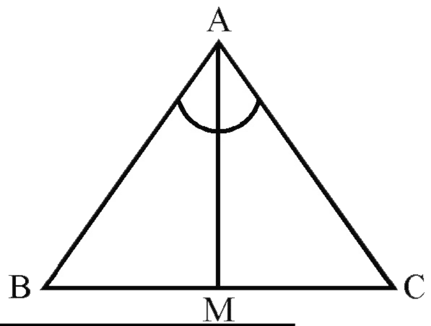
Exercise 10.2

Q.1 Prove that any two medians of an equilateral triangle are equal in measure.



Given

In ΔABC, $\overline{AB} \cong \overline{AC}$ and M is midpoint of $\overline{BC}$

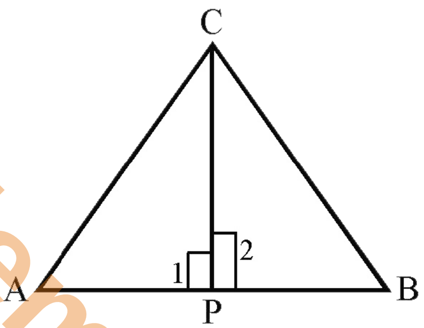
To prove

$\overline{AM}$ bisects $\angle A$ and $\overline{AM}$ is perpendicular to $\overline{BC}$

Proof

Statements	Reasons
In $\Delta ABM \leftrightarrow \Delta ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is midpoint of BC
$\overline{AM} \cong \overline{AM}$	Common
$\therefore \Delta ABM \cong \Delta ACM$	S.S.S $\cong$ S.S.S
So $\angle BAM \cong \angle CAM$	Corresponding angles of congruents triangle
$m\angle AMB + m\angle AMC = 180^\circ$	
$\therefore$ $m\angle AMB = m\angle AMC$	
i.e $\overline{AM}$ is perpendicular to $\overline{BC}$	

Q.2 Prove that a point which is equidistant from the end points of a line segment, is on the right bisector of line segment



Given

$\overline{AB}$ is line segment. The point C is such that $\overline{CA} \cong \overline{CB}$

To prove

Point C lies on the right bisector of $\overline{AB}$

Construction

(i) Take P as midpoint of $\overline{AB}$ i.e. $\overline{AP} \cong \overline{BP}$

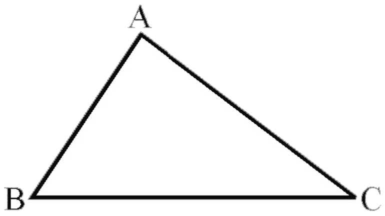
(ii) Joint point C to A, P, B

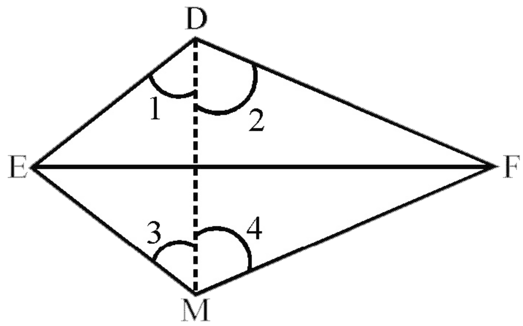
Proof:

Statements	Reasons
In ΔABC	
$\overline{CA} \cong \overline{CB}$	Given
$\angle A \cong \angle B$	Corresponding angles of congruent triangles
Δ CBP $\leftrightarrow \Delta$CAP	
$\overline{CB} \cong \overline{CA}$	
$\Delta CAP \cong \Delta CBP$	S.A.S $\cong$ S.A.S
$\therefore$ $\angle 1 \cong \angle 2$	
$m\angle 1 + m\angle 2 = 180^\circ$	Adjacent angles on one side of a line
Thus m $\angle 1 = m\angle 2 = 90$	
Hence $\overline{CP}$ is right bisector of $\overline{AB}$ and point C lies on $\overline{CB}$	

Theorem 10.1.3

In a correspondence of two triangles if three sides of one triangle are congruent to the corresponding three sides of the other. Then the two triangles are congruent (S.S.S ≅ S.S.S)





Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To prove

$\Delta ABC \cong \Delta DEF$

Construction

Suppose that in ΔDEF the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a ΔMEF in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M. as shown in the above figures we label some of the angles as 1, 2, 3, and 4

Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEM$	Construction
$\overline{AB} \cong \overline{ME}$	Construction
$\therefore \Delta ABC \cong \Delta MEF$	S.A.S Postulate
and $\overline{CA} \cong \overline{FM}$ ____(i)	(Corresponding sides of congruent triangles)
also $\overline{CA} \cong \overline{FD}$ ___(ii)	Given
$\therefore \overline{FM} \cong \overline{FD}$	{ From (i) and (ii) }
In ΔFDM	
$\angle 2 \cong \angle 4$ _____(iii)	$\overline{FM} \cong \overline{FD}$ (proved)
Similarly $\angle 1 \cong \angle 3$ ______(iv)	
$\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	{ from (iii) and iv }
$\therefore m\angle EDF = m\angle EMF$	
Now in $\Delta DEF \leftrightarrow \Delta MEF$	
$\overline{FD} \cong \overline{FM}$	Proved
and $m\angle EDF \cong \angle EMF$	Proved
$\overline{DE} \cong \overline{ME}$	Each one ≅ $\overline{AB}$
$\therefore \Delta DEF \cong \Delta MEF$	S.A.S postulates
also $\Delta ABC \cong \Delta MEF$	Proved
Hence $\Delta ABC \cong \Delta DEF$	Each $\Delta \cong \Delta MEF$ (proved)

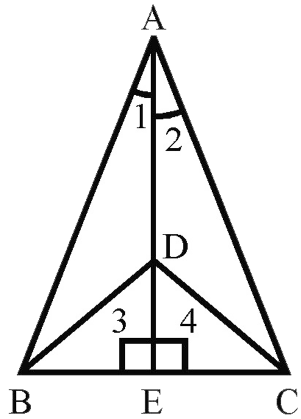
Example 1

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.

Given

ΔABC and ΔDBC formed on the same side of $\overline{BA}$ such that

$\overline{BA} \cong \overline{AC}, \overline{DB} \cong \overline{DC}, \overline{AD}$ meets $\overline{BC}$ at E.



To prove

$\overline{BE} \cong \overline{CE}. \overline{AE} \perp \overline{BC}$

Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow \Delta ADC$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{DB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\therefore \Delta ADB \cong \Delta ADC$	S.S.S $\cong$ S.S.S
$\therefore \angle 1 \cong \angle 2$	Corresponding angles of $\cong \Delta s$
In $\Delta ABE \leftrightarrow \Delta ACE$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\Delta ABE \cong \Delta ACE$	S.A.S postulate
$\overline{AE} \cong \overline{AE}$	Common
$\therefore \overline{BE} \cong \overline{CE}$	Corresponding sides of $\cong \Delta s$
$\angle 3 \cong \angle 4$	Corresponding angles of $\cong \Delta s$
$m\angle 3 + m\angle 4 = 180°$	Supplementary angles postulate
$m\angle 3 = m\angle 4 = 90°$	From I and II
Hence $\overline{AE} \perp \overline{BC}$	