Exercise 9.2

Q.1 **Show whether the points with vertices (5,-2),(5,4) and (-4,1) are the vertices of equilateral triangle or an isosceles triangle**

$$P(5,-2), Q(5,4), R(-4,1)$$

Solution:

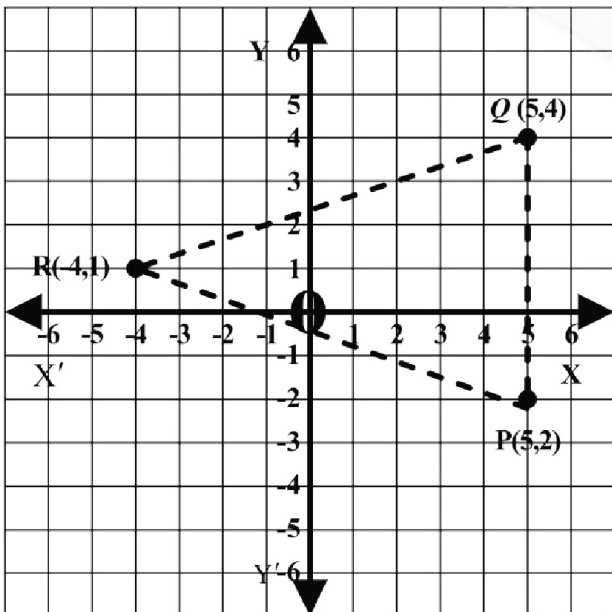We know that the distance formula is

$$= \sqrt{\left|x_2 - x_1\right|^2 + \left|y_2 - y_1\right|^2}$$

We have $P(5,-2), Q(5,4)$

$$|PQ| = \sqrt{\left|5-5\right|^2 + \left|4-(-2)\right|^2}$$

$$|PQ| = \sqrt{(0)^2 + (4+2)^2}$$

$$|PQ| = \sqrt{(6)^2}$$

$$|PQ| = 6$$

$$Q(5,4), R(-4,1)$$

$$|QR| = \sqrt{\left|-4-5\right|^2 + \left|1-4\right|^2}$$

$$|QR| = \sqrt{(-9)^2 + (-3)^2}$$

$$|QR| = \sqrt{81+9}$$

$$|QR| = \sqrt{90}$$

$$|QR| = \sqrt{9 \times 10} = 3\sqrt{10}$$

$$R(-4,1), P(5,-2)$$

$$|RP| = \sqrt{\left|5-(-4)\right|^2 + \left|-2-1\right|^2}$$

$$|RP| = \sqrt{(5+4)^2 + (-3)^2}$$

$$|RP| = \sqrt{(9)^2 + 9}$$

$$|RP| = \sqrt{81+9}$$

$$|RP| = \sqrt{90}$$

$$|RP| = \sqrt{9 \times 10} = 3\sqrt{10}$$

$$|QR| = |PR|$$

Two lengths of triangle are equal

So it is a isosceles triangle

Q.2 **Show whether or not the points with vertices $(-1,1), (2,-2)$ and $(-4,1)$ form a Square**

Solution:

$$P(-1,1) Q(5,4) R(2,-2) S(-4,1)$$

$$\text{Distance} = \sqrt{\left|x_2 - x_1\right|^2 + \left|y_2 - y_1\right|^2}$$

$$|PQ| = \sqrt{\left|5-(-1)^2\right| + \left|4-1\right|^2}$$

$$|PQ| = \sqrt{\left|5+1\right|^2 + \left|3\right|^2}$$

$$|PQ| = \sqrt{6^2 + 9}$$

$$|PQ| = \sqrt{36+9}$$

$$|PQ| = \sqrt{45}$$

$$|PQ| = \sqrt{9 \times 5}$$

$$|PQ| = 3\sqrt{5}$$

$$|QR| = \sqrt{\left|2-5\right|^2 + \left|-2-4\right|^2}$$

$$|QR| = \sqrt{(-3)^2 + (6)^2}$$

$$|QR| = \sqrt{9 + 36}$$

$$|QR| = \sqrt{45}$$

$$|QR| = \sqrt{9 \times 5}$$

$$|QR| = 3\sqrt{5}$$

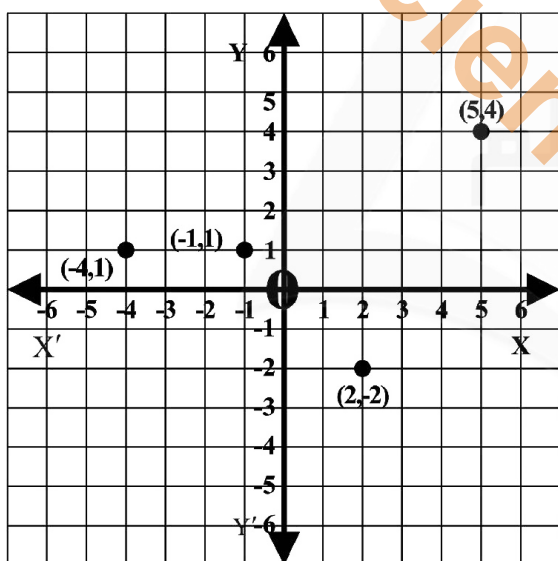$$|RS| = \sqrt{|-4-2|^2 + |1-(-2)|^2}$$

$$|RS| = \sqrt{(-6)^2 + (1+2)^2} = \sqrt{36 + (3)^2}$$

$$|RS| = \sqrt{36 + 9}$$

$$|RS| = \sqrt{45}$$

$$|RS| = \sqrt{9 \times 5}$$

$$|RS| = 3\sqrt{5}$$

$$|SP| = \sqrt{|-4-(-1)|^2 + |1-1|^2}$$

$$|SP| = \sqrt{(-4+1)^2 + (0)^2}$$

$$|SP| = \sqrt{(-3)^2}$$

$$|SP| = \sqrt{9}$$

$$|SP| = 3$$

If all the length are same then it will be a Square all the length are not equal so it is not square.

$$|PQ| = |QR| = |RS| \neq |SP|$$

Q.3 Show whether or not the points with coordinates $(1,3), (4,2)$ and $(-2,6)$ are vertices of a right triangle?

Solution:
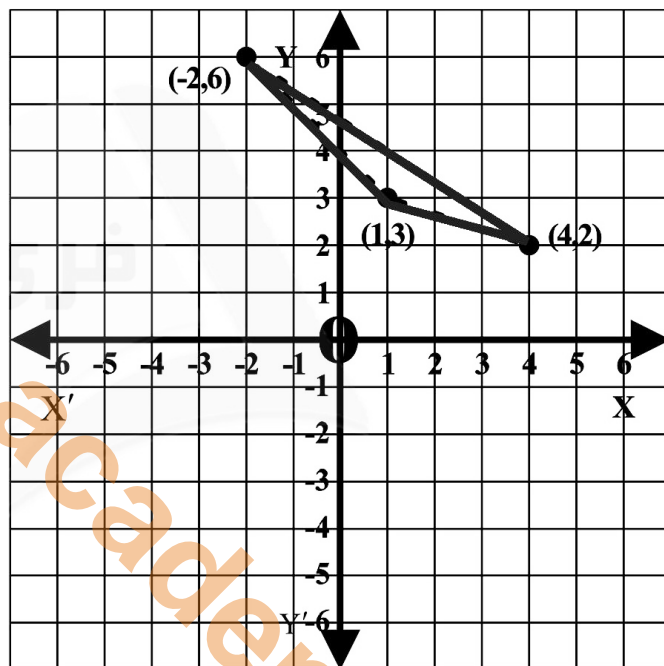
$$A(1,3), B(4,2), C(-2,6)$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|AB| = \sqrt{|4-1|^2 + |2-3|^2}$$

$$|AB| = \sqrt{(3)^2 + (-1)^2}$$

$$|AB| = \sqrt{9+1}$$

$$|AB| = \sqrt{10}$$

$$|BC| = \sqrt{|-2-4|^2 + |6-2|^2}$$

$$|BC| = \sqrt{(-6)^2 + (4)^2}$$

$$|BC| = \sqrt{36 + 16}$$

$$|BC| = \sqrt{52}$$

$$|CA| = \sqrt{|-2-1|^2 + |6-3|^2} = \sqrt{(-3)^2 + (3)^2}$$

$$|CA| = \sqrt{9+9}$$

$$|CA| = \sqrt{18}$$

By Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$$

$$(\sqrt{52})^2 = (\sqrt{18})^2 + (\sqrt{10})^2$$

$52 = 18 + 10$
$52 = 28$
Since $52 \neq 28$
So it not right angle triangle.

Q.4 **Use distance formula to prove whether or not the points $(1,1), (-2,-8)$ and $(4,10)$ lie on a straight line?**

Solution:

$A(1,1), B(-2,-8), C(4,10)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$
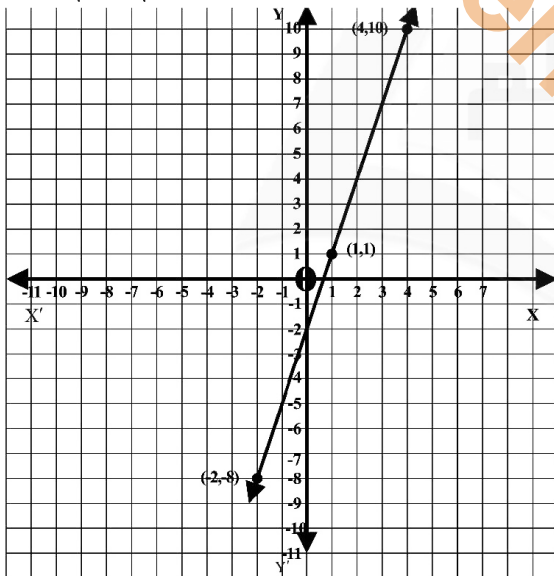
$|AB| = \sqrt{|-2-1|^2 + |-8-1|^2}$

$|AB| = \sqrt{(-3)^2 + (-9)^2}$

$|AB| = \sqrt{9+81}$

$|AB| = \sqrt{90}$

$|AB| = \sqrt{9 \times 10}$

$|AB| = 3\sqrt{10}$

$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$

$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$

$|BC| = \sqrt{(6)^2 + (18)^2}$

$|BC| = \sqrt{36+324}$

$|BC| = \sqrt{360}$

$|BC| = \sqrt{36 \times 10}$

$|BC| = 6\sqrt{10}$

$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$

$|AC| = \sqrt{(3)^2 + (9)^2}$

$|AC| = \sqrt{9+81}$

$|AC| = \sqrt{90}$

$|AC| = \sqrt{9 \times 10}$

$|AC| = 3\sqrt{10}$

$|AC| + |AB| = |BC|$

$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$

$6\sqrt{10} = 6\sqrt{10}$

It means that they lie on same line so they are collinear.

Q.5 **Find K given that point (2, K) is equidistance from (3, 7) and (9,1)**

Solution: $M(2,K), A(3,7)$ and $B(9,1)$

$|\overline{AM}| = |\overline{BM}|$

$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$

$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$

Taking square on both Side

$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$

$K^2 - 14K + 50 = 50 + K^2 - 2K$

$K^2 - 14K + 50 - 50 - K^2 + 2K = 0$

$-12K = 0$

$K = \dfrac{0}{-12}$

$K = 0$

Q.6 **Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear.**

Solution:

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$

$|AB| = \sqrt{(3)^2 + (-12)^2}$

$|AB| = \sqrt{9+144}$

$|AB| = \sqrt{153}$

$|AB| = \sqrt{9 \times 17}$

$52 = 18 + 10$

$52 = 28$

Since $52 \neq 28$

So it not right angle triangle.

Q.4 **Use distance formula to prove whether or not the points $(1,1),(-2,-8)$ and $(4,10)$ lie on a straight line?**

Solution:

$A(1,1), B(-2,-8), C(4,10)$

$d = \sqrt{\left|x_2 - x_1\right|^2 + \left|y_2 - y_1\right|^2}$

$\left|A B\right| = \sqrt{\left|-2 - 1\right|^2 + \left|-8 - 1\right|^2}$

$\left|A B\right| = \sqrt{(-3)^2 + (-9)^2}$

$\left|A B\right| = \sqrt{9 + 81}$

$\left|A B\right| = \sqrt{90}$

$\left|A B\right| = \sqrt{9 \times 10}$

$\left|A B\right| = 3\sqrt{10}$

$\left|B C\right| = \sqrt{\left|4 - (-2)\right|^2 + \left|10 - (-8)\right|^2}$

$\left|B C\right| = \sqrt{(4 + 2)^2 + (10 + 8)^2}$

$\left|B C\right| = \sqrt{(6)^2 + (18)^2}$

$\left|B C\right| = \sqrt{36 + 324}$

$\left|B C\right| = \sqrt{360}$

$\left|B C\right| = \sqrt{36 \times 10}$

$\left|B C\right| = 6\sqrt{10}$

$\left|A C\right| = \sqrt{\left|4 - 1\right|^2 + \left|10 - 1\right|^2}$

$\left|A C\right| = \sqrt{(3)^2 + (9)^2}$

$\left|A C\right| = \sqrt{9 + 81}$

$\left|A C\right| = \sqrt{90}$

$\left|A C\right| = \sqrt{9 \times 10}$

$\left|A C\right| = 3\sqrt{10}$

$\left|A C\right| + \left|A B\right| = \left|B C\right|$

$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$

$6\sqrt{10} = 6\sqrt{10}$

It means that they lie on same line so they are collinear.

Q.5 **Find K given that point (2, K) is equidistant from (3, 7) and (9,1)**

Solution: $M(2,K), A(3,7)$ and $B(9,1)$

$$\begin{array}{ccc} (3,7) & (2,K) & (9,1) \\ A & M & B \end{array}$$

$\left|\overline{AM}\right| = \left|\overline{BM}\right|$

$\sqrt{\left|2 - 3\right|^2 + \left|K - 7\right|^2} = \sqrt{\left|9 - 2\right|^2 + \left|1 - K\right|^2}$

$\sqrt{(-1)^2 + (K - 7)^2} = \sqrt{(7)^2 + (1 - K)^2}$

Taking square on both Side

$\left(\sqrt{1 + K^2 + 49 - 14K}\right)^2 = \left(\sqrt{49 + 1 + K^2 - 2K}\right)^2$

$K^2 - 14K + 50 = 50 + K^2 - 2K$

$K^2 - 14K + 50 - 50 - K^2 + 2K = 0$

$-12K = 0$

$K = \dfrac{0}{-12}$

$K = 0$

Q.6 **Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear.**

Solution:

$d = \sqrt{\left|x_2 - x_1\right|^2 + \left|y_2 - y_1\right|^2}$

$\left|A B\right| = \sqrt{\left|3 - 0\right|^2 + \left|-5 - 7\right|^2}$

$\left|A B\right| = \sqrt{(3)^2 + (-12)^2}$

$\left|A B\right| = \sqrt{9 + 144}$

$\left|A B\right| = \sqrt{153}$

$\left|A B\right| = \sqrt{9 \times 17}$

$$|AB| = 3\sqrt{17}$$

$$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$
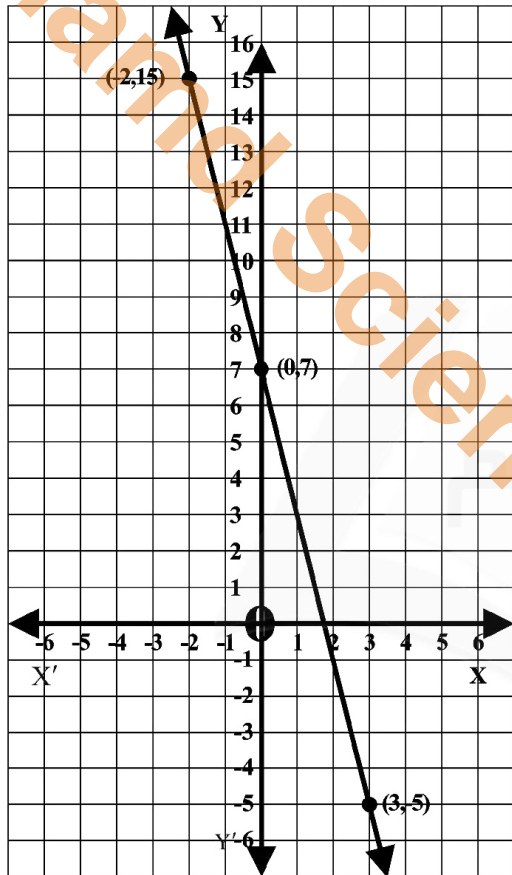
$$|BC| = \sqrt{(-5)^2 + (15+5)^2}$$

$$|BC| = \sqrt{25 + (20)^2}$$

$$|BC| = \sqrt{25 + 400}$$

$$|BC| = \sqrt{425}$$

$$|BC| = \sqrt{25 \times 17}$$

$$|BC| = 5\sqrt{17}$$

$$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$$

$$|AC| = \sqrt{(-2)^2 + (8)^2}$$

$$|AC| = \sqrt{4 + 64}$$

$$|AC| = \sqrt{68}$$

$$|AC| = \sqrt{4 \times 17}$$

$$|AC| = 2\sqrt{17}$$

$$|AB| + |AC| = |BC|$$

$$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 **Verify whether or not the points** $O(0,0)$ $A(\sqrt{3},1), B(\sqrt{3},-1)$ **are the vertices of an equilateral triangle**

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|OA| = \sqrt{|\sqrt{3} - 0|^2 + |0-1|^2}$$

$$|OA| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OA| = \sqrt{3+1}$$

$$|OA| = \sqrt{4}$$

$$|OA| = 2$$

$$|OB| = \sqrt{|\sqrt{3} - 0|^2 + |-1-0|^2}$$

$$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OB| = \sqrt{3+1}$$

$$|OB| = \sqrt{4}$$

$$|OB| = 2$$

$$|AB| = \sqrt{|\sqrt{3} - \sqrt{3}|^2 + |-1-1|^2}$$

$$|AB| = \sqrt{0 + (-2)^2}$$

$$|AB| = \sqrt{4}$$

$$|AB| = 2$$

All the sides are same in length so it is equilateral triangle

Q.8 **Show that the points** $A(-6,-5), B(5,-5), C(5,-8)$ **and** $D(-6,-8)$ **are the vertices of a rectangle find the length of its diagonals are equal**

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$A(-6,-5), B(5,-5)$$

$$|AB| = 3\sqrt{17}$$

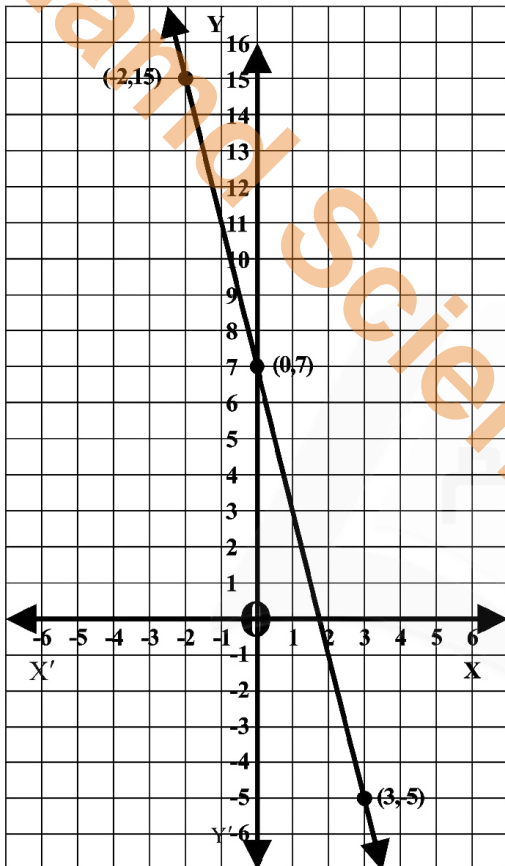$$|BC| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$$

$$|BC| = \sqrt{(-5)^2 + (15+5)^2}$$

$$|BC| = \sqrt{25 + (20)^2}$$

$$|BC| = \sqrt{25 + 400}$$

$$|BC| = \sqrt{425}$$

$$|BC| = \sqrt{25 \times 17}$$

$$|BC| = 5\sqrt{17}$$

$$|AC| = \sqrt{|-2-0|^2 + |15-7|^2}$$

$$|AC| = \sqrt{(-2)^2 + (8)^2}$$

$$|AC| = \sqrt{4 + 64}$$

$$|AC| = \sqrt{68}$$

$$|AC| = \sqrt{4 \times 17}$$

$$|AC| = 2\sqrt{17}$$

$$|AB| + |AC| = |BC|$$

$$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 **Verify whether or not the points** $O(0,0)\ A(\sqrt{3},1), B(\sqrt{3},-1)$ **are the vertices of an equilateral triangle**

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$|OA| = \sqrt{|\sqrt{3}-0|^2 + |0-1|^2}$$

$$|OA| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OA| = \sqrt{3+1}$$

$$|OA| = \sqrt{4}$$

$$|OA| = 2$$

$$|OB| = \sqrt{|\sqrt{3}-0|^2 + |-1-0|^2}$$

$$|OB| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$|OB| = \sqrt{3+1}$$

$$|OB| = \sqrt{4}$$

$$|OB| = 2$$

$$|AB| = \sqrt{|\sqrt{3}-\sqrt{3}|^2 + |-1-1|^2}$$

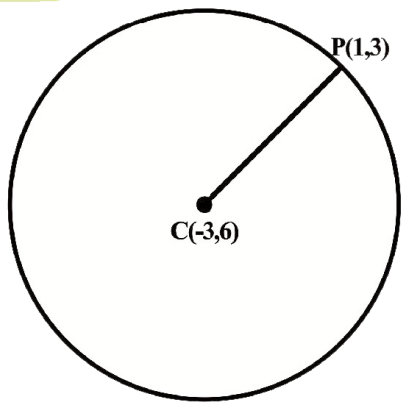$$|AB| = \sqrt{0 + (-2)^2}$$

$$|AB| = \sqrt{4}$$

$$|AB| = 2$$

All the sides are same in length so it is equilateral triangle

Q.8 **Show that the points** $A(-6,-5), B(5,-5), C(5,-8)$ **and** $D(-6,-8)$ **are the vertices of a rectangle find the length of its diagonals are equal**
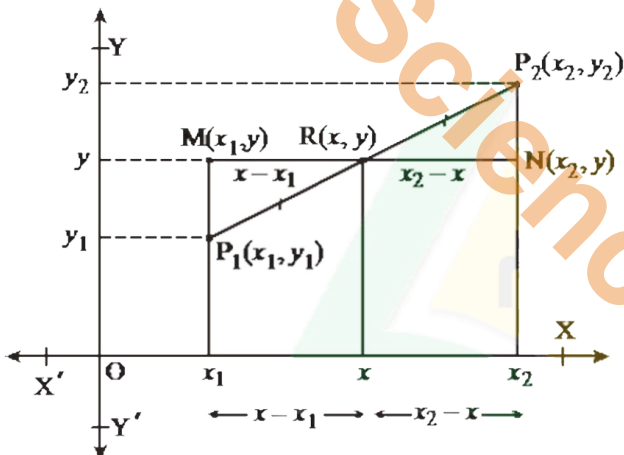
Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$A(-6,-5), B(5,-5)$$

Recognition of the midpoint formula for any two points in the plane

Let $P_1(x, y)$ and $P_2(x_2, y_2)$ be any two points in the plane and $R(x, y)$ be midpoint of point P_1 and P_2 on the line segment P_1P_2 as shown in the figure.

If the line segment MN, parallel to x-axis has its midpoint $R(x, y)$,

then, $x_2 - x = x - x_1$

$x_2 + x_1 = x + x$

$2x = x_1 + x_2 \Rightarrow x = \dfrac{x_1 + x_2}{2}$

Similarly, $y = \dfrac{y_1 + y_2}{2}$

Thus the point R (x, y)

$= R\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the

midpoint of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

Verification of the midpoint formula

$|P_1R| = \sqrt{\left(\dfrac{x_1 + x_2}{2} - x_1\right)^2 + \left(\dfrac{y_1 + y_2}{2} - y_1\right)^2}$

$|P_1R| = \sqrt{\left(\dfrac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\dfrac{y_1 + y_2 - 2y_1}{2}\right)^2}$

$|P_1R| = \sqrt{\left(\dfrac{x_2 - x_1}{2}\right)^2 + \left(\dfrac{y_2 - y_1}{2}\right)^2}$

$|P_1R| = \sqrt{\dfrac{(x_2 - x_1)^2}{4} + \dfrac{(y_2 - y_1)^2}{4}}$

$|P_1R| = \sqrt{\dfrac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{4}}$

$|P_1R| = \dfrac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{2}$

OR

$|P_1R| = \dfrac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \dfrac{1}{2}|P_1P_2|$

and $|P_2R| = \sqrt{\left(\dfrac{x_1 + x_2}{2} - x_2\right)^2 + \left(\dfrac{y_1 + y_2}{2} - y_2\right)^2}$

$|P_2R| = \sqrt{\left(\dfrac{x_1 + x_2 + 2x_2}{2}\right)^2 + \left(\dfrac{y_1 + y_2 - 2y_2}{2}\right)^2}$

$|P_2R| = \sqrt{\left(\dfrac{x_1 - x_2}{2}\right)^2 + \left(\dfrac{y_1 - y_2}{2}\right)^2}$

$|P_2R| = \sqrt{\dfrac{(x_1 - x_2)^2}{4} + \dfrac{(y_1 - y_2)^2}{4}}$

$|P_2R| = \sqrt{\dfrac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{4}}$

$|P_2R| = \dfrac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{2}$

OR

$|P_2R| = \dfrac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\Rightarrow |P_2R| = |P_1R| = \dfrac{1}{2}|P_1R_2|$

Thus it verifies that

$R\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the midpoint

of the line segment P_1RP_2 which lies on the line segment since $|P_1R| + |P_2R| = |P_1P_2|$