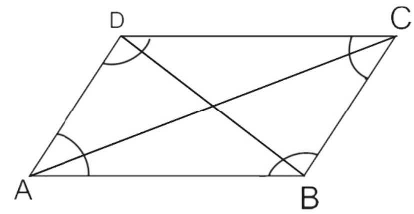
Exercise 11.2

Q.1 Prove that a quadrilateral is a parallelogram if its
(a) Opposite angles are congruent
(b) Diagonals bisects each other

(a) Given

In quadrilateral ABCD
$m\angle A = m\angle C, m\angle B = m\angle D$



To Prove

$ABCD$ is a parallelogram

Statements	Reasons
$m\angle A = m\angle C \ldots$(i)	Given
$m\angle B = m\angle D \ldots$(ii)	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360°$	Angles of quadrilateral
$m\angle A + m\angle B = 180°$	
$m\angle C + m\angle D = 180°$	
$\overline{AD} \parallel \overline{BC}$	
Similarity it can be proved that $\overline{AB} \parallel \overline{DC}$	
Hence ABCD is a parallelogram	

(b) Given

In quadrilateral ABCD, diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other.

i.e. $\overline{OA} = \overline{OC}, \overline{OB} = \overline{OD}$

To prove ABCD is a parallelogram

Proof

Statements	Reasons
In ΔABO $\leftrightarrow$ ΔCDO	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOB \cong \angle COD$	Vertical opposite angles
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\Delta ABO \cong \angle CDO$	$S.A.S \cong S.A.S$
Hence, $\overline{AB} \parallel \overline{CD} \ldots$(i)	$\angle 1 \cong \angle 2$
By taking BOC and is ΔAOD it can be prove that	
$\overline{AD} \parallel \overline{BC} \ldots$(ii)	From (i) and (ii)
Hence ABCD is a parallelogram	

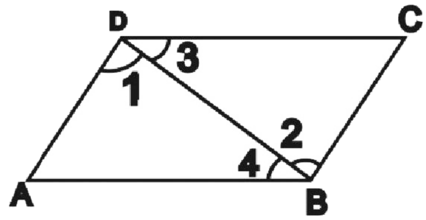
Q.2 **Prove that a quadrilateral is a parallelogram if its opposite sides are congruent**

Given

In quadrilateral $ABCD$

(i) $\overline{AB} \cong \overline{DC}$

(ii) $\overline{AD} \cong \overline{BC}$



To prove

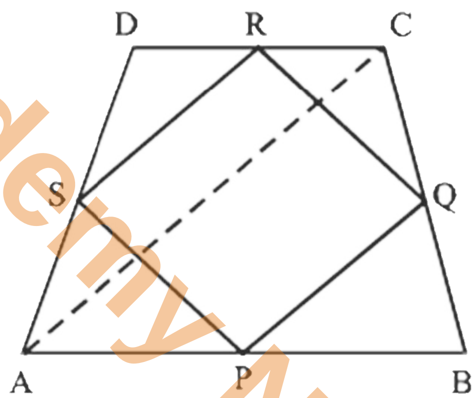
$ABCD$ is a parallelogram i.e. $\overline{AD} \| \overline{BC}$

Prove

Statements	Reasons
$\Delta CDB \leftrightarrow \Delta ABD$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	$S.S.S \cong S.S.S$
Thus, $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 4 \cong \angle 3$	Corresponding angles of congruent triangles
(i) $\overline{AD} \| \overline{BC}$	Alternate angles are congruent
$\overline{AB} \| \overline{DC}$	Alternate angles are congruent
$\therefore$ ABCD is a parallelogram	

Example

The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.



Given

A quadrilateral $ABCD$, in which P is the mid-point of $\overline{AB}$ Q is the mid-point of $\overline{BC}$ R is the mid-point of $\overline{CD}$

S is the mid-point of $\overline{DA}$

P is joined to Q, Q is joined to R.

R is joined to S and S is joined to P.

To prove

PQRS is a parallelogram.

Construction

Join A to C.

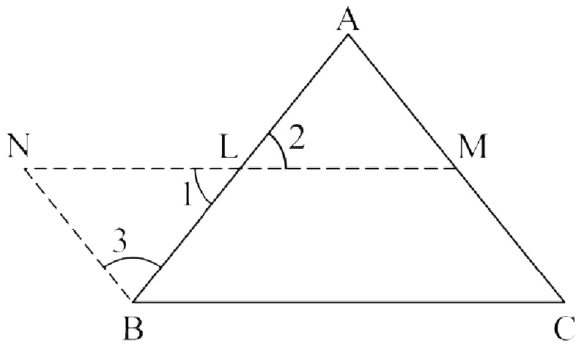
Proof

Statements	Reasons
In ΔDAC,	
$\left.\begin{array}{l}\overline{SR} \| \overline{AC} \\ m\overline{SR} = \frac{1}{2} m\overline{AC}\end{array}\right\}$	S is the midpoint of $\overline{DA}$ R is the midpoint of $\overline{CD}$

In ΔBAC,	
$\overline{PQ} \parallel \overline{AC}$	P is the midpoint of $\overline{AB}$
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$	Q is the midpoint of $\overline{BC}$
$\overline{SR} \parallel \overline{PQ}$	Each $\parallel \overline{AC}$
$m\overline{SR} = m\overline{PQ}$	Each$=\frac{1}{2}\overline{AC}$
Thus $PQRS$ is a parallelogram	$\overline{SR} \parallel \overline{PQ}, m\overline{SR} = m\overline{PQ}$ (proved)

Theorem 11.1.3

The line segment, joining the midpoint of two sides of triangle, is parallel to the third side and is equal to one half of its length.



Given

In ΔABC, the mid-point of $\overline{AB}$ and $\overline{AC}$ are L and M respectively

To prove

$\overline{LM} \parallel \overline{BC}$ and $m\overline{LM} = \frac{1}{2} m\overline{BC}$

Construction

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure, name the angles $\angle 1$, $\angle 2$ and $\angle 3$ as shown.

Proof

Statements	Reasons
In Δ BLN $\leftrightarrow$ Δ ALM	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore$ ΔBLN $\cong$ ΔALM	S.A.S postulate
$\therefore \angle A \cong \angle 3$...(i)	(Corresponding angles of congruent triangles)
And $\overline{NB} \cong \overline{AM}$...(ii)	(Corresponding sides of congruent triangles)
But $\overline{NB} \parallel \overline{AM}$	from (i), alternative $<s$
Thus	
$\overline{NB} \parallel \overline{MC}$ ....(iii)	(M is a point of $\overline{AC}$)

$\overline{MC} \cong AM$ ....(iv)	Given
$\overline{NB} \cong \overline{MC}$ ....(v)	from (ii) and (iv)
$BC\,MN$ is a parallelogram	From (iii) and (v)
$\therefore \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	(Opposite sides of a parallelogram BCMN) (Opposite sides of a parallelogram)
$\overline{BC} \cong \overline{NM}$ ....(vi)	Construction.
$m\overline{LM} = \frac{1}{2} m\overline{NM}$ ....(vii)	
Hence, $m\overline{LM} = \frac{1}{2} m\overline{BC}$	from (vi) and (vii)