

Exercise 7.2

Q1) Identify the following statements as true or

- | | | |
|--------------|---|-------|
| (i) | $ x = 0$ has only one solution | True |
| (ii) | All absolute value equations have two solutions | False |
| (iii) | The equation $ x = 2$ is equivalent to $x = 2$ or $x = -2$ | True |
| (iv) | The equation $ x-4 = -4$ has no solution | True |
| (v) | The equation $ 2x-3 = 5$ is equivalent to $2x - 3 = 5$ or $2x + 3 = 5$ | False |

Q2)

(i) $|3x-5| = 4$
Solution $|3x-5| = 4$
 $3x-5 = \pm 4$
 $3x-5 = 4$ $3x-5 = -4$
 $3x = 4+5$ $3x = -4+5$

$3x = 9$	$3x = 1$
$x = \frac{9}{3}$	$x = \frac{1}{3}$
$x = 3$	

To check

$x = 3$	To check $x = \frac{1}{3}$
$ 3(3)-5 = 4$	$\left 3 \times \frac{1}{3} - 5\right = 4$
$ 9-5 = 4$	$ 1-5 = 4$
$4 = 4$	$ -4 = 4$
	$4 = 4$

Solution Set = $\left\{3, \frac{1}{3}\right\}$

(ii) $\frac{1}{2}|3x+2|-4 = 11$

Solution $\frac{1}{2}|3x+2|-4 = 11$

$\frac{1}{2}|3x+2|-4 = 11$

$\frac{1}{2}|3x+2| = 11+4$

$\frac{1}{2}|3x+2| = 15$

$|3x+2| = 2 \times 15$

$|3x+2| = 30$

$3x+2 = \pm 30$

$3x+2 = 30$

$3x = 30-2$

$3x = 28$

$x = \frac{28}{3}$

Check

$\frac{1}{2}|3x+2|-4 = 11$

$\frac{1}{2}\left|3 \times \frac{28}{3} + 2\right| - 4 = 11$

$\frac{1}{2}|28+2|-4 = 11$

$\frac{1}{2} \times 30 - 4 = 11$

$15 - 4 = 11$

$11 = 11$

$3x+2 = -30$

$3x = -30-2$

$3x = -32$

$x = \frac{-32}{3}$

$\frac{1}{2}\left|3 \times \frac{-32}{3} + 2\right| - 4 = 11$

$\frac{1}{2}|-32+2|-4 = 11$

$\frac{1}{2}|-30|-4 = 11$

$\frac{1}{2}(30) - 4 = 11$

$15 - 4 = 11$

$11 = 11$

Solution Set = $\left\{\frac{28}{3}, \frac{-32}{3}\right\}$

(iii) $|2x+5| = 11$

Solution $|2x+5| = 11$

$2x+5 = \pm 11$

$2x+5 = 11$

$2x = 11-5$

$2x = 6$

$x = \frac{6}{2}$

$x = 3$

$2x+5 = -11$

$2x = -11-5$

$2x = -16$

$x = \frac{-16}{2}$

$x = -8$

To check

$$|2x + 5| = 11$$

$$|2 \times 3 + 5| = 11$$

$$6 + 5 = 11$$

$$11 = 11$$

Solution Set = $\{-8, 3\}$

$$|2(-8) - 8 + 5| = 11$$

$$|-16 + 5| = 11$$

$$|-11| = 11$$

$$11 = 11$$

(iv) $|3 + 2x| = |6x - 7|$

Solution $|3 + 2x| = |6x - 7|$

$$3 + 2x = \pm(6x - 7)$$

$$3 + 2x = 6x - 7$$

$$3 + 7 = 6x - 7$$

$$10 = 4x$$

$$\frac{10}{4} = x$$

$$x = \frac{5}{2}$$

To check

$$|3 + 2x| = |6x - 7|$$

$$\left|3 + 2\left(\frac{5}{2}\right)\right| = \left|6\left(\frac{5}{2}\right) - 7\right|$$

$$\left|3 + 2 \times \frac{1}{2}\right| = \left|6 \times \frac{1}{2} - 7\right|$$

$$|3 + 5| = |15 - 7|$$

$$|8| = |8|$$

$$8 = 8$$

Solution Set = $\left\{\frac{5}{2}, \frac{1}{2}\right\}$

(v) $|x + 2| - 3 = 5 - |x + 2|$

Solution $|x + 2| - 3 = 5 - |x + 2|$

$$|x + 2| + |x + 2| = 5 + 3$$

$$2|x + 2| = 8$$

$$|x + 2| = \frac{8}{2}$$

$$|x + 2| = 4$$

$$x + 2 = \pm 4$$

$$x + 2 = 4$$

$$x = 4 - 2$$

$$x = 2$$

$$x + 2 = -4$$

$$x = -4 - 2$$

$$x = -6$$

To check

$$|x + 2| - 3 = 5 - |x + 2|$$

$$|x + 2| - 3 = 5 - |x + 2|$$

$$|2 + 2| - 3 = 5 - |2 + 2|$$

$$|-6 + 2| - 3 = 5 - |-6 + 2|$$

$$14 - 3 = 5 - |4|$$

$$|-4| - 3 = 5 - |-4|$$

$$4 - 3 = 5 - 4$$

$$4 - 3 = 5 - 4$$

$$1 = 1$$

$$1 = 1$$

Solution Set = $\{-6, 2\}$

(vi) $\frac{1}{2}|x + 3| + 21 = 9$

Solution $\frac{1}{2}|x + 3| + 21 = 9$

$$\frac{1}{2}|x + 3| = 9 - 21$$

$$\frac{1}{2}|x + 3| = -12$$

$$|x + 3| = -12 \times 2$$

$$|x + 3| = -24$$

Value of absolute in never negative so solution is not possible

Solution Set = $\{ \}$

(vii) $\left|\frac{3 - 5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

Solution $\left|\frac{3 - 5x}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$$\left|\frac{3 - 5x}{4}\right| = \frac{2}{3} + \frac{1}{3}$$

$$\left|\frac{3 - 5x}{4}\right| = \frac{2 + 1}{3}$$

$$\left|\frac{3 - 5x}{4}\right| = \frac{3}{3}$$

$$\left|\frac{3 - 5x}{4}\right| = 1$$

$$\frac{3 - 5x}{4} = \pm 1$$

$$\frac{3 - 5x}{4} = 1$$

$$\text{and } \frac{3 - 5x}{4} = -1$$

$$3 - 5x = 4$$

$$3 - 5x = -4$$

$$-5x = 4 - 3$$

$$-5x = -4 - 3$$

$$-5x = 1$$

$$-5x = -7$$

$$x = \frac{1}{-5}$$

$$x = -\frac{1}{5}$$

$$\left| \frac{3-5 \times \left(-\frac{1}{5}\right)}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{3+1}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{4}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\frac{3-1}{3} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

$$\text{Solution Set} = \left\{ \frac{-1}{5}, \frac{7}{5} \right\}$$

$$\text{(viii)} \quad \left| \frac{x+5}{2-x} \right| = 6$$

$$\text{Solution} \quad \left| \frac{x+5}{2-x} \right| = 6$$

$$\frac{x+5}{2-x} = \pm 6$$

$$\frac{x+5}{2-x} = 6$$

$$x+5 = 6(2-x)$$

$$x+5 = 12-6x$$

$$x+6x = 12-5$$

$$7x = 7$$

$$x = \frac{7}{7}$$

$$x = 1$$

$$x = \frac{-7}{-5}$$

$$x = \frac{7}{5}$$

$$\left| \frac{3-5 \times \left(\frac{7}{5}\right)}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{3-7}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$\left| \frac{-4}{4} \right| - \frac{1}{3} = \frac{2}{3}$$

$$|-1| - \frac{1}{3} = \frac{2}{3}$$

$$1 - \frac{1}{3} = \frac{2}{3}$$

$$\frac{3-1}{3} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

$$\frac{x+5}{2-x} = -6$$

$$x+5 = -6(2-x)$$

$$x+5 = -12+6x$$

$$5+12 = 6x-x$$

$$17 = 5x$$

$$\frac{17}{5} = x$$

$$x = \frac{17}{5}$$

To check

$$\left| \frac{x+5}{2-x} \right| = 6$$

$$\left| \frac{1+5}{2-1} \right| = 6$$

$$\left| \frac{6}{1} \right| = 6$$

$$6 = 6$$

$$\left| \left(\frac{17}{5} + 5 \right) \div \left(2 - \frac{17}{5} \right) \right| = 6$$

$$\left| \frac{17+25}{5} \div \frac{10-17}{5} \right| = 6$$

$$\left| \frac{42}{5} \div \frac{-7}{5} \right| = 6$$

$$|-6| = 6$$

$$6 = 6$$

$$\text{Solution Set} = \left\{ 1, \frac{17}{5} \right\}$$

Exercise 7.3

Q1) Solve the following inequalities.

(i) $3x + 1 < 5x - 4$

Solution: $3x + 1 < 5x - 4$

$$3x < 5x - 4 - 1$$

$$3x - 5x < -5$$

$$-2x < -5$$

Case-I When negative is eliminated from both sides of inequality the symbol will be change.

Case-II When negative is transferred from variable to constant side, symbol will also change.

$$x > \frac{-5}{-2}$$

$$x > \frac{5}{2}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{5}{2} \right\}$$

(ii) $4x - 10.3 \leq 21x - 1.8$

Solution: $4x - 10.3 \leq 21x - 1.8$

$$4x - 21x \leq -8.5 + 10.3$$

$$-17x \leq 8.5$$

When negative value is shifted to other side its symbol changes.

$$x \geq \frac{8.5}{-17}$$

$$x \geq -\frac{8.5}{17}$$

$$x \geq -0.5$$

$$\text{Solution Set} = \{x \mid x \geq -0.5\}$$

(iii) $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

Solution: $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$

$$-\frac{1}{2}x - \frac{1}{4} \geq -7 - 4$$

$$\frac{-2x - x}{4} \geq -11$$

$$-3x \geq -44$$

When negative value is shifted the symbol changes

$$x \leq \frac{-44}{-3}$$

$$x \leq \frac{44}{3}$$

$$\text{Solution Set} = \left\{ x \mid x \leq \frac{44}{3} \right\}$$

(iv) $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

Solution: $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

$$x - 10 + 4x \geq 6x - \frac{7}{2}$$

$$5x - 6x \geq -\frac{7}{2} + 10$$

$$-1x \geq \frac{-7 + 20}{2}$$

$$-x \geq -\frac{13}{2}$$

When negative is shifted other side symbol changes

$$x \leq \frac{13}{-1 \times 2}$$

$$x \leq -\frac{13}{2}$$

$$x \leq -6.5$$

$$\text{Solution Set} = \{x \mid x \leq -6.5\}$$

(v) $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

Solution: $\frac{3x+2}{9} - \frac{2x+1}{3} > -1$

$$\frac{3x+2-3(2x+1)}{9} > -1$$

$$3x+2-6x-3 > -9$$

$$-3x > -9+1$$

$$3x > -8$$

Negative value is shifted to other side its symbols changes

$$x < \frac{-8}{-3}$$

$$x < \frac{8}{3}$$

$$\text{Solution Set} = \left\{ x \mid x < \frac{8}{3} \right\}$$

$$\text{(vi)} \quad 3(2x+1) - 2(2x+5) < 5(3x-2)$$

$$\text{Solution: } 3(2x+1) - 2(2x+5) < 5(3x-2)$$

$$6x + 3 - 4x - 10 < 15x - 10$$

$$2x - 7 - 15x < -10$$

$$-13x < -10 + 7$$

$$-13x < -3$$

The value is negative when shifted to other side it changes its symbols

$$x > \frac{-3}{-13}$$

$$x > \frac{3}{13}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{3}{13} \right\}$$

$$\text{(vii)} \quad 3(x-1) - (x-2) > -2(x+4)$$

$$\text{Solution: } 3(x-1) - (x-2) > -2(x+4)$$

$$3x - 3 - x + 2 > -2x - 8$$

$$2x - 1 > -2x - 8$$

$$2x + 2x > -8 + 1$$

$$4x > -7$$

$$x > \frac{-7}{4}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{-7}{4} \right\}$$

$$\text{(viii)} \quad 2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$$

$$\text{Solution: } 2\frac{2}{3}x + \frac{2}{3}(5x-4) > -\frac{1}{3}(8x+7)$$

$$\frac{8}{3}x + \frac{10x-8}{3} > -\frac{(8x+7)}{3}$$

$$\frac{8x+10x-8}{3} > -\frac{8x+7}{3}$$

Multiplying both side by 3

$$\cancel{3} \times \frac{18x-8}{\cancel{3}} > -\cancel{3} \times \frac{8x+7}{\cancel{3}}$$

$$18x-8 > -(8x+7)$$

$$18x-8 > -8x-7$$

$$18x+8x > -7+8$$

$$26x > 1$$

$$x > \frac{1}{26}$$

$$\text{Solution Set} = \left\{ x \mid x > \frac{1}{26} \right\}$$

Q2) Solve the following inequalities

$$\text{(i)} \quad -4 < 3x + 5 < 8$$

$$\text{Solution: } -4 < 3x + 5 < 8$$

$$-4 < 3x + 5 \quad \text{and} \quad 3x + 5 < 8$$

$$-4 - 5 < 3x \quad 3x < 8 - 5$$

$$-9 < 3x \quad 3x < 3$$

$$\frac{-9}{3} < x \quad x < \frac{3}{3}$$

$$-3 < x \quad x < 1$$

$$-3 < x < 1$$

$$\text{Solution Set} = \{x \mid -3 < x < 1\}$$

$$\text{(ii)} \quad -5 \leq \frac{4-3x}{2} < 1$$

$$\text{Solution: } -5 \leq \frac{4-3x}{2} < 1$$

$$-5 \leq \frac{4-3x}{2} \quad \text{and} \quad \frac{4-3x}{2} < 1$$

$$-10 \leq 4-3x \quad 4-3x < 2$$

$$3x-10 \leq 4 \quad -3x < 2-4$$

$$3x \leq 4+10 \quad -3x < -2$$

$$3x \leq 14 \quad x > \frac{-2}{-3}$$

$$x \leq \frac{14}{3} \quad x > \frac{2}{3}$$

$$\frac{2}{3} < x$$

$$\frac{2}{3} < x \leq \frac{14}{3}$$

$$\text{Solution Set} = \left\{ x \mid \frac{2}{3} < x \leq \frac{14}{3} \right\}$$

$$\text{(iii)} \quad -6 < \frac{x-2}{4} < 6$$

$$\text{Solution: } -6 < \frac{x-2}{4} < 6$$

$$-6 < \frac{x-2}{4}$$

$$-24 < x-2$$

$$-24+2 < x$$

$$-22 < x$$

and

$$\frac{x-2}{4} < 6$$

$$x-2 < 24$$

$$x < 24+2$$

$$x < 26$$

$$-22 < x < 26$$

$$\text{Solution Set} = \{x \mid -22 < x < 26\}$$

$$\text{(iv)} \quad 3 \geq \frac{7-x}{2} \geq 1$$

$$\text{Solution: } 3 \geq \frac{7-x}{2} \geq 1$$

$$3 \geq \frac{7-x}{2}$$

$$6 \geq 7-x$$

$$6-7 \geq -x$$

$$-1 \geq -x$$

Negative sign change the symbols

$$1 \leq x$$

and

$$\frac{7-x}{2} \geq 1$$

$$7-x \geq 2$$

$$-x \geq 2-7$$

$$-x \geq -5$$

$$x \leq 5$$

$$1 \leq x \leq 5$$

$$\text{Solution Set} = \{x \mid 1 \leq x \leq 5\}$$

$$\text{(v)} \quad 3x-10 \leq 5 < x+3$$

$$\text{Solution } 3x-10 \leq 5 < x+3$$

$$3x-10 \leq 5 \quad \text{and} \quad 5 < x+3$$

$$3x \leq 5+10 \quad 5-3 < x$$

$$3x \leq 15 \quad 2 < x$$

$$\frac{3x}{3} \leq \frac{15}{3}$$

$$x \leq 5$$

$$2 < x \leq 5$$

$$\text{Solution Set} = \{x \mid 2 < x \leq 5\}$$

$$\text{(vi)} \quad -3 \leq \frac{x-4}{-5} < 4$$

$$\text{Solution } -3 \leq \frac{x-4}{-5} < 4$$

$$-3 \leq \frac{x-4}{-5} \quad \text{and} \quad \frac{x-4}{-5} < 4$$

$$-3 \times -5 \geq x-4$$

$$x-4 > 4(-5)$$

$$15 \geq x-4$$

$$x > -20+4$$

$$15+4 \geq x$$

$$x > -16$$

$$19 \geq x$$

$$-16 < x$$

$$x \leq 19$$

$$-16 < x \leq 19$$

$$\text{Solution Set} = \{x \mid -16 < x \leq 19\}$$

$$\text{(vii)} \quad 1-2x < 5-x \leq 25-6x$$

$$\text{Solution: } 1-2x < 5-x \leq 25-6x$$

$$1-2x < 5-x \quad \text{and}$$

$$5-x \leq 25-6x$$

$$-x+6x \leq 25-5$$

$$6x-x \leq 20$$

$$1-2x+x < 5$$

$$5x \leq 20$$

$$-x < 5-1$$

$$x \leq \frac{20}{5}$$

$$-x < 4$$

$$x \leq 4$$

Due negative sign

Symbol change

$$-4 < x$$

$$-4 < x \leq 4$$

$$\text{Solution Set} = \{x \mid -4 < x \leq 4\}$$

$$\text{(viii)} \quad 3x-2 < 2x+1 < 4x+17$$

$$\text{Solution: } 3x-2 < 2x+1 < 4x+17$$

$$3x-2 < 2x+1$$

$$2x+1 < 4x+17$$

$$3x-2x-2 < +1$$

$$2x-4x < 17-1$$

$$x < 1+2$$

$$-2x < 16$$

$$x < 3$$

$$x > \frac{16}{-2}$$

$$x > -8$$

$$-8 < x$$

$$-8 < x < 3$$

$$\text{Solution Set} = \{x \mid -8 < x < 3\}$$

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