

Exercise 3.3

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

Solution: $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

$$\log \frac{(22)^{\frac{1}{3}}}{5^3} = \log 22^{\frac{1}{3}} - \log 5^3$$

$$= \frac{1}{3} \log 22 - 3 \log 5 \text{ Ans}$$

Q.1 Write the following into sum or difference $\log(A \times B)$

(i) $\log(A \times B)$

Solution: $\log(A \times B)$

$$\log A \times B = \log A + \log B \text{ Ans}$$

(ii) $\log \frac{15.2}{30.5}$

Solution: $\log \frac{15.2}{30.5}$

$$\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5 \text{ Ans}$$

(iii) $\log \frac{21 \times 5}{8}$

Solution: $\log \frac{21 \times 5}{8}$

$$\log \frac{21 \times 5}{8} = \log(21 \times 5) - \log 8 \\ = \log 21 + \log 5 - \log 8 \text{ Ans}$$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

Solution: $\log \sqrt[3]{\frac{7}{15}}$

$$\log \sqrt[3]{\frac{7}{15}} = \log \left(\frac{7}{15} \right)^{\frac{1}{3}}$$

$$= \frac{1}{3} \log \left(\frac{7}{15} \right)$$

$$= \frac{1}{3} (\log 7 - \log 15)$$

$$= \frac{1}{3} \log 7 - \frac{1}{3} \log 15 \text{ Ans}$$

(vi) $\log \frac{25 \times 97}{29}$

Solution: $\log \frac{25 \times 97}{29}$

$$\log \frac{25 \times 47}{29} = \log(25 \times 47) - \log 29$$

$$= \log 25 + \log 47 - \log 29 \text{ Ans}$$

Q.2 Express

$\log x - 2 \log x + 3 \log(x+1) - \log(x^2 - 1)$ as a single logarithm.

Solution:

$$\begin{aligned} &\log x - 2 \log x + 3 \log(x+1) - \log(x^2 - 1) \\ &= \log x - \log x^2 + \log(x+1)^3 - \log(x^2 - 1) \end{aligned}$$

$$= \log \left(\frac{x}{x^2} \right) + \log \frac{(x+1)^3}{x^2 - 1}$$

$$= \log \left(\frac{x}{x^2} \times \frac{(x+1)^3}{x^2 - 1} \right)$$

$$= \log \left(\frac{x(x+1)^3}{x^2(x^2 - 1)} \right)$$

$$= \log \frac{x(x+1)^2 \cancel{(x+1)}}{x \times x(x-1) \cancel{(x+1)}}$$

$$= \log \frac{(x+1)^2}{x(x-1)} \text{ Ans}$$

Q.3 Write the following in the form of a single logarithm.

(i) $\log 21 + \log 5$

Solution: $\log 21 + \log 5$
 $= \log(21 \times 5)$ Ans

(ii) $\log 25 - 2 \log 3$

Solution: $\log 25 - 2 \log 3$
 $= \log 25 - 2 \log 3$
 $= \log 25 - \log 3^2$
 $= \log \frac{25}{3^2}$ Ans

(iii) $2 \log x - 3 \log y$

Solution: $2 \log x - 3 \log y$
 $= 2 \log x - 3 \log y$
 $= \log x^2 - \log y^3$
 $= \log \frac{x^2}{y^3}$ Ans

(iv) $\log 5 + \log 6 - \log 2$

Solution: $\log 5 + \log 6 - \log 2$
 $= \log 5 + \log 6 - \log 2$
 $= \log(5 \times 6) - \log 2$
 $= \log \frac{5 \times 6}{2}$ Ans

Q.4 Calculate the following.

(i) $\log_3 2 \times \log_2 81$

Solution: $\log_3 2 \times \log_2 81$

$$= \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$$

$$= \frac{\log 81}{\log 3}$$

$$= \frac{\log 3^4}{\log 3}$$

$$= \frac{4 \log 3}{\log 3}$$

$$= 4 \text{ Ans}$$

(ii) $\log_3 \times \log_3 25$

Solution: $\log_3 \times \log_3 25$
 $= \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$
 $= \frac{\log 25}{\log 5}$
 $= \frac{\log 5^2}{\log 5}$
 $= \frac{2 \log 5}{\log 5}$
 $= 2 \text{ Ans}$

Q.5 If $\log 2 = 0.3010$, $\log 3 = 0.4771$ and $\log 5 = 0.6990$, then find the values of the following.

(i) $\log 32$

$$\begin{aligned} &= \log 2^5 \\ &\because \text{using } 3^{rd} \text{ law of logarithm} \\ &= 5 \log 2 \\ &\text{By putting the value of } \log 2 \\ &= 5(0.3010) \\ &= 1.5050 \text{ Ans} \end{aligned}$$

(ii) $\log 24$

Solution: $\log 24$
 $= \log(2^3 \times 3)$
 $= \log 2^3 + \log 3$
 $= 3 \log 2 + \log 3$

By putting the value of $\log 2$ and $\log 3$

$$\begin{aligned} &= 3(0.3010) + 0.4771 \\ &= 0.9030 + 0.4771 \\ &= 1.3801 \text{ Ans} \end{aligned}$$

(iii) $\log \sqrt{3 \frac{1}{3}}$

Solution: $\log \sqrt{3 \frac{1}{3}}$
 $= \log \left(\frac{10}{3} \right)^{\frac{1}{2}}$

$$\begin{aligned}
 &= \frac{1}{2} \log \left[\frac{2 \times 5}{3} \right] \\
 &= \frac{1}{2} (\log 2 + \log 5 - \log 3)
 \end{aligned}$$

By putting the values of $\log 2, \log 3$ and $\log 5$

$$\begin{aligned}
 &= \frac{1}{2} (0.3010 + 0.69900 - 0.4771) \\
 &= \frac{1}{2} (1 - 0.4771) \\
 &= \frac{1}{2} (0.5229) \\
 &= 0.26145 \text{ Ans}
 \end{aligned}$$

(iv) $\log \frac{8}{3}$

Solution: $\log \frac{8}{3}$

$$\begin{aligned}
 &= \log \frac{2^3}{3} \\
 &= \log 2^3 - \log 3 \\
 &= 3 \log 2 - \log 3
 \end{aligned}$$

By putting the values of $\log 2$ and $\log 3$

$$\begin{aligned}
 &= 3(0.3010) - 0.4771 \\
 &= 0.9030 - 0.4771 \\
 &= 0.4259 \text{ Ans}
 \end{aligned}$$

(v) $\log 30$

Solution: $\log 30$

$$= \log (5 \times 2 \times 3)$$

\because using first law of logarithm

$$= \log 5 + \log 2 + \log 3$$

By putting the values of $\log 2, \log 3, \log 5$

$$\begin{aligned}
 &= (0.6990) + (0.3010) + (0.4771) \\
 &= 1.4771 \text{ Ans}
 \end{aligned}$$