

Exercise 1.4

Q.1 Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

No, these matrices cannot be multiplied because number of columns of 1st matrix is not equal to the number of rows of 2nd matrix.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

Q.2 If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$ find

(i) AB

Solution: $AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \times 6) + (0 \times 5) \\ (-1 \times 6) + (2 \times 5) \end{bmatrix}$$

$$= \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix} = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) BA (if possible)

Solution:

BA is not possible because number of columns of B not equal to number of rows of A .

Q.3 Find the following products

(i) $[1 \ 2] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $[1 \ 2] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$$= [(1 \times 4) + (2 \times 0)]$$

$$= [4 + 0]$$

$$= [4]$$

(ii) $[1 \ 2] \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Solution: $[1 \ 2] \begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$$= [(1 \times 5) + (2 \times -4)]$$

$$= [5 + (-8)]$$

$$= [5 - 8]$$

$$= [-3]$$

(iii) $[-3 \ 0] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $[-3 \ 0] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$$= [(-3 \times 4) + (0 \times 0)]$$

$$= [-12 + 0]$$

$$= [-12]$$

(iv) $[6 \ 0] \begin{bmatrix} 4 \\ -0 \end{bmatrix}$

Solution: $[6 \ 0] \begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$$[6 \ +0] \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$= [6 \times 4 + (-0)(0)]$$

$$= [24 - 0]$$

$$= [24]$$

(v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

$$= \begin{bmatrix} 1 \times 4 + 2 \times 0 & 1 \times 5 + 2 \times (-4) \\ -3 \times 4 + 0 \times 0 & -3(5) + 0 \times (-4) \\ 6(4) + (-1)(0) & 6(5) + (-1)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 0 & 5 - 8 \\ -12 + 0 & -15 - 0 \\ 24 - 0 & 30 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

$$\begin{bmatrix} 2 \times 2 + (3 \times 3) & (2 \times -1) + (3 \times 0) \\ (1 \times 2) + (1 \times 3) & (1 \times -1) + (1 \times 0) \\ (0 \times 2) + (-2 \times 3) & (0 \times -1) + (-2 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 9 & -2 + 0 \\ 2 + 3 & -1 + 0 \\ 0 + -6 & 0 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ 0-6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix}$$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} (1 \times 1) + (2 \times 3) + (3 \times -1) & (1 \times 2) + (2 \times 4) + (3 \times 1) \\ (4 \times 1) + (5 \times 3) + (6 \times -1) & (4 \times 2) + (5 \times 4) + (6 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+(-3) & 2+8+3 \\ 4+15+(-6) & 8+20+6 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3 & 13 \\ 19-6 & 34 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 13 \\ 13 & 34 \end{bmatrix}$$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$$= \begin{bmatrix} (1 \times 1) + (2 \times 4) & (1 \times 2) + (2 \times 5) & (1 \times 3) + (2 \times 6) \\ (3 \times 1) + (4 \times 4) & (3 \times 2) + (4 \times 5) & (3 \times 3) + (4 \times 6) \\ (-1 \times 1) + (1 \times 4) & (-1 \times 2) + (1 \times 5) & (-1 \times 3) + (1 \times 6) \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1+4 & -2+5 & -3+6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$$

(d) $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

Solution: $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

$$= \begin{bmatrix} (8 \times 2) + (5 \times -4) & (8 \times -\frac{5}{2}) + (5 \times 4) \\ (6 \times 2) + (4 \times -4) & (6 \times -\frac{5}{2}) + (4 \times 4) \end{bmatrix}$$

$$= \begin{bmatrix} 16 + (-20) & \frac{-40}{2} + 20 \\ 12 + (-16) & \frac{-30}{2} + 16 \end{bmatrix}$$

$$= \begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

(e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} (-1 \times 0) + (2 \times 0) & (-1 \times 0) + (2 \times 0) \\ (1 \times 0) + (3 \times 0) & (1 \times 0) + (3 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q.5 Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

and $C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ verify whether

(i) $AB = BA$

Solution: $AB = BA$

L.H.S = AB

R.H.S = BA

L.H.S = AB

$$\begin{aligned}
 &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \\
 &= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix} \\
 &= \begin{bmatrix} -1 + (-9) & -2 + (-15) \\ 2 + 0 & 4 + 0 \end{bmatrix} \\
 &= \begin{bmatrix} -1 - 9 & -2 - 15 \\ 2 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{R.H.S} = \text{BA} &= \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times (-1) + 2 \times 2 & 1 \times 3 + 2 \times 0 \\ -3 \times (-1) + (-5) \times 2 & -3 \times 3 + (-5) \times 0 \end{bmatrix} \\
 &= \begin{bmatrix} -1 + 4 & 3 + 0 \\ 3 - 10 & -9 - 0 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & 3 \\ -7 & -9 \end{bmatrix}
 \end{aligned}$$

Since L.H.S \neq R.H.S
 L.H.S \neq R.H.S
 L.H.S \neq R.H.S

(ii) $A(BC) = (AB)C$

Solution: $A(BC) = (AB)C$

L.H.S = A(BC)

R.H.S = (AB)C

L.H.S

L.H.S = A(BC)

$$\begin{aligned}
 &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \right) \\
 &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2+2 & 1+6 \\ -6+(-5) & -3+(-15) \end{bmatrix} \\
 &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 7 \\ -6-5 & -3-15 \end{bmatrix} \\
 &= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix} \\
 &= \begin{bmatrix} (-1 \times 4) + (3 \times -11) & (-1 \times 7) + (3 \times -18) \\ (2 \times 4) + (0 \times -11) & (2 \times 7) + (0 \times -18) \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} -4 + (-33) & -7 + (-54) \\ 8 + 0 & 14 + 0 \end{bmatrix} \\
 &= \begin{bmatrix} -4 - 33 & -7 - 54 \\ 8 & 14 \end{bmatrix} \\
 &= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}
 \end{aligned}$$

R.H.S = (AB)C

$$\begin{aligned}
 &= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \right) \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} -1 + (-9) & -2 + (-15) \\ 2 + 0 & -4 + 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} -1 - 9 & -2 - 15 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} (-10 \times 2) + (-17 \times 1) & (-10 \times 1) + (-17 \times 3) \\ (2 \times 2) + (4 \times 1) & (2 \times 1) + (4 \times 3) \end{bmatrix} \\
 &= \begin{bmatrix} -20 + (-17) & -10 + (-51) \\ 4 + 4 & 2 + 12 \end{bmatrix} \\
 &= \begin{bmatrix} -20 - 17 & -10 - 51 \\ 8 & 14 \end{bmatrix} \\
 &= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}
 \end{aligned}$$

Since

L.H.S = R.H.S $\Rightarrow A(BC) = (AB)C$

Hence proved

(iii) $A(B+C) = AB + AC$

Solution: $A(B+C) = AB + AC$

L.H.S = A(B+C)

R.H.S = AB + AC

L.H.S

L.H.S = A(B+C)

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 3) + (3 \times -2) & (-1 \times 3) + (3 \times -2) \\ (2 \times 3) + (0 \times -2) & (2 \times 3) + (0 \times -2) \end{bmatrix}$$

$$= \begin{bmatrix} -3 + (-6) & -3 + (-6) \\ 6 + 0 & 6 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 - 6 & -3 - 6 \\ 6 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

R.H.S = AB + AC

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$+ \begin{bmatrix} (-1 \times 2) + (3 \times 1) & (-1 \times 1) + (3 \times 3) \\ (2 \times 2) + (0 \times 1) & (2 \times 1) + (0 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 + (-3) & -2 + (-15) \\ 2 + 0 & 4 + 0 \end{bmatrix} + \begin{bmatrix} -2 + 3 & -1 + 9 \\ 4 + 0 & 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 3 & -2 - 15 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 + 1 & -17 + 8 \\ 2 + 4 & 4 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$$

Since LHS = RHS

A (B + C) = AB + AC

Hence proved

(iv) A(B - C) = AB - AC

Solution: A (B - C) = AB - AC

L.H.S = A (B - C)

R.H.S = AB - AC

L.H.S = A(B - C)

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1-2 & 2-1 \\ -3-1 & -5-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -4 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times -1) + (3 \times -4) & (-1 \times 1) + (3 \times -8) \\ (2 \times -1) + (0 \times -4) & (2 \times 1) + (0 \times -8) \end{bmatrix}$$

$$= \begin{bmatrix} +1 + (-12) & -1 + (-24) \\ -2 + 0 & 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 12 & -1 - 24 \\ -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix}$$

R.H.S = AB - AC

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$- \begin{bmatrix} (-1 \times 2) + (3 \times 1) & (-1 \times 1) + (3 \times 3) \\ (2 \times 2) + (0 \times 1) & (2 \times 1) + (0 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) - (3 \times 3) & (-1 \times 2) + (3 \times -5) \\ (2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$- \begin{bmatrix} (-1 \times 2) + (3 \times 1) & (-1 \times 1) + (3 \times 3) \\ (2 \times 2) + (0 \times 1) & (2 \times 1) + (0 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 - 9 & -2 - 15 \\ 2 + 0 & 4 + 0 \end{bmatrix} - \begin{bmatrix} -2 + 3 & -1 + 9 \\ 4 + 0 & 2 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 1 & -17 - 8 \\ 2 - 4 & 4 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix}$$

Since L.H.S = R.H.S

A (B - C) = AB - AC, Hence **proved**.

Q.6 For the matrices $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$

Solution: $(AB)^t = B^t A^t$

L.H.S = $(AB)^t$

R.H.S = $B^t A^t$

$$(AB) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times 1) + (3 \times -3) & (-1 \times 2) + (3 \times -5) \\ (+2 \times 1) + (0 \times -3) & (2 \times 2) + (0 \times -5) \end{bmatrix}$$

$$= \begin{bmatrix} -1 + (-9) & -2 + (-15) \\ 2 + 0 & 4 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}$$

L.H.S = $(AB)^t$

$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}^t$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$A^t = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

L.H.S = $B^t A^t$

$$= \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times -1) + (-3 \times 3) & (1 \times 2) + (-3 \times 0) \\ (2 \times -1) + (-5 \times 3) & (2 \times 2) + (-5 \times 0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 + (-9) & 2 + 0 \\ -2 + (-15) & 4 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 2 \\ -17 & 4 \end{bmatrix}$$

Since L.H.S = R.H.S

$(AB)^t = B^t A^t$

Hence proved

L.H.S = R.H.S

(ii) $(BC)^t = C^t B^t$

Solution: $(BC)^t = C^t B^t$

L.H.S = $(BC)^t$

R.H.S = $C^t B^t$

To solve L.H.S

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times -2) + (2 \times 3) & (1 \times 6) + (2 \times -9) \\ (-3 \times -2) + (-5 \times 3) & (-3 \times 6) + (-5 \times -9) \end{bmatrix}$$

$$= \begin{bmatrix} -2 + 6 & 6 + (-18) \\ 6 + (-15) & -18 + 45 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 - 18 \\ 6 - 15 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}$$

Taking transpose of BC:-

$$(BC)^t = \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$$

$$\text{L.H.S} = (BC)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

To solve R.H.S =

Taking transpose of matrix C

$$C^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}$$

Taking transpose of matrix B

$$B^t = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

Now, multiplying matrices, $B^t C^t$

$$\text{R.H.S} = C^t B^t = \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} (-2 \times 1) + (3 \times 2) & (-2 \times -3) + (3 \times -5) \\ (6 \times 1) + (-9 \times 2) & (6 \times -3) + (-9 \times -5) \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} -2+6 & 6+(-15) \\ 6+(-18) & -18+45 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 6-15 \\ 6-18 & 27 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \end{aligned}$$

Hence proved
L.H.S = R.H.S

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