Review Exercise 7

Q.1 Choose the correct answer

(i) Which of the following is the solution of the inequality $3-4x \le 11$**?**

(a) -8 **(b)** -2

(c) $-\frac{14}{4}$ **(d)** None of these

(ii) A statement involving any of the symbols $<$, $>$, $\le$or $>$, is called------------

(a) Equation **(b)** Identity

(c) Inequality **(d)** Linear equation

(iii) x **= ------- is a solution of the inequality** $-z < x > \frac{3}{2}$

(a) -5 **(b)** 3

(c) 0 **(d)** $\frac{3}{2}$

(iv) If x **is no larger than 10, then ----------**

(a) $x \le 8$ **(b)** $x \ge 10$

(c) $x < 10$ **(d)** $x > 10$

(v) If the capacity $<$ of an elevator is at most 1600 pounds then ----------

(a) $c < 1600$ **(b)** $c \ge 1600$

(c) $c \le 1600$ **(d)** $c > 1600$

(vi) x = 0 is a solution of the inequality ----------

(a) $x > 0$ **(b)** $3x+5<0$

(c) $x+\frac{z}{2}<0$ **(d)** $x-2<0$

ANSWER KEY

i	ii	iii	iv	v	vi
b	c	c	b	c	d

Q.2 Identify the following statement as true or false

(i) The equation $3x-5=7-x$ is a linear equation. (True)

(ii) The equation $x-0.3x=0.7x$ is an identity (True)

(iii) The equation $-2x+3=8$ is equivalent to $-2x=11$ (False)

(iv) To eliminate fractions we multiply each side of an equation by the L.C.M of denominators (True)

(v) $4(x+3)=x+3$ is a conditional equations (True)

(vi) The equation $2(3x+5)=6x+12$ is an in consistent equation (True)

(vii) To solve $\frac{2}{3}x=12$, we should multiply each side by $\frac{2}{3}$ (False)

(viii) Equations having exactly the same solution are called equivalent equations. (True)

(ix) A solution that does not satisfy the original equation is called extra solution (True)

Q.3 Answer the following short question.

(i) Define a linear inequality in one variable

Ans A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form $ax+b<0, a\neq 0$

(ii) State the trichotomy and transitive properties of in equalities

Ans **Trichotomy Property**

For any $a,b\in R$ one and only one of the following statements in true. $a<b$ or $a=b$, or $a>b$

Transitive Property

Let $a,b,c\in R$.

(a) If $a>b$ and $b>c$, then $a>c$

(b) If $a<b$ and $b<c$, then $a<c$

(iii) The formula relating degree Fahrenheit to degree Celsius is F= $\frac{9}{5}c+32$ for what value of c is F< O was

Ans F= $\frac{9}{5}c+32$

$\frac{9}{5}c+32=\text{F}$

Since F <0

So $\frac{9}{5}c+32<0$

$\frac{9c+160}{5}<0$

Or $9c+160<0\times 5$

Or $9c+160<0$

Or $9c<-160$

Or $c<-\frac{160}{9}$

(iv) Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relation ship

Solution: Let the integer $=y$

Sum of integer and $12=y+12$

Seven times sum of integer and $12=7(y+12)$

According to condition

$50\leq 7(y+12)\leq 60$

$\frac{50}{7}\leq 7\frac{(y+12)}{7}\leq\frac{60}{7}$

$\frac{50}{7}\leq y+12\leq\frac{60}{7}$

$\frac{50}{7}-12\leq y+\cancel{12}-\cancel{12}\leq\frac{60}{7}-12$

$$\frac{50-84}{7} \le y \le \frac{60-84}{7}$$

$$\frac{-34}{7} \le y \le \frac{-24}{7}$$

Solution Set $= \left\{ y \mid \frac{-34}{7} \le y - \frac{24}{7} \right\}$

Q.4 **Solve each of the following and check for extraneous solution if any**

(i) $\sqrt{2t+4} = \sqrt{t-1}$

Solution: $\sqrt{2t+4} = \sqrt{t-1}$

Taking square on both side

$$\left(\sqrt{2t+4}\right)^2 = \left(\sqrt{t-1}\right)^2$$

$$2t+4 = t-1$$

$$2t-t = -1-4$$

$$t = -5$$

To check

$$\sqrt{2t+4} = \sqrt{t-1}$$

When $t = -5$

$$\sqrt{2(-5)+4} = \sqrt{t-5-1}$$

$$\sqrt{-10+4} = \sqrt{-6}$$

$$\sqrt{-6} = \sqrt{-6}$$

L.H.S = R.H.S

Solution Set $= \{-5\}$

(ii) $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

Solution: $\sqrt{3x-1} - 2\sqrt{8-2x} = 0$

$$\sqrt{3x-1} = 2\sqrt{8-2x}$$

Taking square on both side

$$\left(\sqrt{3x-1}\right)^2 = \left(2\sqrt{8-2x}\right)^2$$

$$3x-1 = 4(8-2x)$$

$$3x-1 = 32-8x$$

$$3x+8x = 32+1$$

$$11x = 33$$

$$x = \frac{33}{11}$$

$$x = 3$$

To check

$$\sqrt{3x-1} - 2\sqrt{8-2x} = 0$$

When $x=3$

$\sqrt{3(3)-1}-2\sqrt{8-2(3)}=0$

$\sqrt{9-1}-2\sqrt{8-6}=0$

$\sqrt{8}-2\sqrt{2}=0$

$2\sqrt{2}-2\sqrt{2}=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{3\}$

Q.5 Solve for x

(i) $|3x+14|-2=5x$

Solution: $|3x+14|-2=5x$

$|3x+14|=5x+2$

$3x+14=\pm(5x+2)$

$3x+14=5x+2$

$14-2=5x-3x$

$12=2x$

$\frac{12}{2}=x$

$x=6$

$3x+14=-(5x+2)$

$3x+14=-5x-2$

$3x+5x=-2-14$

$8x=\frac{-16}{8}$

$x=-2$

To check

$|3x+14|-2=5x$

When $x=6$

$|3(6)+14|-2=5(6)$

$|18+14|-2=30$

$32-2=30$

$30=30$

$|3x+14|-2=5x$

when $x=-2$

$|3(-2)+14|-2=5(-2)$

$|-6+14|-2=-10$

$8-2=-10$

$6=-10$

Solution Set $=\{6\}$

(ii) $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

Solution $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

$\frac{2}{3}|x-3|=|x+2|$

$\frac{2}{3}=\frac{|x+2|}{|x-3|}$

$\frac{x+2}{x-3}=\pm\frac{2}{3}$

$$\frac{x+2}{x-3}=\frac{2}{3}$$
$$3(x+2)=2(x-3)$$
$$3x+6=2x-6$$
$$3x-2x=-6-6$$
$$x=-12$$

To check

$$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$$

When $x=-12$

$$\frac{1}{3}|-12-3|=\frac{1}{2}|-12+2|$$
$$\frac{1}{3}|-15|=\frac{1}{2}|-10|$$
$$\frac{1}{\cancel{3}}\left(\cancel{15}^{5}\right)=\frac{1}{\cancel{2}}\left(\cancel{10}^{5}\right)$$
$$5=5$$

and

$$\frac{x+2}{x-3}=-\frac{2}{3}$$
$$3(x+2)=-2(x-3)$$
$$3x+6=-2x+6$$
$$3x+2x=+6-6$$
$$5x=0$$
$$x=\frac{0}{5}\quad\Rightarrow x=0$$
$$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$$

when $x=0$

$$\frac{1}{3}|0-3|=\frac{1}{2}|0+2|$$
$$\frac{1}{3}|-3|=\frac{1}{2}|2|$$
$$\frac{1}{\cancel{3}}\left(\cancel{3}^{1}\right)=\frac{1}{\cancel{2}}\left(\cancel{2}^{1}\right)$$
$$\frac{1}{3}(3)=1$$
$$1=1$$

Solution Set $=\{-12,0\}$

Q.6 Solve the following inequality

(i) $-\frac{1}{3}x+5\le 1$

Solution $-\frac{1}{3}x+5\le 1$

$$-\frac{1}{3}x\le 1-5$$
$$-\frac{1}{3}x\le -4$$
$$x\ge -4\times(-3)$$
$$x\ge 12$$

Solution Set $=\{x\,|\,x\ge 12\}$

(ii) $-3<\frac{1-2x}{5}<1$

Solution $-3<\frac{1-2x}{5}<1$

$$-3<\frac{1-2x}{5}\qquad\qquad \frac{1-2x}{5}<1$$

$$-15 < 1 - 2x \qquad 1 - 2x < 5$$
$$-15 - 1 < -2x \qquad -2x < 5 - 1$$
$$-16 < -2x \qquad -2x < 4$$
$$\frac{-16}{-2} > x \qquad x > \frac{4}{-2}$$
$$8 > x \qquad x > -2$$
$$x < 8 \qquad -2 < x$$
$$-2 < x < 8$$

Solution Set $= \{x \mid -2 < x < 8\}$

Unit 7: Linear Equations and Inequalities

Overview

Linear Equation:

A linear equation in one unknown variable x is an equation of the form ax + b = 0, where a,b $\in$R and a $\neq$ 0.

Example:

(i) $5x - 3 = 0$

(ii) $\frac{1}{2}x + 18 = 0$

Radical equations:

When the variable in an equation occurs under a radical the equation is called a radical equation.

Example:

(i) $\sqrt{2x-3} - 7 = 0$

Absolute value:

The absolute value of a real number ‘a’ denoted by lal, is defined as

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

e.g., $|6| = 6,$

$|0| = 0$

$|-6| = -(-6) = 6$

Extraneous Roots:

If the solutions (roots) obtained from the equation does not satisfy the original equations are called extraneous roots.

Linear inequality:

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form. $ax + b < 0,\ a \neq 0\ a,b \in R$ we may replace the symbol <by>, $\leq$or $\geq$also.

Inconsistent equation:

An inconsistent equation is that whose solution set is ϕ.