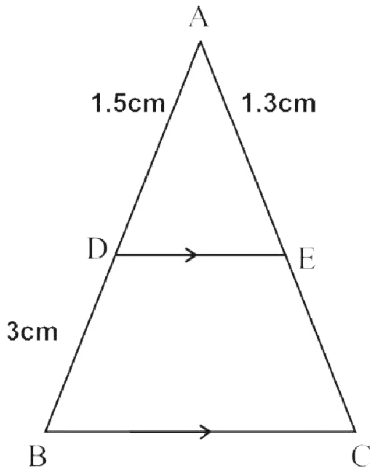
Exercise 14.1

Q.1 In ΔABC

$\overline{DE} \parallel \overline{BC}$



(i) **If** $\overline{AD} = 1.5\text{cm}$ $\overline{BD} = 3\text{cm}$

$\overline{AE} = 1.3\text{cm}$, then find $\overline{CE}$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By substituting the values of $\overline{AD}, \overline{BD}$ and $\overline{AE}$

So

$$\frac{1.5}{3} = \frac{1.3}{\text{EC}}$$

$$\overline{EC}(1.5) = 1.3 \times 3$$

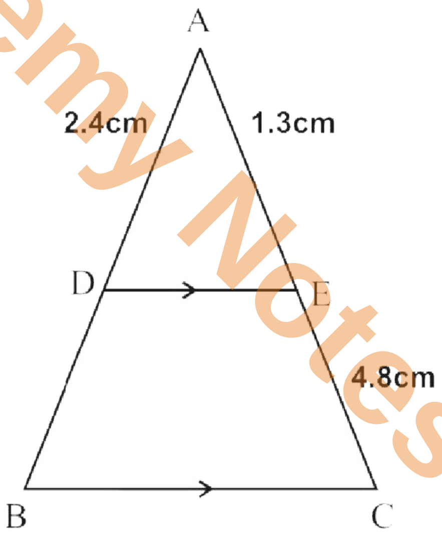
$$\overline{EC} = \frac{1.3 \times 3}{1.5}$$

$$\overline{EC} = \frac{3.9}{1.5}$$

$$\overline{EC} = 2.6\text{cm}$$

(ii) **If AD = 2.4cm** $\overline{AE} = 3.2\text{cm}$

$\overline{EC} = 4.8\text{cm}$ find AB



$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\overline{AC} = \overline{AE} + \overline{EC}$$

$$\overline{AC} = 3.2 + 4.8$$

$$\overline{AC} = 8cm$$

$$\therefore \frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

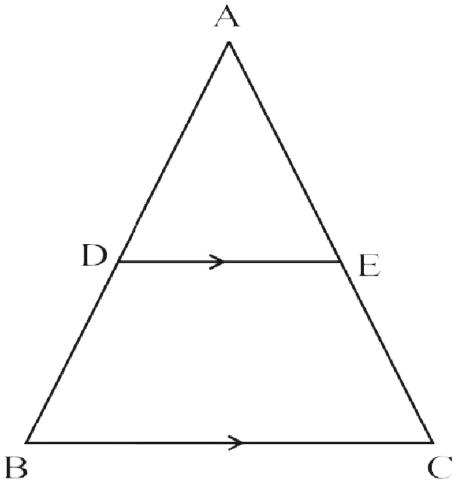
$$\frac{2.4}{\text{AB}} = \frac{3.2}{8}$$

$$2.4 \times 8 = (3.2)\overline{AB}$$

$$\frac{19.2}{3.2} = \overline{AB}$$

$$\overline{AB} = 6\text{cm}$$

(iii) **If** $\frac{\overline{AD}}{\overline{BD}} = \frac{3}{5}\ \overline{AC} = 4.8cm$ **find** $\overline{AE}$



$\overline{AC} = \overline{AE} + \overline{EC}$

$\overline{AC} = \overline{EC} + \overline{AE}$

$\overline{AE} = 4.8 - \overline{EC}$

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AC} - \overline{EC}}{\overline{EC}}$

$\frac{3}{5} = \frac{4.8 - \overline{EC}}{\overline{EC}}$

$3(\overline{EC}) = 5(4.8 - \overline{EC})$

$3(\overline{EC}) = 24 - 5(\overline{EC})$

$3(\overline{EC}) + 5(\overline{EC}) = 24$

$8(\overline{EC}) = 24$

$(\overline{EC}) = \frac{\cancel{24}^{3}}{\cancel{8}}$

$\overline{EC} = 3cm$

$\overline{AE} = \overline{AC} - \overline{EC}$

$= 4.8 - 3$

$= 1.8cm$

(iv) **If** $\overline{AD} = 2.4cm\ \overline{AE} = 3.2cm\ \overline{DE} = 2cm\ \overline{BC} = 5cm$. **Find** $\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}$.

$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}}$

$\frac{2.4}{AB} = \frac{3.2}{AC} = \frac{2}{5}$

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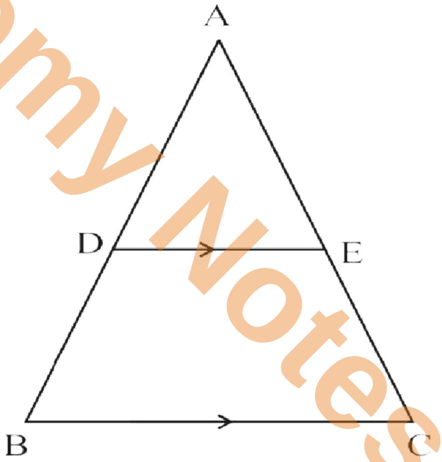
$(2.4)5 = 2(AB)$

$\frac{12.0}{2} = AB$

$\overline{AB} = 6\,cm$



$\frac{3.2}{AC} = \frac{2}{5}$

$16.0 = 2(AC)$

$\frac{\cancel{16}^{8}}{\cancel{2}} = AC$

$\overline{AC} = 8cm$

$\overline{DB} = \overline{AB} - \overline{AD}$

$\overline{DB} = 6 - 2.4$

$\overline{BD} = 3.6\,\text{cm}$

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{6} = \frac{\overline{AE}}{8}$$

$$\overline{AE} = \frac{2.4}{6} \times 8$$

$$\overline{AE} = \frac{19.2}{6}$$

$\overline{AE} = 3.2cm$

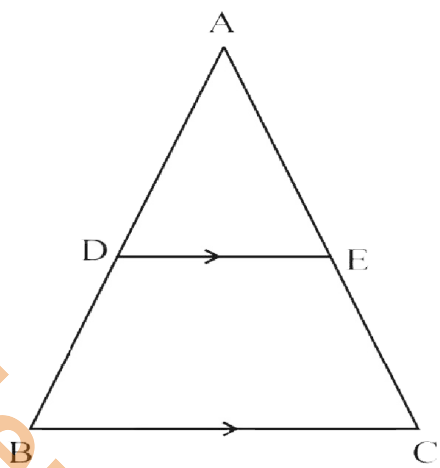
$\overline{CE} = \overline{AC} - \overline{AE}$

$\overline{CE} = 8 - 3.2$

$\overline{CE} = 4.8cm$

If $\overline{AD} = 4x - 3$ $\overline{AE} = 8x - 7$

$\overline{BD} = 3x - 1$ and $\overline{CE} = 5x - 3$ Find the value of x



$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By putting the value of $\overline{AD}, \overline{AE}, \overline{BD}$ and $\overline{CE}$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

By cross multiplying

$(4x - 3)\ (5x - 3) = (8x - 7)\ (3x - 1)$

$20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$0 = 24x^2 - 20x^2 - 29x + 27x + 7 - 9$

$4x^2 - 2x - 2 = 0$

$2(2x^2 - x - 1) = 0$

$$2x^2 - 2x + 1x - 1 = \frac{0}{2}$$

$2x(x - 1) + 1(x - 1) = 0$

$(x - 1)\ (2x + 1) = 0$

$x - 1 = 0$ $\qquad$ $2x + 1 = 0$

$x = 1$ $\qquad$ $2x = -1$

$$x = -\frac{1}{2}$$

Distance is not taken in negative it is always in positive so the value of x = 1.

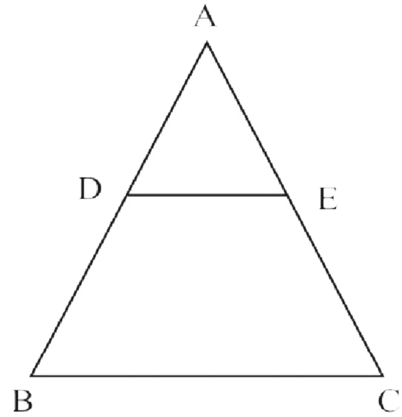
Q.2 **In ΔABC is an isosceles triangle $\angle A$ is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that**

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

Prove that ΔADE is also an isosceles triangle.

Given:

ΔABC is an isosceles triangle, $\angle A$ is vertex and $\overline{DE}$ intersects the sides $\overline{AB}\ and\ \overline{AC}$.



$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$$

To Prove

$$m\overline{AD} = m\overline{AE}$$

Proof

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

Or $\frac{\overline{BD}}{\overline{AD}} = \frac{\overline{EC}}{\overline{AE}}$

Or $\frac{\overline{AD}+\overline{BD}}{\overline{AD}} = \frac{\overline{AE}+\overline{EC}}{\overline{EC}}$

As we know

$$\overline{AB} = \overline{AD} + \overline{BD}$$

$$\overline{AC} = \overline{AE} + \overline{EC}$$

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

From this

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

$$\overline{AD} = \overline{AE}$$

$\overline{AB} = \overline{AC}$ **(Given)**

Q.3 In an equilateral triangle ABC shown in the figure $m\overline{AE}:m\overline{AC} = m\overline{AD}:m\overline{AB}$ find all the three angles of ΔADE and name it also.

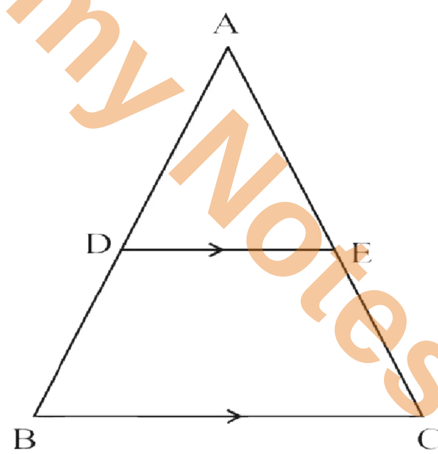
Given

ΔABC is equilateral triangle

To prove

To find the angles of ΔADE

Solution:



$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

All angles are equal as it is an equilateral triangle which are equal to 60° each

$\angle A = \angle B = \angle C$

$m\overline{BC} \| m\overline{DE}$

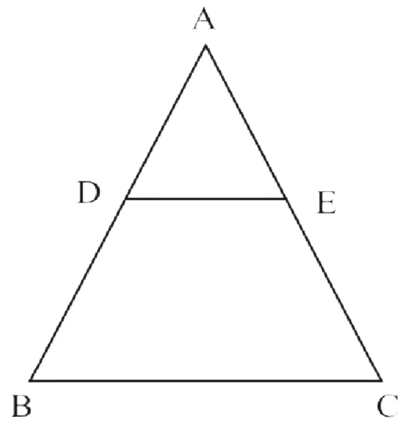
$\angle ADE = \angle ABC = 60^\circ$

$\angle AED = \angle ACB = 60^\circ$

$\angle A = 60^\circ$

ΔADE is an equilateral triangle

Q.4 Prove that line segment drawn through the midpoint of one side of a triangle and parallel to another side bisect the third side



Given

$\overline{AD} = \overline{BD}$

$\overline{DE} \parallel \overline{BC}$

To Prove

$\overline{AE} = \overline{EC}$

In ΔABC

$\overline{DE} \parallel \overline{BC}$

In theorem it is already discussed that

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

As we know $\overline{AD} = \overline{BD}$ or $\overline{BD} = \overline{AD}$

$$\frac{\cancel{\overline{AD}}}{\cancel{\overline{AD}}} = \frac{\overline{AE}}{\overline{EC}}$$

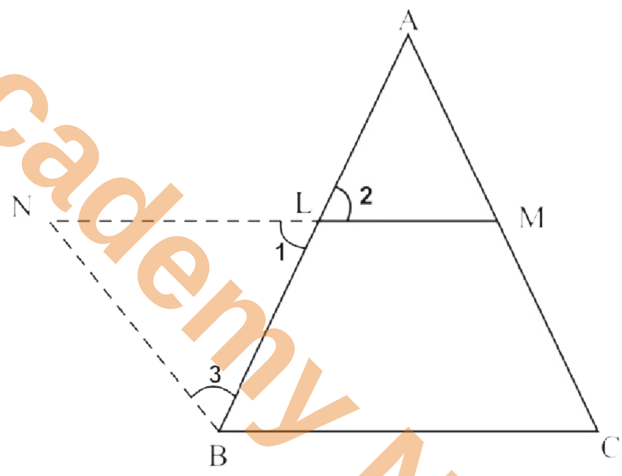
$$1 = \frac{\overline{AE}}{\overline{EC}}$$

$\overline{EC} = \overline{AE}$

Q.5 Prove that the line segment joining the midpoint of any two sides of a triangle is parallel to the third side

Given

ΔABC the midpoint of $\overline{AB}$ and $\overline{AC}$ are L and M respectively



To Prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2}\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure name the angles $\angle 1$, $\angle 2$, and $\angle 3$

Proof

Statements	Reasons
$\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong AL$	Given
$\angle 2 = \angle 1$ or $\angle 1 = \angle 2$	Vertical angles
$\overline{NL} = \overline{ML}$	Construction
$\therefore\ \Delta BLN \cong \Delta ALM$	Corresponding angle of congruent triangles Given
$\therefore \angle A = \angle 3$	
And $\overline{NB} \cong \overline{AM}$	
$\overline{NB} \parallel \overline{AM}$	

$\overline{ML} = \overline{AM}$	Given
$\overline{NB} \cong \overline{ML}$	
$\overline{BC}\,\overline{MN}$ is parallelogram	
$\therefore\ \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	
$\overline{BC} \cong \overline{NM}$	(Opposite side of parallelogram BCMN)
$m\overline{LM} = \frac{1}{2} m\overline{NM}$	(Opposite side of parallelogram)
Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	

Theorem 14.1.3

The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.

Given

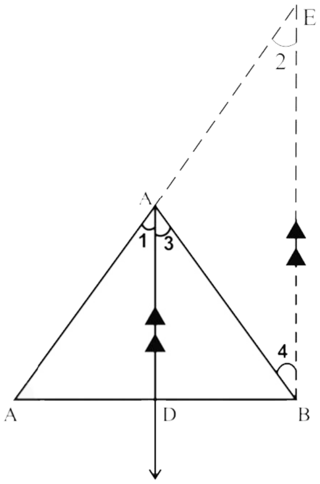
In ΔABC internal angle bisector of $\angle A$ meets $\overline{CB}$ at the points D.

To prove

$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction

Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet $\overline{CA}$ Produced at E

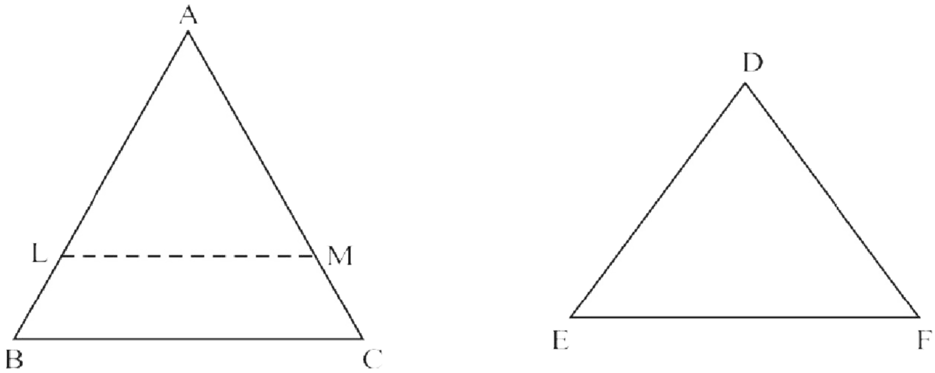


Proof

Statements	Reasons
$\because \overline{AD} \parallel \overline{EB}$ and $\overline{EC}$ intersect them	Construction
$m\angle 1 = m\angle 2$..........(i)	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and $\overline{AB}$ intersects them	
$\therefore\ m\angle 3 = m\angle 4$ ....(ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore\ m\angle 2 = m\angle 4$	From (i) and (ii)
And $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ, the sides opposite to congruent angles are also congruent
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	$m\overline{EA} = m\overline{AB}$ (proved)
Thus $m\overline{BD} : m\overline{DC} = m\overline{AB} : \overline{AC}$	

Theorem 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional



Given

$\Delta ABC \sim \Delta DEF$

i.e $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction

(I) Suppose that $m\overline{AB} > m\overline{DE}$

(II) $m\overline{AB} \leq m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$

Join L and M by the line segment LM

Proof

Statements	Reasons
In $\Delta ALM \leftrightarrow \Delta DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\Delta ALM \cong \Delta DEF$	S.A.S Postulate
And $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$ ....(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$, we can prove that $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$ ....(ii)	

Thus $\frac{m\overline{DE}}{m\overline{AB}}=\frac{m\overline{DF}}{m\overline{AC}}=\frac{m\overline{EF}}{m\overline{BC}}$	By (i) and (ii)
Or $\frac{m\overline{AB}}{m\overline{DE}}=\frac{m\overline{AC}}{m\overline{DF}}=\frac{m\overline{BC}}{m\overline{EF}}$	By taking reciprocals
If $m\overline{AB}=m\overline{DE}$	
Then in $\Delta ABC \leftrightarrow \Delta DEF$	
(II) If $m\overline{AB}<m\overline{DE}$, it can similarly be proved by taking intercepts on the sides of ΔDEF	
$\angle A \cong \angle D$	
$\angle B \cong \angle E$	
And $\overline{AB}\cong\overline{DE}$	
So $\Delta\ ABC \cong \Delta DEF$	$A.S.A \cong A.S.A$
Thus $\frac{m\overline{AB}}{m\overline{DE}}=\frac{m\overline{AC}}{m\overline{DF}}=\frac{m\overline{BC}}{m\overline{EF}}=1$	$\overline{AC}\cong\overline{DF}$, $\overline{BC}\cong\overline{EF}$
Hence the result is true for all the cases.	