Exercise 12.2

Q.1 In a quadrilateral ABCD $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}, \overline{CD}$ meet each other at point N. Prove that $\overline{BN}$ is a bisector of $\angle ABC$

Given

In the quadrilateral ABCD

$\overline{AB} \cong \overline{BC}$

$\overline{NM}$ is right bisector of $\overline{CD}$

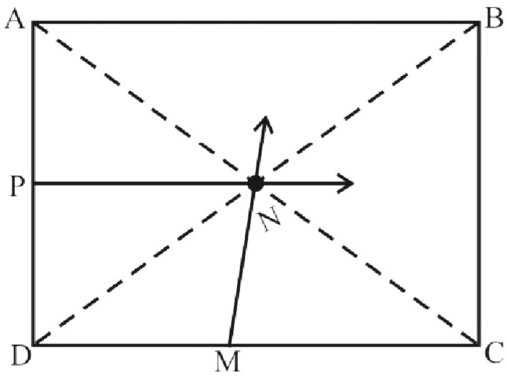
$\overline{PN}$ is right bisector of $\overline{AD}$

They meet at N

To prove

$\overline{BN}$ is the bisector of angle ABC

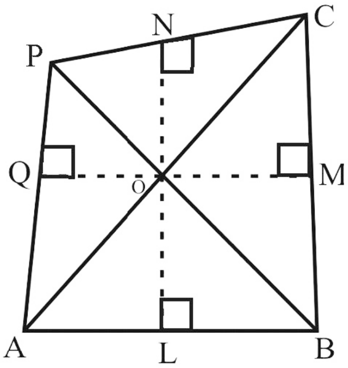
Construction join N to A,B,C,D



Proof

Statements	Reasons
$\overline{ND} \cong \overline{NA}$ ________ (i)	N is an right bisector of $\overline{AD}$
$\overline{ND} \cong \overline{NC}$ ________ (ii)	N is on right bisector of $\overline{DC}$
$\overline{NA} = \overline{NC}$ ________ (iii)	from (i) and (ii)
$\Delta BNC \leftrightarrow \Delta ANB$	
$\overline{NC} = \overline{NA}$	Already proved (from iii)
$\overline{AB} \cong \overline{CB}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\therefore \Delta BNA \cong \Delta BNC$	$S.S.S \cong S.S.S$
Hence $\angle ABN \cong \angle NBC$	Corresponding angles of congruent triangles
Hence $\overline{BN}$ is the bisector of $\angle ABC$	

Q.2 The bisectors of $\angle A, \angle B$ and $\angle C$ of a quadrilateral ABCP meet each other at point O. Prove that the bisector of $\angle P$ will also pass through the point O.



Given

ABCP is quadrilateral. $\overline{AO}, \overline{BO}, \overline{CO}$ are bisectors of $\angle A, \angle B$ and $\angle C$ meet at point O.

To prove

$\overline{PO}$ is bisector of $\angle P$

Construction:

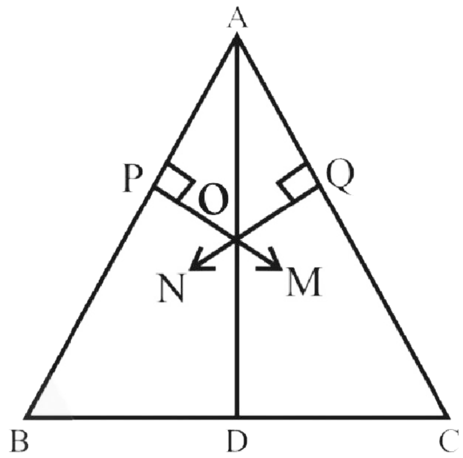
Join P to O.

Draw $\overline{OQ} \perp \overline{AP}$, $\overline{ON} \perp \overline{PC}$ and $\overline{OL} \perp \overline{AB}$, $\overline{OM} \perp \overline{BC}$

Proof:

Statements	Reasons
$\overline{OM} \cong \overline{ON}$ ________ (i)	O is on the bisector of $\angle C$
$\overline{OL} \cong \overline{OM}$ ________ (ii)	O is on the bisector of $\angle B$
$\overline{OL} \cong \overline{OQ}$ ________ (iii)	O is on the bisector of $\angle A$
$\overline{OQ} \cong \overline{ON}$	From i, ii, iii
Point O lines on the bisector of $\angle P$	
$\therefore \overline{OP}$ is the bisector of angle P	

Q.3 **Prove that the right bisector of congruent sides of an isosceles triangle and its altitude are concurrent.**



Given

ΔABC

$\overline{AB} \cong \overline{AC}$ due to isosceles triangle $\overline{PM}$ is right bisector of $\overline{AB}$

$\overline{QN}$ is right bisector of $\overline{AC}$

$\overrightarrow{PM}$ and $\overrightarrow{QN}$ intersect each other at point O

Required

The altitude of ΔABC lies at point O

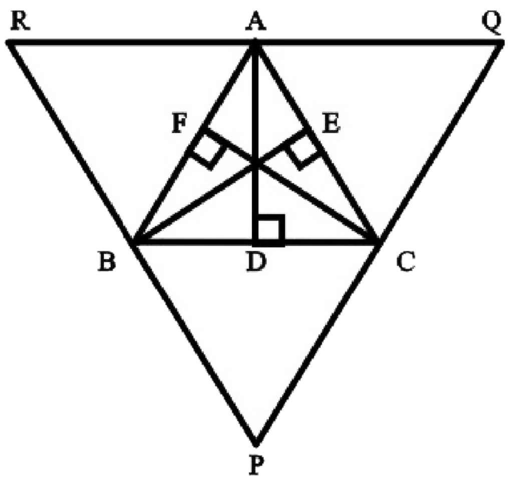
Join A to O and extend it to cut $\overline{BC}$ at D.

Proof

Statements	Reasons
$m\overline{AB} \cong m\overline{AC}$	Given
$\frac{1}{2}m\overline{AB} = \frac{1}{2}m\overline{AC}$	Dividing both side by 2
$\overline{AQ} \cong \overline{AP}$	
In $\Delta AQO \leftrightarrow \Delta APO$	
$\angle APO \cong \angle AQO$	Each 90^o (Given)
$\overline{AQ} \cong \overline{AP}$	Already Proved
$\overline{AO} \cong \overline{AO}$	Common
$\Delta APO \cong \Delta AQO$	$H.S \cong H.S$
$\angle PAO \cong \angle QAO$ (i)	Corresponding angles of congruent triangles
$\Delta BAD \leftrightarrow \Delta CAD$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD} \cong \overline{AD}$	Common

$\angle BAD \cong \angle CAD$	Proved from (i)
$\Delta BAD \cong \Delta CAD$	$S.A.S \cong S.A.S$
$\angle ODB \cong \angle ODC$	Each angle is 90^o (Given)
$m\angle ODM + m\angle ODC = 180^o$	Supplementary angle
$\therefore \overline{AD} \perp \overline{BC}$	
Point 0 lies on altitude $\overline{AD}$	

Q.4 Prove that the altitudes of a triangle are concurrent.



Given

In ΔABC

$\overline{AD}, \overline{BE}, \overline{CF}$ are its altitudes

i.e $\overline{AD} \perp \overline{BC}, \overline{BE} \perp \overline{AC}, \overline{CF} \perp \overline{AB}$

Required $\overline{AD}, \overline{BE}$ and $\overline{CF}$ are concurrent

Construction:

Passing through A, B, C take

$\overline{RQ} \parallel \overline{BC}, \overline{RP} \parallel \overline{AC}$ and $\overline{QP} \parallel \overline{AB}$ respectively forming a ΔPQR

Proof

Statements	Reasons
$\overline{BC} \parallel \overline{AQ}$	Construction
$\overline{AB} \parallel \overline{QC}$	Construction
$\therefore ABCQ$ is a $\parallel^{gm}$	
Hence $\overline{AQ} \cong \overline{BC}$	
Similarly $\overline{AB} \cong \overline{QC}$	
Hence point A is midpoint RQ	
And $\overline{AD} \perp \overline{BC}$	Given
$\overline{BC} \parallel \overline{RQ}$	Opposite sides of parallelogram ABCQ
$\overline{AD} \parallel \overline{RQ}$	
Thus $\overline{AD} \perp$ is right bisector of $\overline{RQ}$	
similarly $\overline{BE}$ is a right bisector of $\overline{RP}$ and	
$\overline{CF}$ is right bisector of PQ	
$\therefore \perp^s$ $\overline{AD}, \overline{BE}, \overline{CF}$ are right bisector of sides of ΔPQR	
$\therefore \overline{AD}, \overline{BE}$ and $\overline{CF}$ are Concurrent	

Theorem12.1.6

The bisectors of the angles of a triangle are concurrent

Given

ΔABC

To Prove

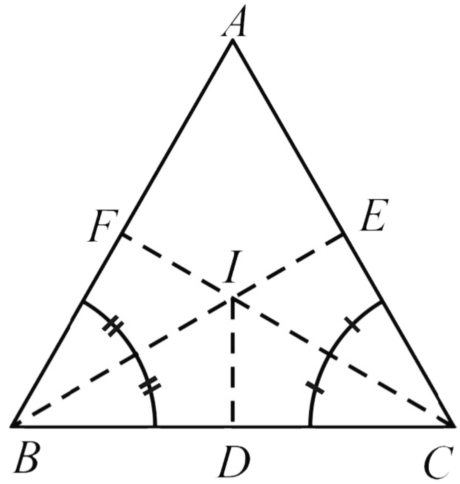
The bisector of $\angle A$, $\angle B$, and $\angle C$ are concurrent

Construction:

Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I. From I, draw

$\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$

Proof



Statements	Reasons
$\overline{ID} \cong \overline{IF}$	(Any point on bisector of an angle is equidistance from its arms.
Similarly $ID \cong IE$ $\therefore \overline{IE} \cong \overline{IF}$	Each $\cong$ ID
So the point I is on the bisector of $\angle A$ … (i)	
Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$ … (ii)	Construction
Thus the bisector of $\angle A$, $\angle B$ and $\angle C$ are concurrent at I	{From (i) and (ii)}