Exercise 13.1

Q.1 Two sides of a triangle measure 10cm and 15 cm which of the following measure is possible for the third side?

(a) 5*cm*
(b) 20 *cm*
(c) 25 *cm*
(d) 30 *cm*

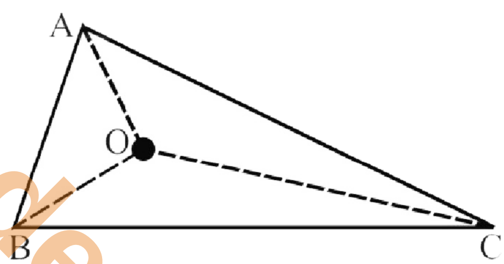
Solution

Lengths of two sides are 15 and 10 cm.
So, sum of two lengths of triangle = 10 + 15 = 25 m
$10 + 15 > 20$
$10 + 20 > 15$
$15 + 20 > 10$
$\therefore$ 20 cm is possible for third side
Or
Sum of length of two sides is always greater than the third sides of a triangle.

Given

Q.2 Point O is interior of ΔABC
Show that

$$m\overline{OA} + m\overline{OB} + m\overline{OC} > \frac{1}{2}\left(m\overline{AB} + m\overline{BC} + m\overline{CA}\right)$$



Given

Point O is interior of ΔABC

To prove:

$$m\overrightarrow{OA} + m\overline{OB} + m\overline{OC} < \frac{1}{2}\left(m\overline{AB} + m\overline{BC} + m\overline{AC}\right)$$

Construction

Join O with A, B and C.
So that we get three triangle ΔOAB, ΔOBC and ΔOAC

Proof

Statements	Reasons
In ΔOAB $m\overline{OA} + \overline{OB} > m\overline{AB}$ _____(i)	In any triangle the sum of length of two sides is greater then the third sides.
In ΔOAC $m\overline{OC} + m\overline{OA} > m\overline{AC}$ _____(ii)	As in (i)
In ΔOBC $m\overline{OB} + \overline{OC} > m\overline{BC}$ _____(iii)	As in (i)
Adding equation i, ii and iii $\overline{OA} + \overline{OC} + \overline{OA} + \overline{OB} + \overline{OB} + \overline{OC} > \overline{AC} + \overline{AB} + \overline{BC}$ $2\overline{OA} + 2\overline{OC} + 2\overline{OB} > \overline{AB} + \overline{BC} + \overline{CA}$ $2(OA + OC + OB) > \overline{AB} + \overline{BC} + \overline{CA}$	

$\frac{\not{2}(OA+OC+OB)}{\not{2}} > \frac{\overline{AB}+\overline{BC}+\overline{CA}}{2}$ $(OA+OC+OB) > \frac{1}{2}(\overline{AB}+\overline{BC}+\overline{CA})$	Dividing both sides by 2

Q.3 In the ΔABC m∠B = 70° and m∠C = 45° which of the sides of the triangle is longest and which is the shortest.

Solution

Sum of three angle in a triangle is 180°

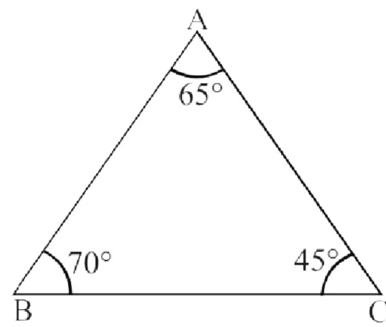
$\angle A + \angle B + \angle C = 180$

$\angle A + 70 + 45 = 180$

$\angle A + 115 = 180$

$\angle A = 180 - 115$

$\angle A = 65^\circ$



Sides of the triangle depend upon the angles largest angle has largest opposite side and smallest angle has smallest opposite side here $\angle B$ is largest so, $\overline{AC}$ is largest $\angle C$ is smallest, so $\overline{AB}$ is smallest side.

Q.4 Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

Solution

Sum of three angles in a triangle is equal to 180°. So in a triangle one angle will be equal to 90° and rest of two angles are acute angle (less than 90°)

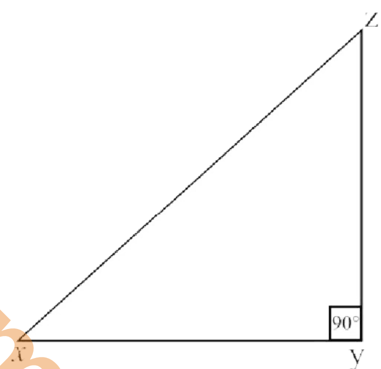
$\therefore\ m\angle y = 90$

And $m\angle x + m\angle z = 90$

So $m\angle x$ and $m\angle z$ are acute angle

$\therefore$ Opposite to $m\angle y = 90^\circ$ is hypotenuse

It is largest side.



Q.5 In the triangular figure $\overline{AB} > \overline{AC}$. $\overline{BD}$ and $\overline{CD}$ are the bisectors of ∠B and ∠C respectively prove that $\overline{BD} > \overline{DC}$

Given

In ΔABC

$\overline{AB} > \overline{AC}$

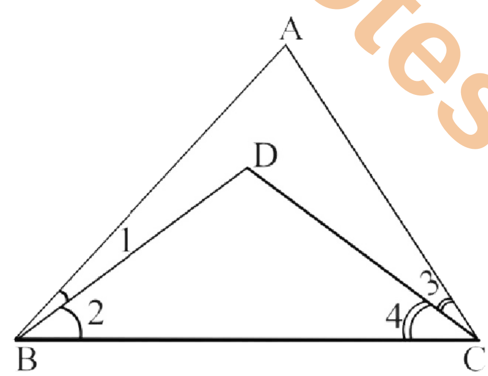
$\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$

To prove

$\overline{BD} > \overline{CD}$

Construction

Label the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$

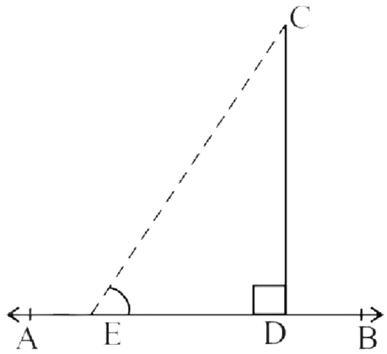


Proof

Statements	Reasons
In ΔABC	
$\overline{AB} > \overline{AC}$	Given
$\overline{BD}$ is the bisector of $\angle B$	
$\frac{1}{2}m\angle ACB > \frac{1}{2}m\angle ABC$	
$m\angle ABC$	
$m\angle 2 < m\angle 4$	
$\overline{CD}$ is the bisector of $\angle C$	Given
InΔBCD	
$\overline{BD} > \overline{DC}$	Side opposite to greater angle is greater

Theorem 13.1.4

From a point, out side a line, the perpendicular is the shortest distance from the point to the line.



Given:

A line AB and a point C

(Not lying on $\overleftrightarrow{AB}$)and a point D on $\overleftrightarrow{AB}$ such that $\overline{CD} \perp \overleftrightarrow{AB}$

To prove

$m\overline{CD}$ is the shortest distance from the point C to $\overleftrightarrow{AB}$

Construction

Take a point E on $\overleftrightarrow{AB}$ .Join C and E to form a ΔCDE

Proof

Statements	Reasons
In ΔCDE	
$m\angle CDB > m\angle CED$	(An exterior angle of a triangle is greater than non adjacent interior angle)
But $m\angle CDB = m\angle CDE$	Supplement of right angle
$\therefore$ $m\angle CDE > m\angle CED$	
Or $m\angle CED < m\angle CDE$	
Or $m\overline{CD} < m\overline{CE}$	Side opposite to greater angle is greater.
But E is any point on $\overleftrightarrow{AB}$	
Hence $m\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$	