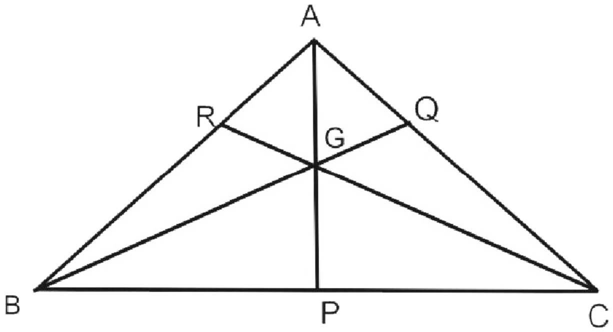
Exercise 11.4

Q.1 The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm. 1.4 cm and 1.6 cm. Find the length of its medians.

Let ΔABC with the point of concurrency of medians at G



$\overline{AG} = 1.2cm,\ \overline{BG} = 1.4cm\ and\ \overline{CG} = 1.6cm$

$\overline{AP} = \frac{3}{2}\overline{AG} = \frac{3}{2}\times 1.2 = 1.8cm$

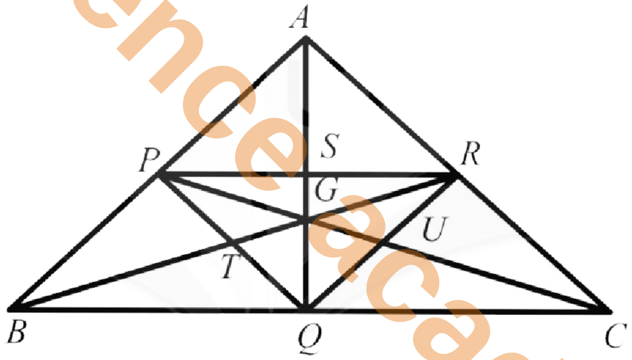
$\overline{BQ} = \frac{3}{2}\overline{BG} = \frac{3}{2}\times 1.4 = 2.1cm$

$\overline{CR} = \frac{3}{2}\overline{CG} = \frac{3}{2}\times 1.6 = 2.4cm$

Q.2 Prove that the point of concurrency of the medians of a triangle and the triangle which is made by joining the midpoint of its sides to the same.

Given

In ΔABC, AQ, CP, BR are medians which meet at G.



To prove

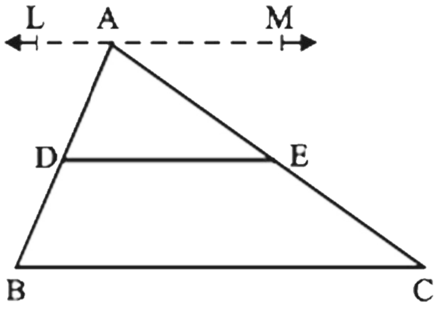
G is the point of concurrency of the medians of ΔABC and ΔPQR

Proof

Statements	Reasons
$\overline{PR} \parallel \overline{BC}$	P, R are midpoint of $\overline{AB}, \overline{AC}$
$\overline{BQ} \parallel \overline{PR}$	
Similarly $\overline{QR} \parallel \overline{BP}$	
$\therefore PBQR$ is a parallelogram it diagonals $\overline{BR}$ and $\overline{PQ}$ bisector each other at T	
Similarly U is the midpoint of QR and S is midpoint of $\overline{PR}$	
$\therefore \overline{PU}, \overline{QS}, \overline{RT}$ are medians of ΔPQR	
(i) $\overline{AQ}$ and $\overline{SQ}$ pass through G	
(ii) $\overline{BR}$ and $\overline{TR}$ pass through G	
(iii) $\overline{UP}$ and $\overline{CP}$ pass through G	
Hence G is point of concurrency of medians of ΔPQR and ΔABC	

Example

A line, through the mid-point of one side, parallel to another side of a triangle, bisects the third side.



Given

In $\Delta ABC, D$ is the mid-point of $\overline{AB}$.

$\overline{DE} \parallel \overline{BC}$ which cuts $\overline{AC}$ at E.

To prove

$\overline{AE} \cong \overline{BC}$

Construction

Through A, draw $\overleftrightarrow{LM} \parallel \overline{BC}$.

Proof

Statements	Reasons
Intercepts cut by $\overleftrightarrow{LM}, \overline{DE}, \overline{BC}$ on $\overline{AC}$ are congruent. i.e., $\overline{AE} \cong \overline{EC}$.	Intercepts cut by parallels $\overleftrightarrow{LM}, \overline{DE}$. $\overline{BC}$ on $\overline{AB}$ are congruent (given)

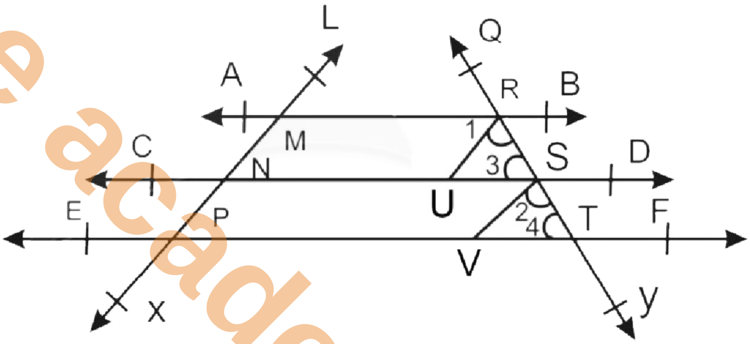
Theorem 11.1.5

Statement: In three or more parallel lines make congruent segments on a traversal they also intercept congruent segments on any other line that cuts them.

Given

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. The transversal $\overleftrightarrow{QY}$ intersects them at point R, S and T respectively.



Prove

$\overline{RS} \cong \overline{ST}$

Construction

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U, from S draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V. as shown in the figure let the angles be labeled as $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Proof

Statements	Reasons
$MNUR$ is parallelogram	$\overline{RU} \parallel \overline{LX}$ (Construction) $\overline{AB} \parallel \overline{CO}$ (given)
$\therefore \overline{MN} \cong \overline{RU}$ (i)	(Opposite side of parallelogram).
Similarly.	
$\overline{NP} \cong \overline{SV}$ (ii)	
But $\overline{MN} \cong \overline{NP}$ (iii)	Given
$\therefore \overline{RU} \cong \overline{SV}$	{from (i) (ii) and (iii)} each is $\parallel$ $\overline{LX}$ (construction)

Also $\overline{RU} \parallel \overline{SV}$	
$\therefore \angle 1 \cong \angle 2$	Corresponding angles
and $\angle 3 \cong \angle 4$	Corresponding angles
In $\Delta RUS \leftrightarrow \Delta SVT$	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore \Delta RUS \cong \Delta SVT$	$S.A.A \cong S.A.A$
Hence $\overline{RS} \cong \overline{ST}$	(Corresponding sides of congruent triangles)