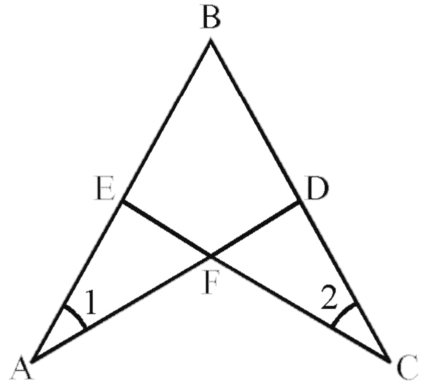
Exercise 10.1

Q.1 **In the given figure**

$\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

Prove that

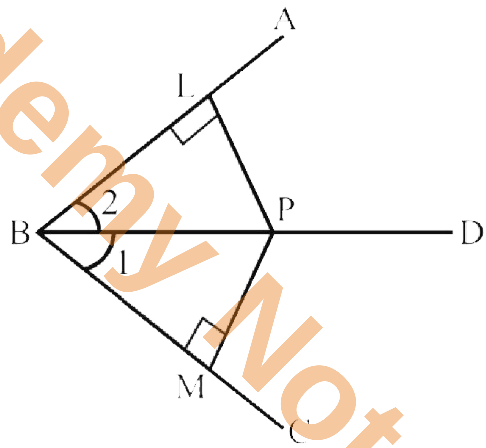
$\Delta ABD \cong \Delta CBE$



Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\Delta ABD \cong \Delta CBE$	S.A.A ≅ S.A.A

Q.2 **From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.**



Given

$\overline{BD}$ is bisector of $\angle ABC$. P is point on $\overline{BD}$ and $\overline{PL}$ are $\overline{PM}$ are perpendicular to $\overline{AB}$ and $\overline{CB}$ respectively

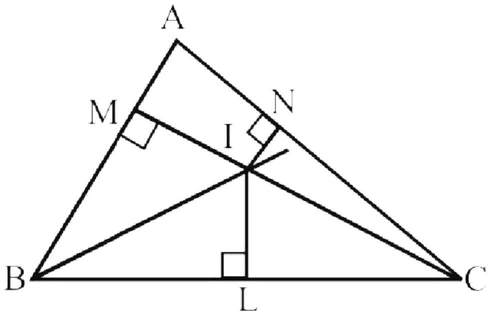
To prove

$\overline{PL} \cong \overline{PM}$

Proof

Statements	Reasons
In $\Delta BLP \leftrightarrow \Delta BMP$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle BLP \cong \angle BMP$	Each right angle (given)
$\angle LBP \cong \angle MBP$	Given $\overline{BD}$ is bisector of angle B
$\therefore\ \Delta BLP \cong \Delta BMP$	S.A.A ≅ S.A.A
So $\overline{PL} \cong \overline{PM}$	Corresponding sides of congruent triangles

Q.3 **In a triangle ABC, the bisects of $\angle B$ and $\angle C$ meet in point I prove that I is equidistant from the three sides by ΔABC**



Given

In ΔABC, the bisector of $\angle B$ and $\angle C$ meet at I and $\overline{IL}$, $\overline{IM}$, and $\overline{IN}$ are perpendiculars to the three sides of ΔABC.

To prove

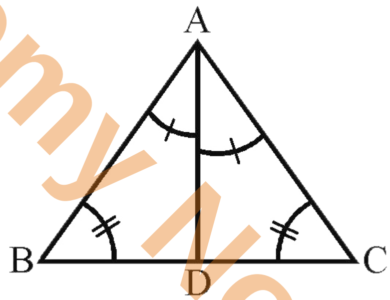
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$

Proof

Statements	Reasons
In $\Delta ILB \leftrightarrow \Delta IMB$	
$\overline{BI} \cong \overline{BI}$	Common
$\angle IBL \cong \angle IBM$	Given BI is bisector of $\angle B$
$\angle ILB \cong \angle IMB$	Given each angle is rights angles
$\Delta ILB \cong \Delta IMB$	SAA $\cong$ S.A.A
$\therefore$ $\overline{IL} \cong \overline{IM}$ ______ (i)	Corresponding sides of $\cong$ Δ's
Similarly	
$\Delta IAC \cong \Delta INC$	
So $\overline{IL} \cong \overline{IN}$ ______ (ii)	Corresponding sides of $\cong$ Δs
from (i) and (ii)	
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
$\therefore$ I is equidistant from the three sides of ΔABC.	

Theorem 10.1.2

If two angles of a triangles are congruent then the sides opposite to them are also congruent



Given

In ΔABC, $\angle B \cong \angle C$

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Draw the bisector of $\angle A$, meeting $\overline{BC}$ at point D

Proof

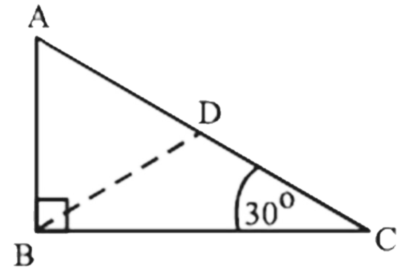
Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\Delta ABD \cong \Delta ACD$	S.A.A $\cong$ S.A.A
Hence $\overline{AB} \cong \overline{AC}$	(Corresponding sides of congruent triangles)

Example 1

If one angle of a right triangle is of $30°$, the hypotenuse is twice as long as the side opposite to the angle.

Given

In $\Delta ABC, m\angle B = 90°$ and $m\angle C = 30°$



To prove

$m\overline{AC} = 2m\overline{AB}$

Constructions

At, B construct $\angle CBD$ of $30°$

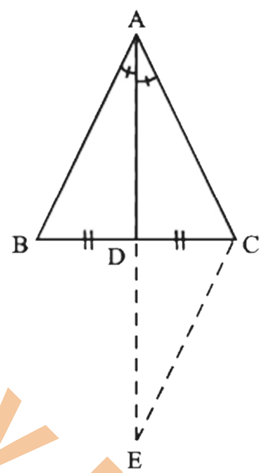
Let $\overline{BD}$ cut $\overline{AC}$ at the point D.

Proof

Statements	Reasons
In $\Delta ABD, m\angle A = 60°$	$m\angle ABC = 90°, m\angle C = 30°$
$m\angle ABD = m\angle ABC$, $mCBD = 60°$	$m\angle ABC = 90°, m\angle CBD = 30°$
$\therefore mADB = 60°$	Sum of measures of $\angle$s of a Δ is $180°$
$\therefore \Delta ABD$ is equilateral	Each of its angles is equal to $60°$
$\therefore \overline{AB} \cong \overline{BD} \cong \overline{CD}$	Sides of equilateral Δ
In $\Delta BCD, \overline{BD} \cong \overline{CD}$	**$\angle C = \angle CBD$ (each of 30),**
Thus $m\overline{AC} = m\overline{AD} + m\overline{CD} = m\overline{AB} + m\overline{AB} = 2(m\overline{AB})$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$

Example 2

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.



Given

In $\Delta ABC, \overline{AD}$ bisect $\angle A$ and $\overline{BD} \cong \overline{CD}$

To prove

$\overline{AB} \cong \overline{AC}$

Construction

Produce $\overline{AD}$ to E, and take $\overline{ED} \cong \overline{AD}$

Joint C to E

Proof

Statements	Reasons
In $\Delta ADB \leftrightarrow EDC$	
$\overline{AD} \cong \overline{ED}$	Construction
$\angle ADB \cong \angle EDC$	Vertical angles
$\overline{BD} \cong \overline{CD}$	Given
$\therefore \Delta ADB \cong \Delta EDC$	S.A.S. Postulate
$\therefore \overline{AB} \cong \overline{EC}$ ...(i)	Corresponding sides
and $\angle BAD \cong \angle E$	Corresponding angles
But $\angle BAD \cong \angle CAD$	Given
$\therefore \angle E \cong \angle CAD$	Each $\cong \angle BAD$
In $\Delta ACE, \overline{AC} \cong \overline{EC}$ ...(ii)	$\angle E \cong \angle CAD$ (proved)
Hence $\overline{AB} \cong \overline{AC}$	From (i) and (ii)