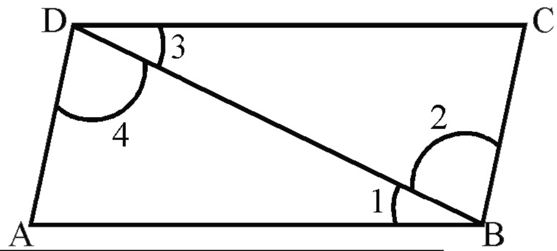
Exercise 10.3

Q.1 In the figure, $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$ prove that $\angle A = \angle C$, $\angle ABC \cong \angle ADC$



Given

In the figure $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$

To prove

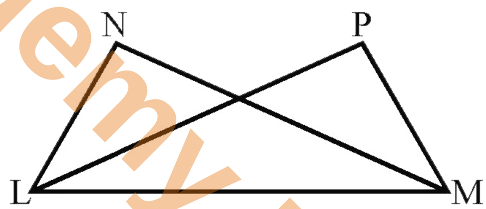
$\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	S.S.S ≅ S.S.S
$\therefore$ Hence $\angle A \cong \angle C$	Corresponding angles of congruent triangles
$\angle 1 \cong \angle 3$	Corresponding angles of congruent triangles
$\angle 2 \cong \angle 4$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	
or $m\angle ABC = m\angle ADC$	
$\angle ABC \cong \angle ADC$	

Q.2 In the figure $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$ prove that $\angle N \cong \angle P$, $\angle NML \cong \angle PLM$



Given

In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$

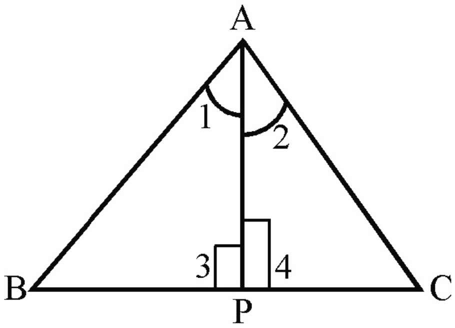
To prove

$\angle N \cong \angle P$ and $\angle NML \cong \angle PLM$

Proof

Statements	Reasons
$\Delta LMN \leftrightarrow \Delta MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common
$\Delta LMN \cong \Delta MLP$	S.S.S ≅ S.S.S
$\angle N \cong \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

Q.3 **Prove that median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base**



Given

ΔABC

(i) $\overline{AB} \cong \overline{AC}$

(ii) Point P is mid point of $\overline{BC}$ i.e : $\overline{BP} = \overline{CP}$

P is joined to A, i.e. $\overline{AP}$ is median

To prove

$\angle 1 \cong \angle 2$

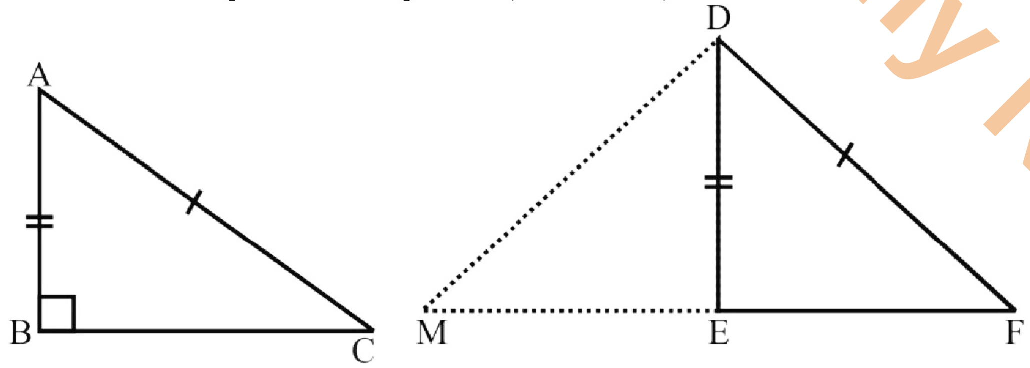
$\overline{AP} \perp \overline{BC}$

Proof

Statements	Reasons
$\Delta ABP \leftrightarrow \Delta ACP$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BP} \cong \overline{CP}$	Given
$\overline{AP} \cong \overline{AP}$	Common
$\Delta ABP \cong \Delta ACP$	S.S.S ≅ S.S.S
$\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
$\angle 3 \cong \angle 4$ ______(i)	
$m\angle 3 + m\angle 4 = 180^o$ _____(ii)	Corresponding angles of congruent triangles
Thus $m\angle 3 = m\angle 4 = 90$	
$\because \overline{AP} \perp \overline{BC}$	From equation (i) and (ii)

Theorem 10.1.4

If in the corresponding of the two right angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other them the triangles are congruent (H.S ≅ H.S)



Given

$\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (right angles)

$\overline{CA} \cong \overline{FD}, \overline{AB} \cong \overline{DE}$

To Prove

$\Delta ABC \cong \Delta DEF$

Construction

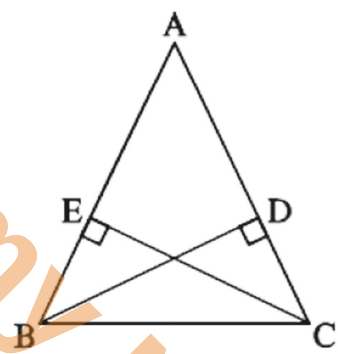
Prove $\overline{FE}$ to a point M such that $\overline{EM} \cong \overline{BC}$ and join the point D and M

Proof

Statements	Reasons
$m\angle DEF + \angle DEM = 180^\circ$ ____(i)	Supplementary angles
Now $m\angle DEF = 90^\circ$ ________(ii)	Given
$\therefore m\angle DEM = 90^\circ$	{ from (i) and (ii) }
In $\Delta ABC \leftrightarrow \Delta DEM$	
$\overline{BC} \cong \overline{EM}$	Construction
$\angle ABC \cong \angle DEM$	(Each angle equal to 90°)
$\overline{AB} \cong \overline{DE}$	Given
$\Delta ABC \cong \Delta DEM$	SAS postulate
ad $\angle C \cong \angle M$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{MD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\overline{MD} \cong \overline{FD}$	Each is congruent to $\overline{CA}$
In DMF	
$\angle F \cong \angle M$	$\overline{MD} \cong \overline{FD}$ (proved)
But $\angle C \cong \angle M$	(Proved)
$\angle C \cong \angle F$	Each is congruent to $\angle M$
	Given
$\angle ABC \cong \angle DEF$	Given
$\overline{AB} \cong \overline{DE}$	(Proved)
Hence $\Delta ABC \cong \Delta DEF$	(S.A.A $\cong$ S.A.A)

Example

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.



Given

In $\Delta ABC, \overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

To prove

$\overline{AB} \cong \overline{AC}$

Proof

Statements	Reasons
In $\Delta BCD \leftrightarrow \Delta CBC$	
$\angle BDC \cong \angle BEC$	$\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ given $\Rightarrow$ each angle $= 90^\circ$
$\overline{BC} \cong \overline{BC}$	Common hypotenuse
$\overline{BD} \cong \overline{CE}$	Given
$\Delta BCD \cong \Delta CBE$	H.S $\cong$ H.S
$\angle BCA \cong \angle CBE$	Corresponding angles Δ s
Thus $\angle BCA \cong \angle CBA$	
Hence $\overline{AB} \cong \overline{AC}$	In $\Delta ABC, \angle BCA \cong \angle CBA$