Review Exercise 1

Q.1 Select the correct answer in each of the following.

(i) **The order of matrix** $\begin{bmatrix} 2 & 1 \end{bmatrix}$ **is….**

(a) 2-by-1 **(b)** 1-by-2
(c) 1-by-1 **(d)** 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called …matrix.**

(a) Zero **(b)** Unit
(c) Scalar **(d)** Singular

(iii) **Which is order of a square matrix?**

(a) 2-by-2 **(b)** 1-by-2
(c) 2-by-1 **(d)** 3-by-2

(iv) **Order of transpose of** $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is…**

(a) 3-by-2 **(b)** 2-by-3
(c) 1-by-3 **(d)** 3-by-1

(v) **Adjoint of** $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ **is…**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) **Product of** $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **is…**

(a) $\begin{bmatrix} 2x+y \end{bmatrix}$ **(b)** $\begin{bmatrix} x-2y \end{bmatrix}$

(c) $\begin{bmatrix} 2x-y \end{bmatrix}$ **(d)** $\begin{bmatrix} x+2y \end{bmatrix}$

(vii) **If** $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$**=0, then** x **is equal to…**

(a) 9 **(b)** -6
(c) 6 **(d)** -9

(viii) **If** $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$**, then X is equal to…**

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
b	c	a	b	a	c	a	d

Q.2 Complete the follwoing:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ... matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ... matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is....

(iv) In matrix multiplication, in gereral, AB ...BA.

(v) Matrix A+B may be found if order of A and B is...

(vi) A matrix is called ... matrix if number of rows and columns are equal.

ANSWER KEY

i	ii	iii	iv	v	vi
Null	Unit	$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$	$\neq$	Same	Square

Q.3 If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$**, then find *a* and *b*.**

Solution: $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

$a+3=-3$ $b-1=2$

$a=-3-3$ $b=2+1$

$a=-6$ $b=3$ **Ans**

Q.4 If $A=\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, B\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$**, then find the following.**

(i) $2A+3B$

(ii) $-3A+2B$

(iii) $-3(A+2B)$

(iv) $\frac{2}{3}(2A-3B)$

Solution: (i)

$2A+3B = 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3\begin{bmatrix} 5 & 4 \\ -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$

$= \begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix}$

$= \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix}$ **Ans**

Solution: (ii)

$-3A+2B = -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$

$= \begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix}$

$$=\begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix}$$ **Ans**

Solution: (iii)

$$-3\left(A+2B\right)=-3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$$

$$=-3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+\begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$=-3\begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix}$$

$$=\begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix}$$ **Ans**

Solution: (iv) $\frac{2}{3}\left(2A-3B\right)$

$$=\frac{2}{3}\left(2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}-3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$$

$$=\frac{2}{3}\left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}-\begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}\right)$$

$$=\frac{2}{3}\begin{bmatrix} 4-15 & 6-(-12) \\ 22(-6) & 0-(-3) \end{bmatrix}$$

$$=\frac{2}{3}\begin{bmatrix} -11 & 6+12 \\ 2+6 & 0+3 \end{bmatrix}$$

$$=\frac{2}{3}\begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} -11\times\frac{2}{3} & 18\times\frac{2}{3} \\ 8\times\frac{2}{3} & 3\times\frac{2}{3} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix}$$ **Ans**

Q.5 Find the value of X, if

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}+X=\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}.$$

Solution: Given that

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}+X=\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$$

$$X=\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}-\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2-(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 2 & -3 \\ -4 & -2+3 \end{bmatrix}$$

$$X=\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$ **Ans**

Q.6 If $A=\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B=\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$,

then prove that

(i) $AB\neq BA$

(ii) $A\left(BC\right)=\left(AB\right)C$

Solution: Given that

$$A=\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B=\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

(i) $AB\neq BA$

L.H.S= $AB=\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$

$$=\begin{bmatrix} 0\times(-3)+1\times 5 & 0\times 4+1\times(-2) \\ 2\times(-3)+(-3)\times 5 & 2\times 4+(-3)\times(-2) \end{bmatrix}$$

$$=\begin{bmatrix} 0+5 & 0-2 \\ -6-15 & 8+6 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & -2 \\ -21 & 14 \end{bmatrix} \quad \rightarrow (i)$$

R.H.S= $BA=\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$

$$=\begin{bmatrix} -3(0)+4(2) & -3(1)+4(-3) \\ 5(0)+(-2)(2) & 5(1)+(-2)(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 0+8 & -3-12 \\ 0-4 & 5+6 \end{bmatrix}$$

$$=\begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix} \quad \rightarrow (ii)$$

From (i) and (ii) , we get

$L.H.S \neq R.H.S$

$AB \neq BA$

Hence proved

(ii) $A(BC)=(AB)C$

Solution:

We cannot solve because matrix C is not given.

Q.7 **If** $A=\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 4\\ -3 & -5\end{bmatrix}$, **then verify that**

(i) $(AB)^t = B^t A^t$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

Solution: Given that

$A=\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 4\\ -3 & -5\end{bmatrix}$

(i) $(AB)^t = B^t A^t$

$AB=\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}\begin{bmatrix}2 & 4\\ -3 & -5\end{bmatrix}$

$=\begin{bmatrix}3(2)+2(-3) & 3(4)+2(-5)\\ 1(2)+(-1)(-3) & 1(4)+(-1)(-5)\end{bmatrix}$

$=\begin{bmatrix}6-6 & 12-10\\ 2+3 & 4+5\end{bmatrix}$

$=\begin{bmatrix}0 & 2\\ 5 & 9\end{bmatrix}$

L.H.S$=(AB)^t=\begin{bmatrix}0 & 2\\ 5 & 9\end{bmatrix}^t$

$=\begin{bmatrix}0 & 5\\ 2 & 9\end{bmatrix} \quad \rightarrow (i)$

$A^t=\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}^t$

$=\begin{bmatrix}3 & 1\\ 2 & -1\end{bmatrix}$

$B^t=\begin{bmatrix}2 & 4\\ -3 & -5\end{bmatrix}^t=\begin{bmatrix}2 & -3\\ 4 & -5\end{bmatrix}$

R.H.S$=B^tA^t=\begin{bmatrix}2 & -3\\ 4 & -5\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & -1\end{bmatrix}$

$=\begin{bmatrix}2\times3+(-3)\times2 & 2(1)+(-3)(-1)\\ 4\times3+(-5)\times2 & 4(1)+(-5)(-1)\end{bmatrix}$

$=\begin{bmatrix}6-6 & 2+3\\ 12-10 & 4+5\end{bmatrix}$

$=\begin{bmatrix}0 & 5\\ 2 & 9\end{bmatrix} \quad \rightarrow (ii)$

From equal (i) and (ii) we get

L.H.S=R.H.S

$(AB)^t = B^tA^t$

Hence proved

(ii) $(AB)^{-1}=B^{-1}A^{-1}$

$|AB|=\begin{vmatrix}0 & 2\\ 5 & 9\end{vmatrix}$

$=0\times9-2\times5$

$=0-10$

$=-10$ (Non singular)

Inverse exists

$Adj(AB)=\begin{bmatrix}9 & -2\\ -5 & 0\end{bmatrix}$

L.H.S$=(AB)^{-1}=\frac{1}{|AB|}Adj(AB)$

$=\frac{1}{-10}\begin{bmatrix}9 & -2\\ -5 & 0\end{bmatrix}$

$=\begin{bmatrix}\frac{9}{10} & \frac{-2}{-10}\\ \frac{-5}{-10} & \frac{0}{-10}\end{bmatrix}$

$=\begin{bmatrix}\frac{-9}{10} & \frac{1}{5}\\ \frac{1}{2} & 0\end{bmatrix} \quad \rightarrow (i)$

$B=\begin{bmatrix}2 & 4\\ -3 & -5\end{bmatrix}$

$|B|=\begin{vmatrix}2 & 4\\ -3 & -5\end{vmatrix}$

$=2(-5)-4\times(-3)$

$=-10+12$

$=2$ (non singular)

$\therefore B^{-1}$ exists

$$AdjB=\begin{bmatrix}-5 & -4\\ 3 & 2\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}AdjB$$

$$=\frac{1}{2}\begin{bmatrix}-5 & -4\\ 3 & 2\end{bmatrix}$$

$$A=\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}$$

$$|A|=\begin{vmatrix}3 & 2\\ 1 & -1\end{vmatrix}$$

$$=3(-1)-2\times 1$$

$$=-3-2$$

$=-5$ (non singular)

$\therefore A^{-1}$ exists

$$AdjA=\begin{bmatrix}-1 & -2\\ -1 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times AdjA$$

$$=\frac{1}{-5}\begin{bmatrix}-1 & -2\\ -1 & 3\end{bmatrix}$$

R.H.S=$B^{-1}A^{-1}$

$$=\left(\frac{1}{2}\begin{bmatrix}-5 & -4\\ 3 & 2\end{bmatrix}\right)\times\left(\frac{1}{-5}\begin{bmatrix}-1 & -2\\ -1 & 3\end{bmatrix}\right)$$

$$=\frac{1}{2}\left(-\frac{1}{5}\right)\begin{bmatrix}-5 & -4\\ 3 & 2\end{bmatrix}\begin{bmatrix}-1 & -2\\ -1 & 3\end{bmatrix}$$

$$=-\frac{1}{10}\begin{bmatrix}-5(-1)+(-4)(-1) & -5(-2)+(-4)(3)\\ 3(-1)+2(-1) & 3(-2)+2(3)\end{bmatrix}$$

$$=-\frac{1}{10}\begin{bmatrix}5+4 & 10-12\\ -3-2 & -6+6\end{bmatrix}$$

$$=-\frac{1}{10}\begin{bmatrix}9 & -2\\ -5 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\frac{9}{-10} & \frac{-2}{-10}\\ \frac{-5}{-10} & \frac{0}{-10}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-9}{10} & \frac{1}{5}\\ \frac{1}{2} & 0\end{bmatrix} \qquad \rightarrow(ii)$$

From equation (i) and (ii) we get

L.H.S =R.H.S

$$(AB)^{-1}=B^{-1}A^{-1}$$

Hence proved