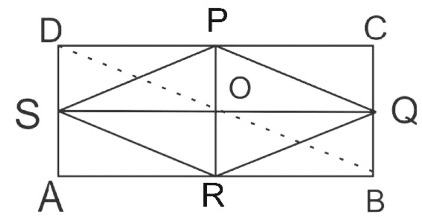
Exercise 11.3

Q.1 Prove that the line segments joining the midpoint of the opposite side of a quadrilateral bisect each other.

Given

$ABCD$ is quadrilaterals point $QRSP$ are the mid point of the sides $\overline{RP}$ and $\overline{SQ}$ are joined they meet at O.



$\overline{OP} \cong \overline{OR}$ $\quad \overline{OQ} \cong \overline{OS}$

Construction

Join P, Q, R and S in order join C to A or A to C

Proof

Statements	Reasons
$SP \parallel AC$ … (i)	In $\Delta ADC, S, P$ are mid point of AD, DC
$m\overline{SP} = \frac{1}{2} m\overline{AC}$...(ii)	
$\overline{AC} \parallel \overline{RQ}$...(iii)	In ΔABC, Q, R are midpoint of $\overline{BC}$, $\overline{AB}$
$m\overline{RQ} = \frac{1}{2}\overline{AC}$...(iv)	
$m\overline{SP} \parallel \overline{RQ}$...(v)	
and $\overline{RQ} = \overline{SP}$...(vi)	From (ii) and (iv)
Now $\overline{RP}$ *and* $\overline{QS}$ diagonals of parallelogram PQRS intersect at O.	
$\therefore$ $\overline{OP} \cong \overline{OR}$ $\overline{OS} \cong \overline{OQ}$	Diagonals of a parallelogram bisects each other.

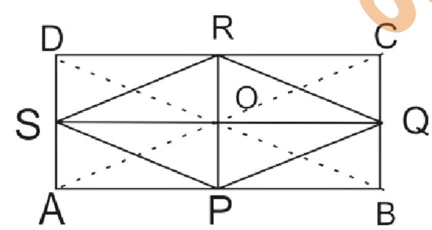
Q.2 Prove that the line segments joining the midpoint of the opposite sides of a rectangle are the right bisectors of each other.
[Hint: Diagonals of a rectangle are congruent]

Given

(i) $ABCD$ is a rectangle

(ii) $P, Q.R.S$ are the midpoints of $\overline{AB}$, $\overline{CD}$ and $\overline{DA}$

(iii) $\overline{SQ}$ and $\overline{RP}$ cut each other at point O



$\overline{OS} \cong \overline{OQ}$

$\overline{OP} \cong \overline{OR}$

Construction

Join P to Q and Q to R and R to S and S to P

Join A to C and B to D

Proof

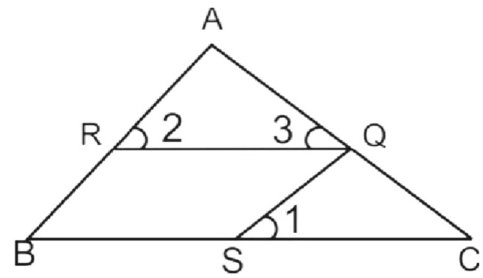
Statements	Reasons
Midpoint of $\overline{BC}$ is Q	Given
Midpoint of $\overline{AB}$ is P	Given
$\therefore \overline{AC} \parallel \overline{PQ}$................(i)	
$\frac{1}{2}\overline{AC} = \overline{PQ}$............(ii)	
In ΔADC	
$\overline{AC} \parallel \overline{SR}$................(iii)	
$\frac{1}{2}\overline{AC} = \overline{SR}$............(iv)	
$\overline{PQ} = \overline{SR}$	From equation (i) and (ii) each are parallel to $\overline{AC}$ each are half of $\overline{DB}$
$\overline{SP} = \overline{RQ}$	
By joined B to D we can prove	
$\overline{RQ} \parallel \overline{SP}$	
$m\overline{SR} \parallel m\overline{PQ}$	Each of them = $\frac{1}{2}\overline{AC}$
$m\overline{AC} \parallel m\overline{BD}$	
PQRS is a parallelogram all it sides are equal	
$\overline{OP} \cong \overline{OR}$	
$\overline{OS} \cong \overline{OQ}$	
$\Delta OQR \leftrightarrow \Delta OQP$	
$\overline{OR} \cong \overline{OP}$	Proved
$\overline{OQ} \cong \overline{OQ}$	Common
$\overline{RQ} \cong \overline{PQ}$	Adjacent
$\therefore \Delta OQR \cong \Delta OQP$	
$\angle ROQ \cong \angle POQ$..............(vii)	
$\angle ROQ + \angle POQ = 180$........(viii)	Supplementary angle
$\angle ROQ = \angle POQ = 90°$	From (vii) and (viii)
Thus $\overline{PR} \perp \overline{QS}$	

Q.3 Prove that line segment passing the midpoint of one side and parallel to other side of a triangle also bisects the third side.

Given

In $\Delta ABC, R$ is the midpoint of $\overline{AB}, \overline{RQ} \parallel \overline{BC}$

$\overline{RQ} \parallel \overline{BS}$



To prove

$\overline{AQ} = \overline{QC}$

Construction

$\overline{QS} \parallel \overline{AB}$

Proof

Statements	Reasons
$\overline{RQ} \parallel \overline{BS}$	Given
$\overline{QS} \parallel \overline{BR}$	Construction
$RBSQ$ is a Parallelogram	
$\overline{QS} \cong \overline{BR}$...(i)	Opposite side
$\overline{AR} \cong \overline{RB}$...(ii)	Given
$\overline{QS} \cong \overline{AR}$...(iii)	From (i) and (ii)
$\angle 1 \cong \angle B$ and	
$\angle 1 \cong \angle 2$...(iv)	
$\Delta ARQ \leftrightarrow \Delta QSC$	
$\angle 2 \cong \angle 1$	From (iv)
$\angle 3 \cong \angle C$	
$\overline{AR} \cong \overline{SQ}$	From (iii)
Hence, $\Delta ARQ \cong \Delta QSC$	$A.A.S \cong A.A.S$
$\overline{AQ} \cong \overline{QC}$	Corresponding sides

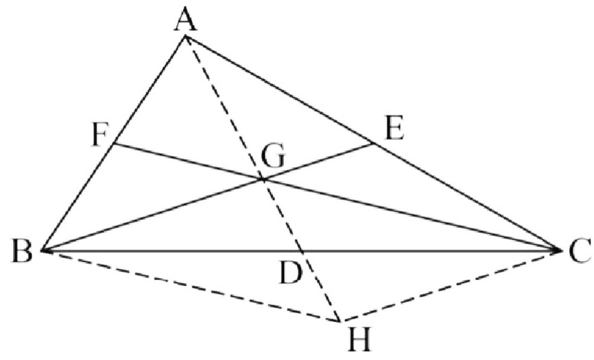
Theorem: 11.1.4

Statement: The median of triangle are concurrent and their point of concurrency is the point of trisection of each median.

Given ΔABC

To prove

The medians of the ΔABC are concurrent and the point of concurrency is the point of trisection of each median



Construction

Draw two medians $\overline{BE}$ and $\overline{CF}$ of the ΔABC which intersect each other at point G. Join A to G and produce it to the point H such that $AG \simeq \overline{GH}$ Join H to the points B and C $\overline{AH}$ Intersects $\overline{BC}$ at the point D.

Proof

Statements	Reasons
In Δ ACH,	
$\overline{GE} \parallel \overline{HC}$	G and E are mid-points of sides $\overline{AH}$ and $\overline{AC}$ respectively
Or $\overline{BE} \parallel \overline{HC}$ ....(i)	G is point of $\overline{BE}$ diagonals $\overline{BC}$
Similarly $\overline{CF} \parallel \overline{HB}$...(ii)	
$\therefore$ BHCG is a parallelogram	From (i) and (ii)
And	
$m\overline{GD} = \frac{1}{2} m\overline{GH}$...(iii)	Diagonals $\overline{BC}$ and $\overline{GH}$ of a parallelogram $BHCG$ intersect each other at point D.
$\overline{BD} = \overline{CD}$	
$\overline{AD}$ is a median of ΔABC medians $\overline{AD}, \overline{BE}$ and $\overline{CF}$ pass through the point G	G is the interesting point of $\overline{BE}$, $\overline{CF}$ and $\overline{AD}$ pass through it.
Now $\overline{GH} \cong \overline{AG}$...(iv)	Construction
$m\overline{GD} = \frac{1}{2} m\overline{AG}$	From (iii) and (iv)
and G is the point of trisection of $\overline{AD}$...(v)	
similarly it can be proved that G is also the point of trisection of $\overline{CF}$ and $\overline{BE}$	