ bisect each other.

Solution:

$ABCD$ is parallelogram which vertices are

$A(1,2), B(4,2), C(-1,-3) D(-4,-3)$

Let $\overline{BD}$ and $\overline{AC}$ the diagonals of parallelogram they intersect at point M

$A(1,2), C(-1,-3)$ midpoint of AC

Midpoint formula

$$M_1(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$M_1(x,y)=M_1\left(\frac{1-1}{2},\frac{2-3}{2}\right)$$

$$M_1(x,y)=M_1\left(\frac{0}{2},\frac{-1}{2}\right)=\left(0,\frac{-1}{2}\right)$$

Midpoint of BD,

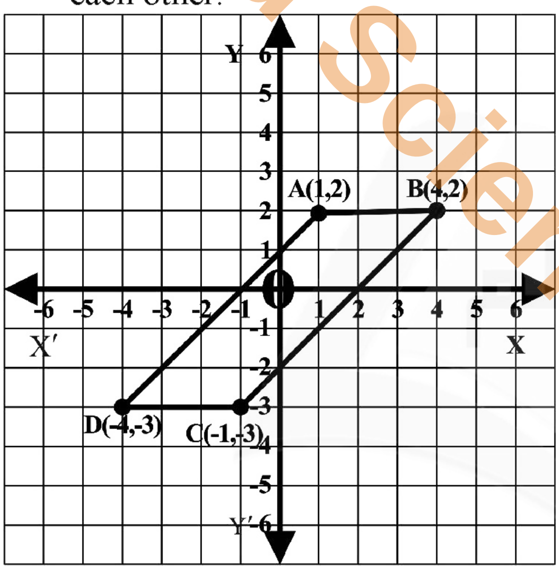
$$M_2(x,y)=M_2\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$M_2(x,\ y)=M_2\left(\frac{4-4}{2},\frac{2-3}{2}\right)$$

$$M_2(x,y)=M_2\left(\frac{0}{2},\frac{-1}{2}\right)$$

$$M_2(x,y)=M_2\left(0,\frac{-1}{2}\right)$$

As M_1 and M_2 Coincide the diagonals of the parallelogram bisect each other.



Q.6 **The vertices of a triangle are $P(4,6), Q(-2,-4)$ and $R(-8,2)$. Show that the length of the line segment joining the midpoints of the line segments $\overline{PR}, \overline{QR}$ is $\frac{1}{2}\overline{PQ}$**

Solution:

M_1 the midpoint of QR is

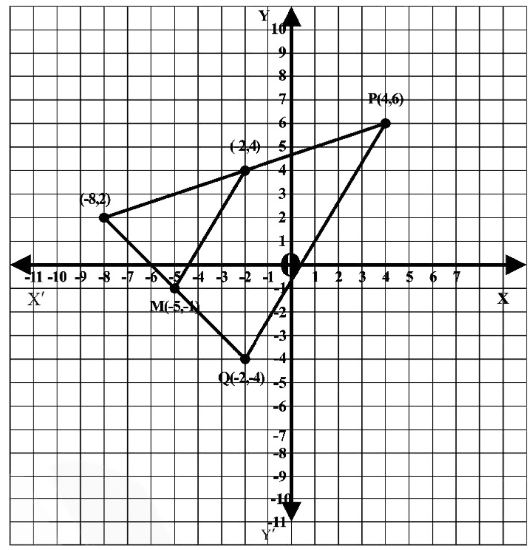
$$Q(-2,-4), R(-8,2)$$

$$M_1(x,y)=M_1\left(\frac{-2-8}{2},\frac{-4+2}{2}\right)$$

$$=M_1\left(\frac{-10}{2},\frac{-2}{2}\right)$$

$$=M_1(-5,-1)$$

$$M_1(-5,-1)$$



M_2 the midpoint of PR is

$$P(4,6), Q(-8,+2)$$

$$M_2(x,y)=M\left(\frac{4-8}{2},\frac{6+2}{2}\right)$$

$$M_2(x,y)=M_2\left(\frac{-4}{2},\frac{8}{2}\right)$$

$$M_2(x,y)=M_2(-2,4)$$

$$M_2(-2,4)$$

$$|M_1M_2|=\sqrt{|-5+2|^2+|4+1|^2}$$

$$|M_1M_2|=\sqrt{(-3)^2+(5)^2}$$

$$|M_1M_2|=\sqrt{9+25}$$

$$|M_1M_2|=\sqrt{34}$$

$$|PQ|=\sqrt{|4+2|^2+|6+4|^2}$$

$$|PQ|=\sqrt{(6)^2+(10)^2}=\sqrt{36+100}$$

$$|PQ|=\sqrt{136}$$

$$|PQ| = \sqrt{4 \times 34}$$

$$|PQ| = 2\sqrt{34}$$

$$\frac{|PQ|}{2} = \sqrt{34}$$

OR

$$\frac{1}{2}|PQ| = \sqrt{34}$$

Hence we proved that

$$|M_1M_2| = \frac{1}{2}|PQ|$$