Review Exercise 2

Q.1 Multiple choice questions. Choose the correct answer.

(i) $\left(27x^{-1}\right)^{-\frac{2}{3}}$ ____________

(a) $\frac{\sqrt[3]{x^2}}{9}$ **(b)** $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$ **(d)** $\frac{\sqrt{x^3}}{8}$

(ii) **Write** $\sqrt[7]{x}$ **in the exponential form** ___________

(a) x **(b)** x^7

(c) $x^{\frac{1}{7}}$ **(d)** $x^{\frac{7}{2}}$

(iii) **Write** $4^{\frac{2}{3}}$ **with radical sing** ___________

(a) $\sqrt[3]{4^2}$ **(b)** $\sqrt[2]{4^3}$

(c) $\sqrt[2]{4^3}$ **(d)** $\sqrt{4^6}$

(iv) **In** $\sqrt[3]{35}$ **the radicand is;**

(a) 3 **(b)** $\frac{1}{3}$

(c) 35 **(d)** None

(v) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} =$ ___________

(a) $\frac{5}{4}$ **(b)** $\frac{4}{5}$

(c) $-\frac{5}{4}$ **(d)** $-\frac{4}{5}$

(vi) **The conjugate of** $5+4i$ **is** ___________

(a) $-5+4i$ **(b)** $-5-4i$

(c) $5-4i$ **(d)** $5+4i$

(vii) **The value of** i^9 **is;**

(a) 1 **(b)** -1

(c) i **(d)** $-i$

(viii) **Every real number is __________**

(a) Positive integer (b) A rational number
(c) A negative integer (d) A complex number

(ix) **Real point of** $2ab\left(i+i^2\right)$ **is __________**

(a) $2ab$ (b) $-2ab$
(c) $2abi$ (d) $-2abi$

(x) **Imaginary part of** $-i(3i+2)$ **is __________**

(a) -2 (b) 2
(c) 3 (d) -3

(xi) **Which of the following sets have the closure property w.r.t addition __________**

(a) $\{0\}$ (b) $\{0,1\}$
(c) $\{0,1\}$ (d) $\left\{1,\sqrt{2},\frac{1}{2}\right\}$

(xii) **Name the property of real number used in** $\left[-\frac{\sqrt{5}}{2}\right]\times 1=-\frac{\sqrt{5}}{2}$ __________

(a) Additive identity (b) Additive inverse
(c) Multiplicative identity (d) Multiplicative inverse

(xiii) **If** $x,y,z\in R, z<0,$ **then** $x<y\Rightarrow> \ldots$

(a) $xz<yz$ (b) $xz>yz$
(c) $xz=yz$ (d) None of these

(xiv) **IF** $a,b\in\ \in\ R,$ **only one of** $a=b$ **or** $a<b$ or $a>b$ **hold is called __________**

(a) Trichotomy property (b) Transitive property
(c) Additive property (d) Multiplicative property

(xv) **A non-terminating, non-recurring decimal represents …**

(a) A natural number (b) A rational number
(c) An irrational number (d) A prime number

ANSWER KEY

i	**a**	**vi**	**c**	**xi**	**a**
ii	**c**	**vii**	**c**	**xii**	**c**
iii	**a**	**viii**	**d**	**xiii**	**b**
iv	**c**	**ix**	**b**	**xiv**	**a**
v	**b**	**x**	**a**	**xv**	**c**

Q.2 True or False? Identity

(i) Division is not an associative operation. **True**
(ii) Every whole number is a natural number. **False**
(iii) Multiplicative inverse of 0.02 is 50. **True**
(iv) π is rational number. **False**
(v) Every integer is a rational number. **True**
(vi) Subtraction is a commutative operation. **False**
(vii) Every real number is a rational number. **False**
(viii) Decimal representation of a rational number is either terminating or recurring. **True**
(ix) $1.\bar{8} = 1 + \frac{8}{9}$ **True**

Q.3 Simplify the following

(i) $\sqrt[4]{81y^{-12}x^{-8}}$

Solution:

$$= \left(3^4 y^{12} x^{-8}\right)^{\frac{1}{4}}$$

$$= 3^{4\times\frac{1}{4}} y^{-12^{3}\times\frac{1}{4}} x^{-8^{2}\times\frac{1}{4}}$$

$$= 3y^{-3}x^{-2}$$

$$\sqrt[4]{81y^{-12}x^{-8}} = \frac{3}{y^3x^2}$$ **Ans**

(ii) $\sqrt{25x^{10}y^{8m}}$

Solution:

$$= \sqrt{25x^{10n}y^{8m}}$$

$$= \left(5^2 x^{10n} y^{8m}\right)^{\frac{1}{2}}$$

$$= 5^{2\times\frac{1}{2}}.x^{10n^{5}\times\frac{1}{2}}.y^{8m^{4}\times\frac{1}{2}}$$

$$\sqrt{25x^{10}y^{8m}} = 5x^{5n}.y^{4m}$$ **Ans**

(iii) $\left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}}$

Solution:

$$= \left(x^{3+2}.y^{4+1}.z^{5+5}\right)^{\frac{1}{5}}$$

$$= \left(x^5y^5z^{10}\right)^{\frac{1}{5}}$$

$$= x^{5\times\frac{1}{5}} \times y^{5\times\frac{1}{5}} \times z^{10^{2}\times\frac{1}{5}}$$

$$\left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}} = x.y.z^2$$ **Ans**

(iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$

Solution:

$$= \left(\frac{2^5x^{-4}y^{-4}z}{5^4x^4yz^{-4}}\right)^{\frac{2}{5}}$$

$$= \left[\frac{2^5z^{1+4}}{5^4x^{4+6}\times y^{1+4}}\right]^{\frac{2}{5}}$$

$$= \left[\frac{2^5z^5}{5^4x^{10}y^5}\right]^{\frac{2}{5}}$$

$$= \frac{2^{5\times\frac{2}{5}} \times z^{5\times\frac{2}{5}}}{5^{4\times\frac{2}{5}} \times x^{10^{2}\times\frac{2}{5}} \times y^{5\times\frac{2}{5}}}$$

$$= \frac{2^2\times z^2}{5^{\frac{8}{5}}\times x^4\times y^2}$$

$$= \frac{4z^2}{5^{\frac{5}{5}+\frac{3}{5}}\times x^4y^2}$$

$$= \frac{4z^2}{5^{1+\frac{3}{5}}\times x^4y^2}$$

$$\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}} = \frac{4z^2}{5\times5^{\frac{3}{5}}x^4y^2}$$ **Ans**

Q.4 **Simplify** $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

Solution: $\sqrt{\dfrac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

$$=\sqrt{\dfrac{(6^3)^{\frac{2}{3}}\times(5^2)^{\frac{1}{2}}}{\left(\dfrac{4}{100}\right)^{-\frac{3}{2}}}}$$

$$=\sqrt{\dfrac{6^2\times5}{\left(\dfrac{100}{4}\right)^{\frac{3}{2}}}}$$

$$=\sqrt{\dfrac{6^2\times5}{(5^2)^{\frac{3}{2}}}}$$

$$=\sqrt{\dfrac{6^2\times5}{(5)^3}}$$

$$=\sqrt{\dfrac{6^2}{5^3\times5^{-1}}}$$

$$=\sqrt{\dfrac{6^2}{5^{3-1}}}$$

$$=\sqrt{\dfrac{6^2}{5^2}}$$

$$=\sqrt{\left(\dfrac{6}{5}\right)^2}$$

$$=\left(\dfrac{6}{5}\right)^{\not 2\times\frac{1}{\not 2}}$$

$$=\dfrac{6}{5}\ \textbf{Ans}$$

Q.5 $\left(\dfrac{a^p}{a^q}\right)^{p+q}\times\left(\dfrac{a^q}{a^r}\right)^{q+r}\div5\left(a^q a^r\right)^{p-r}$

Solution:

$$=\dfrac{\left(a^{p-q}\right)^{p+q}\left(a^{q-r}\right)^{q+r}}{5\left(a^{p+r}\right)^{p-r}}$$

$$=\dfrac{a^{(p-q)(p+q)}a^{(q-r)(q+r)}}{5a^{(p+r)(p-r)}}$$

$$=\dfrac{a^{p^2-q^2}a^{q^2-r^2}}{5a^{p^2-r^2}}$$

$$=\dfrac{a^{p^2-q^2+q^2-r^2}}{5a^{p^2-r^2}}$$

$$=\dfrac{a^{p^2-r^2-p^2+r^2}}{5}$$

$$=\dfrac{a^0}{5}$$

$$\left(\dfrac{a^p}{a^q}\right)^{p+q}\times\left(\dfrac{a^q}{a^r}\right)^{q+r}\div5\left(a^q a^r\right)^{p-r}$$

$$=\dfrac{1}{5}\ \textbf{Ans}$$

Q.6 **Simplify** $\left(\dfrac{a^{2l}}{a^{l+m}}\right)\left(\dfrac{a^{2m}}{a^{m+n}}\right)\left(\dfrac{a^{2n}}{a^{n+2}}\right)$

Solution:

$$=a^{2l-l-m}a^{2m-m-n}a^{2n-n-n}$$

$$=a^{l-m}a^{m-n}a^{n-l}$$

$$=a^{l-m+m-n+n-l}$$

$$=a^0$$

$$\left(\dfrac{a^{2l}}{a^{l+m}}\right)\left(\dfrac{a^{2m}}{a^{m+n}}\right)\left(\dfrac{a^{2n}}{a^{n+2}}\right)=1\ \textbf{Ans}$$

Q.7 **Simplify** $\sqrt[3]{\dfrac{a^l}{a^m}}\times\sqrt[3]{\dfrac{a^m}{a^n}}\times\sqrt[3]{\dfrac{a^n}{a^r}}$

Solution:

$$=\sqrt[3]{a^{l-m}}\sqrt[3]{a^{m-n}}\sqrt[3]{a^{n-l}}$$

$$=\left(a^{l-m}\right)^{\frac{1}{3}}\times\left(a^{m-n}\right)^{\frac{1}{3}}\times\left(a^{n-l}\right)^{\frac{1}{3}}$$

$$=a^{\frac{l-m}{3}}\times a^{\frac{m-n}{3}}\times a^{\frac{n-l}{3}}$$

$$=a^{\frac{l-m}{3}+\frac{m-n}{3}+\frac{n-l}{3}}$$

$$=a^{\frac{l-m+m-n+n-l}{3}}$$

$$=a^{\frac{0}{3}}$$

$$=a^0$$

$$\sqrt[3]{\dfrac{a^l}{a^m}}\times\sqrt[3]{\dfrac{a^m}{a^n}}\times\sqrt[3]{\dfrac{a^n}{a^r}}=1\ \textbf{Ans}$$

Unit 2: Real and Complex Numbers

Overview

Natural Numbers:

The numbers $1, 2, 3, \ldots$ which we use for counting certain objects are called natural numbers or positive integers. The set natural numbers is denoted by N.

i.e. $N = \{1, 2, 3, \ldots\}$

Whole Numbers:

If we include 0 in the set of natural number, the resulting set is the set of whole numbers, denoted by W,

i.e. $W = \{0, 1, 2, 3, \ldots\}$

Integers:

The set of integers consist of positive integers, 0 and negative integers and is denoted by Z i.e. $Z\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational Numbers:

All numbers of the form $\frac{p}{q}$ where p, q are integers and q is not zero are called rational numbers. The set of rational numbers is denoted by Q,

i.e. $Q = \left\{\frac{p}{q} \mid p, q \in Z \wedge q \neq 0, (p, q) = 1\right\}$

Irrational Numbers:

The numbers which cannot be expressed as quotient of integers are called irrational numbers. The set of irrational numbers is denoted by Q',

i.e. $Q' = \left\{x \mid x \neq \frac{p}{q}, p, q \in Z \wedge q \neq 0\right\}$

The union of the set of rational numbers and irrational numbers is known as the set of real numbers. It is denoted by R,

i.e. $R = Q \cup Q'$

Types of Rational Numbers:

(i) Terminating Decimal Fractions

The decimal fraction in which there are finite number of digits in its decimal part is called a terminating decimal fraction. For example $\frac{2}{5} = 0.4$ and $\frac{3}{8} = 0.375$.

(ii) Recurring and Non-terminating Decimal Fractions:

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called recurring decimal fraction.

For example $\frac{2}{9} = 0.2222...$ and $\frac{4}{11} = 0.363636...$

Concept of Radicals and Radicands:

In the radical $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called the radical sign, n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Base and Exponent:

In the exponential notation of (read as a to the nth power) we call 'a' as the base and 'n' as the exponent or the power t which the base is raised.

Definition of a Complex Number:

A number of the form $z = a + bi$ where a and b are real numbers and $i - \sqrt{-1}$, is called a complex number and is represented by z i.e., $z = a + ib$

Conjugate of a Complex Number:

If we change i to $-i$ in $z = a + bi$, we obtain another complex number $a - bi$ called the complex conjugate of z and is denoted by $\bar{z}$ (read z bar).