

Exercise 7.3

Q1) Solve the following inequalities.

(i) $3x + 1 < 5x - 4$

Solution: $3x + 1 < 5x - 4$

$$3x < 5x - 4 - 1$$

$$3x - 5x < -5$$

$$-2x < -5$$

Case-I When negative is eliminated from both sides of inequality the symbol will be change.

Case-II When negative is transferred from variable to constant side, symbol will also change.

$$x > \frac{-5}{-2}$$

$$x > \frac{5}{2}$$

Solution Set = $\left\{ x \mid x > \frac{5}{2} \right\}$

(ii) $4x - 10.3 \leq 21x - 1.8$

Solution: $4x - 10.3 \leq 21x - 1.8$

$$4x - 21x \leq -8.5 + 10.3$$

$$-17x \leq 8.5$$

When negative value is shifted to other side its symbol changes.

$$x \geq \frac{8.5}{-17}$$

$$x \geq -\frac{8.5}{17}$$

$$x \geq -0.5$$

Solution Set = $\{x \mid x \geq -0.5\}$

(iii) $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

Solution: $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

$$-\frac{1}{2}x - \frac{1}{4}x \geq -7 - 4$$

$$\frac{-2x - x}{4} \geq -11$$

$$-3x \geq -44$$

When negative value is shifted the symbol changes

$$x \leq \frac{-44}{-3}$$

$$x \leq \frac{44}{3}$$

Solution Set = $\{x \mid x \leq \frac{44}{3}\}$

(iv) $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

Solution: $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

$$x - 10 + 4x \geq 6x - \frac{7}{2}$$

$$5x - 6x \geq -\frac{7}{2} + 10$$

$$-1x \geq \frac{-7 + 20}{2}$$

$$-x \geq -\frac{13}{2}$$

When negative is shifted other side symbol changes

$$x \leq \frac{13}{-1 \times 2}$$

$$x \leq -\frac{13}{2}$$

$$x \leq -6.5$$

Solution Set = $\{x \mid x \leq -6.5\}$

(v) $\frac{3x + 2}{9} - \frac{2x + 1}{3} > -1$

Solution: $\frac{3x + 2}{9} - \frac{2x + 1}{3} > -1$

$$\frac{3x + 2 - 3(2x + 1)}{9} > -1$$

$$3x + 2 - 6x - 3 > -9$$

$$-3x > -9 + 1$$

$$x > -8$$

Negative value is shifted to other side its symbols changes

$$x < \frac{-8}{-3}$$
$$x < \frac{8}{3}$$

$$\text{Solution Set} = \left\{ x \mid x < \frac{8}{3} \right\}$$

(vi) $3(2x+1) - 2(2x+5) < 5(3x-2)$

Solution: $3(2x+1) - 2(2x+5) < 5(3x-2)$

$$6x + 3 - 4x - 10 < 15x - 10$$
$$2x - 7 - 15x < -10$$
$$-13x < -10 + 7$$
$$-13x < -3$$

The value is negative when shifted to other side it changes its symbols

$$x > \frac{-3}{-13}$$
$$x > \frac{3}{13}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{3}{13} \right\}$$

(vii) $3(x-1) - (x-2) > -2(x+4)$

Solution: $3(x-1) - (x-2) > -2(x+4)$

$$3x - 3 - x + 2 > -2x - 8$$
$$2x - 1 > -2x - 8$$
$$2x + 2x > -8 + 1$$
$$4x > -7$$
$$x > \frac{-7}{4}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{-7}{4} \right\}$$

(viii) $2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$

Solution: $2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$

$$\frac{8}{3}x + \frac{10x-8}{3} > -\frac{(8x+7)}{3}$$

$$\frac{8x+10x-8}{3} > -\frac{8x+7}{3}$$

Multiplying both side by 3

$$3 \times \frac{18x-8}{3} > -3 \times \frac{8x+7}{3}$$

$$18x - 8 > -(8x + 7)$$

$$18x - 8 > -8x - 7$$

$$18x + 8x > -7 + 8$$

$$26x > 1$$

$$x > \frac{1}{26}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{1}{26} \right\}$$

Q2) Solve the following inequalities

(i) $-4 < 3x + 5 < 8$

Solution: $-4 < 3x + 5 < 8$

$$\begin{aligned} -4 &< 3x + 5 & \text{and} & 3x + 5 < 8 \\ -4 - 5 &< 3x & 3x &< 8 - 5 \\ -9 &< 3x & 3x &< 3 \\ \frac{-9}{3} &< x & x &< \frac{3}{3} \\ -3 &< x & x &< 1 \\ -3 &\leq x & & \end{aligned}$$

$$\text{Solution Set} = \left\{ x \mid -3 < x < 1 \right\}$$

(ii) $-5 \leq \frac{4-3x}{2} < 1$

Solution: $-5 \leq \frac{4-3x}{2} < 1$

$$\begin{aligned} -5 &\leq \frac{4-3x}{2} & \text{and} & \frac{4-3x}{2} < 1 \\ -10 &\leq 4 - 3x & 4 - 3x &< 2 \end{aligned}$$

$$3x - 10 \leq 4$$

$$3x \leq 4 + 10$$

$$3x \leq 14$$

$$x \leq \frac{14}{3}$$

$$\frac{2}{3} < x$$

$$-3x < 2 - 4$$

$$-3x < -2$$

$$x > \frac{-2}{-3}$$

$$x > \frac{2}{3}$$

$$\frac{2}{3} < x \leq \frac{14}{3}$$

Solution Set = $\{x \mid \frac{2}{3} < x \leq \frac{14}{3}\}$

(iii) $-6 < \frac{x-2}{4} < 6$

Solution: $-6 < \frac{x-2}{4} < 6$

$$-6 < \frac{x-2}{4}$$

$$-24 < x - 2$$

$$-24 + 2 < x$$

$$-22 < x$$

and

$$\frac{x-2}{4} < 6$$

$$x - 2 < 24$$

$$x < 24 + 2$$

$$x < 26$$

$$-22 < x < 26$$

Solution Set = $\{x \mid -22 < x < 26\}$

(iv) $3 \geq \frac{7-x}{2} \geq 1$

Solution: $3 \geq \frac{7-x}{2} \geq 1$

$$3 \geq \frac{7-x}{2}$$

$$6 \geq 7 - x$$

$$6 - 7 \geq -x$$

$$-1 \geq -x$$

Negative sign change the symbols

$$1 \leq x$$

and

$$\frac{7-x}{2} \geq 1$$

$$7 - x \geq 2$$

$$-x \geq 2 - 7$$

$$-x \geq -5$$

$$x \leq 5$$

$$1 \leq x \leq 5$$

Solution Set = $\{x \mid 1 \leq x \leq 5\}$

(v) $3x - 10 \leq 5 < x + 3$

Solution: $3x - 10 \leq 5 < x + 3$

$$3x - 10 \leq 5 \quad \text{and} \quad 5 < x + 3$$

$$3x \leq 5 + 10 \quad 5 - 3 < x$$

$$3x \leq 15 \quad 2 < x$$

$$\frac{3x}{3} \leq \frac{15}{3}$$

$$x \leq 5$$

$$2 < x \leq 5$$

Solution Set = $\{x \mid 2 < x \leq 5\}$

(vi) $-3 \leq \frac{x-4}{-5} < 4$

Solution: $-3 \leq \frac{x-4}{-5} < 4$

$$-3 \leq \frac{x-4}{-5} \quad \text{and} \quad \frac{x-4}{-5} < 4$$

$$-3 \times -5 \geq x - 4$$

$$x - 4 > 4(-5)$$

$$15 \geq x - 4$$

$$x > -20 + 4$$

$$15 + 4 \geq x$$

$$x > -16$$

$$19 \geq x$$

$$-16 < x$$

$$x \leq 19$$

$$-16 < x \leq 19$$

Solution Set = $\{x \mid -16 < x \leq 19\}$

(vii) $1 - 2x < 5 - x \leq 25 - 6x$

Solution: $1 - 2x < 5 - x \leq 25 - 6x$

$$1 - 2x < 5 - x \quad \text{and} \quad 5 - x \leq 25 - 6x$$

$$-x + 6x \leq 25 - 5 \quad 6x - x \leq 20$$

$$1 - 2x + x < 5 \quad 5x \leq 20$$

$$-x < 5 - 1$$

$$x \leq \frac{20}{5}$$

$$-x < 4$$

$$x \leq 4$$

Due negative sign

Symbol change

$$-4 < x$$

$$-4 < x \leq 4$$

Solution Set = $\{x \mid -4 < x \leq 4\}$

(viii) $3x - 2 < 2x + 1 < 4x + 17$

Solution: $3x - 2 < 2x + 1 < 4x + 17$

$$3x - 2 < 2x + 1 \quad 2x + 1 < 4x + 17$$

$$3x - 2x - 2 < +1 \quad 2x - 4x < 17 - 1$$

$$x < 1 + 2$$

$$-2x < 16$$

$$x < 3$$

$$x > \frac{16}{-2}$$

$$x > -8$$

$$-8 < x$$

$$-8 < x < 3$$

Solution Set = $\{x \mid -8 < x < 3\}$

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