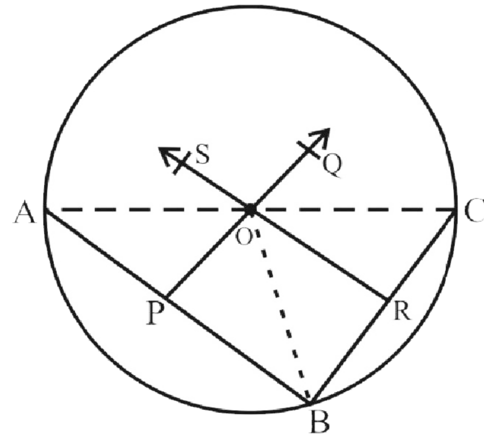
Exercise 12.1

Q.1 Prove that the centre of a circle is on the right bisectors of each of its chords.

Given

A, B, C are the three non-collinear points.

Required: To find the centre of the circle passing through A,B,C



Construction

Join B to C, A take $\overrightarrow{PQ}$ is right bisector of $\overline{AB}$ and $\overline{RS}$ right bisector of BC, they intersect at O.

Join O to A, O to B, O to C.

$\therefore$ O is the centre of circle.

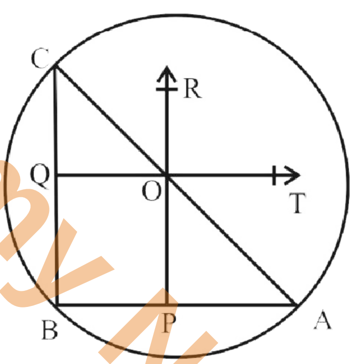
Proof

Statements	Reasons
$\overline{OB} \cong \overline{OC}$ ___________ (i)	O is the right bisector of $\overline{BC}$
$\overline{OA} \cong \overline{OB}$ ___________ (ii)	O is the right bisector of $\overline{AB}$
$\overline{OA} = \overline{OB} = \overline{OC}$	From (i) and (ii)
Hence is equidistant from the A,B,C	
$\therefore O$ is center of circle which is required	

Q.2 Where will the center of a circle passing through three non-collinear points? And Why?

Given

A.B.C are three non collinear points and circle passing through these points.



To prove

Find the center of the circle passing through vertices A, B and C.

Construction

(i) Join B to A and C.

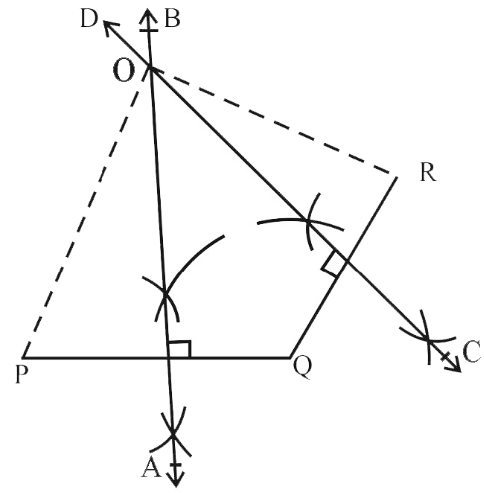
(ii) Take $\overrightarrow{QT}$ right bisector of $\overline{BC}$ and take also $\overrightarrow{PR}$ right bisector of $\overline{AB}$.

$\overrightarrow{PR}$ and $\overrightarrow{QT}$ intersect at point O. joint O to A,B and C. O is the center of the circle.

Proof

Statements	Reasons
$\overline{QO}$ is right bisector $\overline{BC}$	
$\overline{OB} \cong \overline{OC} \ldots$(i)	
$\overline{PO}$ is right bisector of $\overline{AB}$	
$\overline{OA} \cong \overline{OB} \ldots$(ii)	
So	
$\overline{OA} \cong \overline{OC} \cong \overline{OB}$	From (i) and (ii)
$\therefore$ It is proved that O is the center of the circle.	

Q.3 Three village P,Q and R are not on the same line. The people of these villages want to make a children park at such a place which is equidistant from these three villages. After fixing the place of children park prove that the park is equidistant from the three villages.



Given

P,Q,R are three villages not on the same straight line.

To prove

The point equidistant from P,R,Q.

Construction

(i) Joint Q to P and R.

(ii) Take $\overleftrightarrow{AB}$ right bisector of $\overline{PQ}$ and $\overleftrightarrow{CD}$ right bisector of $\overline{QR}$. $\overline{AB}$ and $\overleftrightarrow{CD}$ intersect at O.

(iii) Join 0 to P, Q, R

The place of children part at point O.

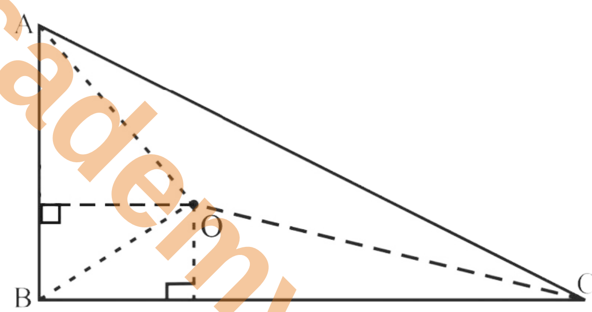
Proof

Statements	Reasons
$\overline{OQ} \cong \overline{OR}$ ________ (i)	O is on the right bisector of $\overline{QR}$
$\overline{OP} \cong \overline{OQ}$ ________ (ii)	O is on the right bisector of $\overline{PQ}$
$\overline{OP} \cong \overline{OQ} \cong \overline{OR}$ ________ (iii)	From (i) and (ii)
$\therefore O$ is on the bisector of $\angle P$	
Hence $\overline{PO}$ is bisector of $\angle P$	

O is equidistant from P,Q and R

Theorem 12.1.3

The right bisectors of the sides of a triangle are concurrent.



Given

ΔABC

To prove

The right bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ are concurrent.

Construction

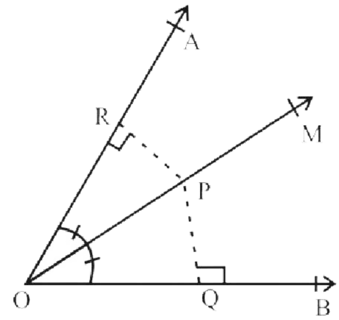
Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

Proof

Statements	Reasons
$\overline{OA} \cong \overline{OB}$ ⟶ (i)	(Each point on right bisector of a segment is equidistant from its end points)
$\overline{OB} \cong \overline{OC}$ ⟶ (ii)	As in (i)
$\overline{OA} \cong \overline{OC}$	from (i) and (ii)
$\therefore$ Point O is on the right bisector of $\overline{CA}$ → (iv)	(O is equidistant from A and C)
But point O is on the right bisector of $\overline{AB}$ and of $\overline{BC}$ ⟶ (v)	**Construction**
Hence the right bisectors of the three sides of triangle are concurrent at O	{from (iv) and (v)}

Theorem 12.1.4

Any point on the bisector of an angle is equidistant from its arms.



Given

A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$

To Prove

$\overline{PQ} \cong \overline{PR}$ i.e P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Construction

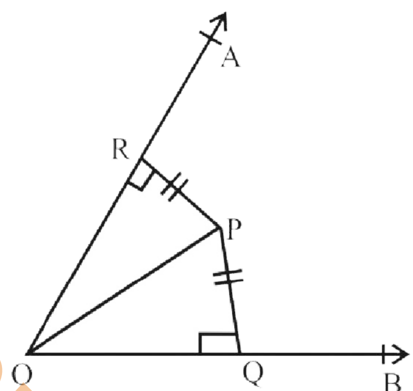
Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$

Proof

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	Construction
$\angle POQ \cong \angle POR$	Given
$\therefore \Delta POQ \cong \Delta POR$	$S.A.A \cong S.A.A$
Hence $\overline{PQ} \cong \overline{PR}$	(Corresponding sides of congruent triangles)

Theorem 12.1.5 (Converse of Theorem 12.1.4)

Any point inside an angle, equidistant from its arms, is on the bisector of it.



Given

Any point P lies inside $\angle AOB$, such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$

To prove

Point P is on the bisector of $\angle AOB$

Construction

Join P to O

Proof

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	Given (Right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore \Delta POQ \cong \Delta POR$	$H.S \cong H.S$
Hence $\angle POQ \cong \angle POR$	(Corresponding angles of congruent triangles)
i.e, P is on the bisector of $\angle AOB$	