

Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$$

To write the determinant form

$$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$$

$$= (-1)(0) - (2)(1)$$

$$= 0 - 2$$

$$= -2$$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$$

$$= (1)(-2) - (2)(3)$$

$$= -2 - 6$$

$$= -8$$

(iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

Solution:

$$C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$$

To write in determinant form

$$\begin{aligned} |C| &= \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} \\ &= (3)(2) - (3)(2) \\ &= 6 - 6 \\ &= 0 \end{aligned}$$

(iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Solution:

$$D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

To write in determinant form

$$|D| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$$

$$= (3)(4) - (2)(1)$$

$$= 12 - 2$$

$$= 10$$

Q.2 Find which of the following matrices are singular or non-singular?

(i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$$

$$|A| = (3)(4) - (2)(6)$$

$$|A| = 12 - 12$$

$$|A| = 0$$

It is a singular matrix.

(ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$$

$$|B| = (4)(2) - (3)(1)$$

$$|B| = 8 - 3$$

$$|B| = 5$$

It is non-singular matrix.

$$(iii) \quad C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$$

Solution:

$$C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$$

To write in determinant form

$$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$$

$$|C| = (7)(5) - (3)(-9)$$

$$|C| = 35 + 27$$

$$|C| = 62$$

It is not equal to zero so

It is non-singular matrix.

$$(iv) \quad D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$$

Solution:

$$D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$$

To write in determinant form

$$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$$

$$|D| = (5)(4) - (-2)(-10)$$

$$|D| = 20 - 20$$

$$|D| = 0$$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

$$(i) \quad A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$$

$$|A| = (-1)(0) - (2)(3)$$

$$|A| = 0 - 6$$

$$|A| = -6 \neq 0 \text{ (Non-Singular)}$$

A^{-1} exists

To write in Adj A

$$Adj A = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adj A$$

Putting the values

$$A^{-1} = \frac{1}{-6} \times \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 \times \frac{1}{-6} & -3 \times \frac{1}{-6} \\ -2 \times \frac{1}{-6} & -1 \times \frac{1}{-6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{0}{-6} & \frac{+3}{+6} \\ \frac{+2}{+6} & \frac{+1}{+6} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

$$(ii) \quad B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

Solution:

$$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix}$$

$$|B| = (-1)(-5) - (-3)(2)$$

$$|B| = -5 + 6$$

$$|B| = 1 \neq 0 \text{ (Non-Singular)}$$

B^{-1} exists

$$\text{Adj}B = \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times \text{Adj}B$$

Putting the values

$$B^{-1} = \frac{1}{1} \times \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \times -5 & \frac{1}{1} \times -2 \\ \frac{1}{1} \times 3 & \frac{1}{1} \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5}{1} & \frac{-2}{1} \\ \frac{3}{1} & \frac{1}{1} \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix}$$

$$\text{(iii)} \quad C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

Solution:

To write in determinant form

$$|C| = \begin{vmatrix} -2 & 6 \\ 3 & -9 \end{vmatrix}$$

$$|C| = (-2)(-9) - (3)(6)$$

$$|C| = 18 - 18$$

$$|C| = 0 \text{ Singular}$$

C^{-1} Does not exist.

$$\text{(iv)} \quad D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$$

Solution:

To write in determinant form

$$D = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix}$$

$$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix} = \frac{1}{2} \times 2 - \frac{3}{4} \times 1$$

$$= 1 - \frac{3}{4}$$

$$= \frac{4-3}{4}$$

$$|D| = \frac{1}{4} \neq 0 \text{ (Non Singular)}$$

D^{-1} exists

$$\text{Adj}D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$D^{-1} = \frac{1}{|D|} \times \text{Adj}D$$

By putting the values

$$= \frac{1}{\frac{1}{4}} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \div \frac{1}{4} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \times \frac{4}{1} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 4 \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q.4 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$,

then

Then verify that

$$(i) \quad A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$$

$$\text{Solution: } A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$\text{Adj}A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix}$$

$$= 1 \times 6 - 2 \times 4$$

$$= 6 - 8$$

$$= -2$$

$$A(\text{Adj}A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & (-2)+2 \\ 24-4 & -8+6 \end{bmatrix}$$

$$A(\text{Adj}A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad \text{--- (i)}$$

$$(\text{Adj}A)A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$(\text{Adj}A)A = \begin{bmatrix} (6) \times 1 + (-2) \times 4 & (6) \times 2 + (-2) \times 6 \\ (-4) \times 1 + (1) \times 4 & (-4) \times 2 + (1) \times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$$

$$(\text{Adj}A)A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad \text{--- (ii)}$$

$$(\det A)I = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times 1 & 0 \times 2 \\ -2 \times 0 & 1 \times -2 \end{bmatrix}$$

$$(\det A)I = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad \text{--- (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

$$A(\text{Adj}A) = (\text{Adj}A)A = (\det A)I$$

$$(ii) \quad BB^{-1} = I = B^{-1}B$$

$$\text{Solution: } BB^{-1} = I = B^{-1}B$$

To write in determinant form

$$|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$$

$$= -6 - (-2)$$

$$= -6 + 2$$

$$= -4 \neq 0 \text{ (None singular)}$$

$$= B^{-1} \text{ exists.}$$

To write in AdjB

$$\text{Adj}B = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj}B$$

$$B^{-1} = \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

New

$$BB^{-1} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \times \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= -\frac{1}{4} \begin{bmatrix} -6+2 & 3-3 \\ -4+4 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{-4} & 0 \\ 0 & \frac{4}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

and

$$B^{-1}B = \frac{1}{-4} \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -6+2 & 2-2 \\ -6+6 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4} \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$|B| = 4 - (-2)$$

$$|B| = 4 + 2$$

$$|B| = 6$$

To write in Adj B

$$AdjB = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times AdjB$$

By putting value

$$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$$|A| = 8 - (-0)$$

$$|A| = 8$$

To write in Adj A

$$AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

To solve R.H.S

$$B^{-1}A^{-1} = \frac{1}{6} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{6} \times \frac{1}{8} \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$$

$$= \frac{1}{48} \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \times \frac{1}{48} & 8 \times \frac{1}{48} \\ -6 \times \frac{1}{48} & -16 \times \frac{1}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

Hence proved

L.H.S = R.H.S