Exercise 2.6

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Q.1 Identify the following statement as true or false.

(i) $\sqrt{-3}\sqrt{-3}=3$ **False**

(ii) $i^{73}=-i$ **False**

(iii) $i^{10}=-1$ **True**

(iv) Complex conjugate of $\left(-6i+i^2\right)$ is $\left(-1+6i\right)$ **True**

(v) Difference of a complex number $z=a+bi$ and its conjugate is a real number. **False**

(vi) If $(a-1)-(b+3)i=5+8i$, then $a=6$ and $b=-11$. **True**

(vii) Product of a complex number and its conjugate is always a non-negative real number. **True**

Q.2 Express the each complex number in the standard form $a+bi$, where a and b are real number.

(i) $(2+3i)+(7-2i)$

Solution:

$=2+3i+7+2i$

$=2+7+3i-2i$

$=9+i$ **Ans**

(ii) $2(5+4i)-3(7+4i)$

Solution: $2(5+4i)-3(7+4i)$

$=10+8i-21-12i$

$=10-21+8i-12i$

$=-11-4i$ **Ans**

(iii) $(-3+5i)-(4+9i)$

Solution: $(-3+5i)-(4+9i)$

$=+3-5i-4-9i$

$=3-4-5i-9i$

$=-1-14i$ **Ans**

(iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

Solution: $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

$=2(-1)+6i^2.i+3(i^2)^8-6(i^2)^9.i+4(i^2)^{12}.i$

$=-2+6(-1)i+3(-1)^8-6(-1).i+4(-1)^{12}.i$

$=-2-6i+3-6(-1)i+4(+1)i$

$=1-\cancel{6i}+\cancel{6i}+4i$

$=1+4i$ **Ans**

Q.3 Simplify and write your answer in the form $a+bi$

(i) $(-7+3i)(-3+2i)$

Solution: $(-7+3i)(-3+2i)$

$=-7(-3+2i)+3i(-3+2i)$

$=21-14i-9i+6i^2$

$=21-23i+6(-1)$

$=21-23i-6$

$=21-6-23i$

$=15-23i$ **Ans**

(ii) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

Solution: $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

$$=\left(2-\sqrt{4\times -1}\right)\left(3-\sqrt{4\times -1}\right)$$
$$=\left(2-\sqrt{4i^2}\right)\left(3-\sqrt{4i^2}\right)$$
$$=(2-2i)(3-2i)$$
$$=2(3-2i)-2i(3-2i)$$
$$=6-4i-6i+4i^2$$
$$=6-10i+4(-1)$$
$$=6-10i-4$$
$$=2-10i \text{ Ans}$$

(iii) $\left(\sqrt{5}-3i\right)^2$

Solution: $\left(\sqrt{5}-3i\right)^2$

$$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$$
$$=5+9i^2-6\sqrt{5}i$$
$$=5+9(-1)-6\sqrt{5}i$$
$$=5-9-6\sqrt{5}i$$
$$=-4-6\sqrt{5}i \text{ Ans}$$

(iv) $(2-3i)\left(\overline{3-2i}\right)$

Solution: $(2-3i)\left(\overline{3-2i}\right)$

$$=(2-3i)(3+2i)$$
$$=2(3+2i)-3i(3+2i)$$
$$=6+4i-9i-6i^2$$
$$=6-5i-6(-1)$$
$$=6-5i+6$$
$$=6+6-5i$$
$$=12-5i \text{ Ans}$$

Q.4 **Simplify and write your answer in the form** $a+bi$.

(i) $\dfrac{-2}{1+i}$

Solution: $\dfrac{-2}{1+i}$

$$=\frac{-2}{1+i}\times\frac{1-i}{1-i}$$
$$=\frac{-2(1-i)}{(1)^2-(i)^2}$$
$$=\frac{-2+2i}{1-i^2}$$
$$=\frac{-2+2i}{1-(-1)}$$
$$=\frac{-2+2i}{1+1}$$
$$=\frac{-2+2i}{2}$$
$$=-\frac{2}{2}+\frac{2i}{2}$$
$$=-1+i \text{ Ans}$$

(ii) $\dfrac{2+3i}{4-i}$

Solution: $\dfrac{2+3i}{4-i}$

$$=\frac{2+3i}{4-i}\times\frac{4+i}{4+i}$$
$$=\frac{(2+3i)(4+i)}{(4)^2-(i)^2}$$
$$=\frac{2(4+i)+3i(4+i)}{16-(-1)}$$
$$=\frac{8+2i+12i+3i^2}{16+1}$$
$$=\frac{8+4i+3(-1)}{17}$$
$$=\frac{8+14i-3}{17}$$
$$=\frac{8-3+14i}{17}$$
$$=\frac{5+14i}{17}$$
$$=\frac{5}{17}+\frac{14}{17}i \text{ Ans}$$

(iii) $\dfrac{9-7i}{3+i}$

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Solution: $\dfrac{9-7i}{3+i}$

$$=\frac{9-7i}{3+i}\times\frac{3-i}{3-i}$$

$$=\frac{(9-7i)(3-i)}{(3)^2-(i)^2}$$

$$=\frac{9(3-i)-7i(3-i)}{9-(-1)}$$

$$=\frac{27-9i-21i+7i^2}{9+1}$$

$$=\frac{27-30i+7(-1)}{10}$$

$$=\frac{27-30i-7}{10}$$

$$=\frac{27-7-30i}{10}$$

$$=\frac{20-30i}{10}$$

$$=\frac{20}{10}-\frac{30i}{10}$$

$=2-3i$ **Ans**

(iv) $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

Solution: $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

$$=\frac{2-6i-(4+i)}{3+i}$$

$$=\frac{2-6i-4-i}{3+i}$$

$$=\frac{2-4-6i-i}{3+i}$$

$$=\frac{-2-7i}{3+i}$$

$$=\frac{-2-7i}{3+i}\times\frac{3-i}{3-i}$$

$$=\frac{-2-7i}{3+i}\times\frac{3-i}{3-i}$$

$$=\frac{-2(3-i)-7i(3-i)}{(3)^2-(i)^2}$$

$$=\frac{-6+2i-21i+7i^2}{9-(-1)}$$

$$=\frac{-6-19i+7(-1)}{9+1}$$

$$=\frac{-6-19i-7}{10}$$

$$=\frac{-6-7-19i}{10}$$

$$=\frac{-13-19i}{10}$$

$=\dfrac{-13}{10}-\dfrac{19i}{10}$ **Ans**

(v) $\left[\dfrac{1+i}{1-i}\right]^2$

Solution: $\left[\dfrac{1+i}{1-i}\right]^2$

$$=\frac{(1+i)^2}{(1-i)^2}$$

$$=\frac{(1)^2+(i)^2+2ab}{(1)^2+(i)^2-2ab}$$

$$=\frac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$$

$$=\frac{1+(-1)+2i}{1+(-1)-2i}$$

$$=\frac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$$

$$=\frac{\cancel{2i}}{-\cancel{2i}}=-1$$

$=-1$

$=-1+0i$ **Ans**

(vi) $\dfrac{1}{(2+3i)(1-i)}$

Solution: $\dfrac{1}{(2+3i)(1-i)}$

$$=\frac{1}{2(1-i)+3i(1-i)}$$

$$=\frac{1}{2-2i+3i-3i^2}$$

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$$= \frac{1}{2+i-3(-1)}$$

$$= \frac{1}{2+i+3}$$

$$= \frac{1}{2+3+i}$$

$$= \frac{1}{5+i}$$

$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{1(5-i)}{(5)^2-(i)^2}$$

$$= \frac{5-i}{25-(-1)}$$

$$= \frac{5-i}{25+1}$$

$$= \frac{5-i}{26}$$

$$= \frac{5}{26} - \frac{1i}{26} \text{ Ans}$$

Q.5 Calculate $(a)\bar{z}\,(b)\,z+\bar{z}\,(c)\,z-\bar{z}\,(d)\,z\bar{z}$ **for each of the following.**

(i) $z = -i$

Solution: $z = -i$

(a) $\bar{z} = +i$

(b) $z + \bar{z} = -i + i$

$= 0$

(c) $z - \bar{z} = (-i) - (i)$

$= -2i$

(d) $z\bar{z} = (-i)(i)$

$= -i^2$

$= -(-1)$

$= 1$ **Ans**

(ii) $z = 2+i$

Solution: $z = 2+i$

$z + 2i$

(a) $\bar{z} = 2-i$

(b) $z + \bar{z} = (2+i)+(2-i)$

$= 2 + \not{i} + 2 - \not{i}$

$= 2+2$

$= 4$

(c) $z - \bar{z} = (2+i)-(2-i)$

$= \not{2} + i - \not{2} + i$

$= i + i$

$= 2i$

(d) $z\bar{z} = (2+i)(2-i)$

$= (2)^2 - (i)^2$

$= 4 - i^2$

$= 4 - (-1)$

$= 4 + 1$

$= 5$ **Ans**

(iii) $z = \frac{1+i}{1-i}$

Solution: $z = \frac{1+i}{1-i}$

$$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{1(1+i)+i(1+i)}{(1-i)(1+i)}$$

$$= \frac{1+i+i+(-1)}{(1)^2-(i)^2}$$

$$= \frac{1+2i+(-1)}{1-(-1)}$$

$$= \frac{\not{1}+2i-\not{1}}{1+1}$$

$$= \frac{\not{2}i}{\not{2}}$$

$= i$

$z = i$

(a) $\bar{z} = -i$

(b) $z + \bar{z} = i + (-i)$

$= \not{i} - \not{i}$

$= 0$

(c) $z - \bar{z} = i - (-i)$

$= i + i$

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$= 2i$

(d) $z\bar{z} = (i)(-i)$

$= -i^2$

$= -(-1)$

$= +1$ **Ans**

(iv) $z = \dfrac{4-3i}{2+4i}$

Solution: $z = \dfrac{4-3i}{2+4i}$

$z = \dfrac{4-3i}{2+4i} \times \dfrac{2-4i}{2-4i}$

$= \dfrac{4(2-4i)-3i(2-4i)}{(2+4i)(2-4i)}$

$= \dfrac{8-16i-6i+12i^2}{(2)^2-(4i)^2}$

$= \dfrac{8-22i+12(-1)}{4-16i^2}$

$= \dfrac{8-22i-12}{4-16(-1)}$

$= \dfrac{8-12-22i}{4+16}$

$= \dfrac{-4-22i}{20}$

$= \dfrac{-4}{20} - \dfrac{22}{20}i$

$= -\dfrac{1}{5} - \dfrac{11}{10}i$

(a) $\bar{z} = \dfrac{-1}{5} + \dfrac{11}{10}i$

(b)

$z + \bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right) + \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\dfrac{1}{5} - \dfrac{11}{10}i - \dfrac{1}{5} + \dfrac{11}{10}i$

$= -\dfrac{1}{5} - \dfrac{1}{5}$

$= \dfrac{-1-1}{5}$

$= -\dfrac{2}{5}$

(c)

$z - \bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right) - \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\dfrac{1}{5} - \dfrac{11}{10}i + \dfrac{1}{5} - \dfrac{11}{10}i$

$= -\dfrac{11}{10}i - \dfrac{11}{10}i = \dfrac{-11i-11i}{10}$

$= -\dfrac{22^{11}i}{10^{5}}$

$= -\dfrac{11}{5}i$

(d) $z\bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right)\left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= \left(-\dfrac{1}{5}\right)^2 - \left(\dfrac{11}{10}i\right)^2$

$= \dfrac{1}{25} - \dfrac{121}{100}i^2$

$= \dfrac{1}{25} - \dfrac{121}{100}(-1)$

$= \dfrac{1}{25} + \dfrac{121}{100}$

$= \dfrac{4+121}{100}$

$= \dfrac{125}{100}$

$= \dfrac{5}{4}$ **Ans**

Q.6 If $z = 2+3i$ and show that.

(i) $\overline{z+w} = \bar{z} + \bar{w}$

Solution: $\overline{z+w} = \bar{z} + \bar{w}$

$z + w = 2+3i+5-4i$

$= 2+5+3i-4i$

$= 7-i$

L.H.S $= \overline{z+w}$

$= \overline{7-i}$

$= 7+i$ ... (i)

R. H. S $= \bar{z} + \bar{w}$

$= (\overline{2+3i}) + (\overline{5-4i})$

$= 2-3i+5+4i$

$= 2+5-3i+4i$

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$= 7 + i$ ... (ii)

From (i) and (ii) we get
L.H.S=R.H.S

$\overline{z+w} = \overline{z} + \overline{w}$

Hence proved

(ii) $\overline{z-w} = \overline{z} - \overline{w}$

Solution: $\overline{z-w} = \overline{z} - \overline{w}$

$z - w = (2+3i) - (5-4i)$

$= 2 + 3i - 5 + 4i$

$= 2 - 5 + 3i + 4i$

$= -3 + 7i$

L.H.S $= \overline{z-w}$

$= \overline{-3+7i}$

$= -3 - 7i$ ...(i)

R.H.S $= \overline{z} - \overline{w}$

$= \left(\overline{2+3i}\right) - \left(\overline{5-4i}\right)$

$= 2 + 3i - (5 + 4i)$

$= 2 - 3i - 5 - 4i$

$= -3 - 7i$

From (i) and (ii) we get
L.H.S=R.H.S

$\overline{z-w} = \overline{z-w}$

Hence proved

(iii) $\overline{zw} = \overline{z}\,\overline{w}$

Solutions: $\overline{zw} = \overline{z}\,\overline{w}$

$zw = (2+3i)(5+4i)$

$= 2(5-4i) + 3i(5-4i)$

$= 10 - 8i + 15i - 12i^2$

$= 10 + 7i - 12(-1)$

$= 10 + 7i + 12$

$= 22 + 7i$

L.H.S $= \overline{zw}$

$= \overline{22+7i}$

$= 22 - 7i$

R.H.S $= \overline{zw}$

$= \left(\overline{2+3i}\right)\left(\overline{5-4i}\right)$

$= (2-3i)(5+4i)$

$= 2(5+4i) - 3i(5+4i)$

$= 10 + 8i - 15i - 12i^2$

$= 10 - 7i - 12(-1)$

$= 10 - 7i + 12$

$= 22 - 7i$

From (i) and (ii) we get
L.H.S=R.H.S

$\overline{zw} = \overline{z}\overline{w}$

Hence proved

(iv) $\overline{\left[\frac{z}{w}\right]} = \frac{\overline{z}}{\overline{w}}$, where $\neq 0$

Solutions: $\overline{\left[\frac{z}{w}\right]} = \frac{\overline{z}}{\overline{w}}$

$\frac{z}{w} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i}$

$= \frac{2(5+4i) + 3i(5+4i)}{(5-4i)(5+4i)}$

$= \frac{10 + 8i + 15i + 12i^2}{(5)^2 - (4i)^2}$

$= \frac{10 + 23i + 12(-1)}{25 - 16i^2}$

$= \frac{10 + 23i - 12}{25 - (6(-1))}$

$= \frac{10 + 23i - 12}{25 + 16}$

$= \frac{-2 + 23i}{41}$

L.H.S= $\left(\overline{\frac{z}{w}}\right)$

$= \left(\overline{\frac{-2+23i}{41}}\right)$

$= \frac{-2}{41} - \frac{23}{41}i$ ... (i)

R.H.S $= \frac{\overline{z}}{\overline{w}}$

$= \frac{\left(\overline{2+3i}\right)}{\left(\overline{5-4i}\right)}$

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$$=\frac{2-3i}{5+4i}$$
$$=\frac{2-3i}{5+4i}\times\frac{5-4i}{5-4i}$$
$$=\frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)}$$
$$=\frac{10-8i-15i+12i^2}{(5)^2-(4i)^2}$$
$$=\frac{10-23i+12(-1)}{25-16i^2}$$
$$=\frac{10-23i+12(-1)}{25-16(-1)}$$
$$=\frac{10-23i-12}{25+16}$$
$$=\frac{-2-23i}{41}$$
$$=\frac{-2}{41}-\frac{23}{41}i \quad \ldots \text{(ii)}$$

From (i) and (ii) we get
L.H.S=R.H.S
Hence Proved

$$\overline{\left[\frac{z}{w}\right]}=\frac{\bar{z}}{\bar{w}}$$

(v) $\frac{1}{2}(z+\bar{z})$ **is the real part of** z **.**

Solution: $\frac{1}{2}(z+\bar{z})$

$$=\frac{1}{2}\left[(2+3i)+\left(\overline{2+3i}\right)\right]$$
$$=\frac{1}{2}[(2+3i)+(2-3i)]$$
$$=\frac{1}{2}[2+\cancel{3i}+2-\cancel{3i}]$$
$$=\frac{1}{2}[2+2]$$
$$=\frac{1}{\cancel{2}}\left[\cancel{4}^{2}\right]$$
$$=2=\operatorname{Re}(z)$$

$\frac{1}{2}(z+\bar{z})$ is the real part of z . **Ans**

(vi) $\frac{1}{2}(z-\bar{z})$ **is the imaginary part of** z **.**

Solution: $\frac{1}{2}(z-\bar{z})$

$$\frac{1}{2}(z-\bar{z})=$$
$$=\frac{1}{2}\left[(2+3i)-\left(\overline{2+3i}\right)\right]$$
$$=\frac{1}{2}[(2+3i)-(2-3i)]$$
$$=\frac{1}{2}[\cancel{2}+3i-\cancel{2}+3i]$$
$$=\frac{1}{\cancel{2}}\left[{}^{3}\cancel{6}i\right]$$
$$=3i$$
$$=\text{Imaginary}(z)$$

$\frac{1}{2}(z-\bar{z})$ is the imaginary part of z . **Ans**

Q.7 Solve the following equations for real x **and** y **.**

(i) $(2-3i)(x+yi)=4+i$

Solution: $(2-3i)(x+yi)=4+i$

$$x+yi=\frac{4+i}{2-3i}$$
$$x+yi=\frac{4+i}{2-3i}\times\frac{2+3i}{2+3i}$$
$$=\frac{4(2+3i)+i(2+3i)}{(2-3i)(2+3i)}$$
$$=\frac{8+12i+2i+3i^2}{(2)^2-(3i)^2}$$
$$=\frac{8+14i+3(-1)}{4-9i^2}$$
$$=\frac{8+14i-3}{4-9(-1)}$$
$$=\frac{8-3+14i}{4+9}$$

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$$= \frac{5+14i}{13}$$

$$x + yi = \frac{5}{13} + \frac{14}{13}i$$

$$x = \frac{5}{13}, y = \frac{14}{13}$$ **Ans**

(ii) $(3-2i)(x+yi) = 2(x-2yi)+2i-1$

Solution:

$$(3-2i)(x+yi) = 2(x-2yi)+2i-1$$

$$3(x+yi)-2i(x+yi) = 2x-4yi+2i-1$$

$$3x+3yi-2xi-2yi^2 = (2x-1)+i(2-4y)$$

$$3x+(3x-2x)i-2y(-1) = (2x-1)+i(2-4y)$$

$$3x+(3y-2x)i+2y = (2x-1)+i(2-4)$$

$$(3x+2y)+(3y-2x)i = (2x-1)+(2-4y)i$$

Comparing the real and imaginary parts.

$$3x+2y = 2x-1 \quad ,$$

$$3y-2x = 2-4y$$

$$3x-2x+2y = -1 \quad ,$$

$$3y-2x = 2-4y$$

$$x+2y = -1 \quad ,$$

$$-2x+3y+4y = 2$$

$$-2x+7y = 2$$

$x+2y=-1$ ____________ (i)

$-2x+7y=2$ ____________ (ii)

Multiply equation (i) with (2)

$$2(x+2y) = -1\times 2$$

$2x+4y=-2$ ____________ (iii)

$$\cancel{2x}+4y = \cancel{-2}$$

$$\cancel{-2x}+7y = \cancel{2}$$

$$11y = 0$$

$$y = \frac{0}{11}$$

$$y = 0$$

Putting $y=0$ in equation (i)

$$x+2y = -1$$

$$x+2(0) = -1$$

$$x+0 = -1$$

$$x = -1+0$$

$x=-1$ **Ans**

(iii) $(3+4i)^2 - 2(x-yi) = x+yi$

Solution: $(3+4i)^2 - 2(x-yi) = x+yi$

$$(3+4i)^2 - 2(x-yi) = x+yi$$

$$9+24i+16i^2-2x+2yi = x+yi$$

$$9+24i+16(-1)-2x+2yi = x+yi$$

$$9+24i-16-2x+2yi = x+yi$$

$$9+24i-16-2x = x+2yi-yi = 0$$

$$9+24i-16-3x+yi = 0$$

$$-3x+yi = -9-24i+16$$

$$-3x+yi = 16-9-24i$$

$$-3x+yi = 7-24i$$

Comparing the real and imaginary parts.

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$-3x = 7$

$y = -24$

$x = \frac{-7}{3}$

$y = -24$ **Ans**

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