

## Exercise 2.4

**Q.1** Use laws of exponents to simplify.

$$(i) \quad \frac{(243)^{\frac{2}{3}} (32)^{\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

$$\begin{aligned} \text{Solution:} & \frac{(243)^{\frac{2}{3}} (32)^{\frac{1}{5}}}{\sqrt{(196)^{-1}}} \\ &= \frac{(243)^{\frac{2}{3}} (32)^{\frac{1}{5}}}{\sqrt{(196)^{-1}}} \\ &= \frac{(3^5)^{\frac{2}{3}} \times (2^5)^{\frac{1}{5}}}{\sqrt{[(14)^2]^{-1}}} \\ &= \frac{(3)^{\frac{10}{3}} \times 2^{-1}}{\sqrt{[(14)^{-1}]^2}} \\ &= \frac{(3)^{\frac{10}{3}} \times 2^{-1}}{(14)^{-1}} \\ &= \frac{14}{(3)^{\frac{10}{3}} \times 2} \\ &= \frac{7}{\frac{10}{3^3}} \\ &= \frac{7}{\sqrt[3]{3^{10}}} \\ &= \frac{7}{\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}} \\ &= \frac{7}{\sqrt[3]{3^3 \times 3^3 \times 3^3 \times 3}} \\ &= \frac{7}{\sqrt[3]{3^3 \times 3^3 \times 3^3 \times 3^3}} \end{aligned}$$

$$\begin{aligned} &= \frac{7}{3 \times 3 \times 3 \times \sqrt[3]{3}} \\ &= \frac{7}{27\sqrt[3]{3}} \text{ Ans} \end{aligned}$$

$$(ii) \quad (2x^5y^{-4})(-8x^{-3}y^2)$$

$$\begin{aligned} \text{Solution:} & (2x^5y^{-4})(-8x^{-3}y^2) \\ &= -16x^{5-3}y^{-4+2} \\ &= -16x^2y^{-2} \\ &= \frac{-16x^2}{y^2} \text{ Ans} \end{aligned}$$

$$(iii) \quad \left[ \frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right]^{-3}$$

$$\begin{aligned} \text{Solution:} & \left[ \frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0} \right]^{-3} \\ &= [x^{-2-4}y^{-1+3}z^{-4-0}]^{-3} \\ &= (x^{-6}y^2z^{-4})^{-3} \\ &= (x^{-6})^{-3} (y^2)^{-3} (y^{-4})^{-3} \\ &= x^{18}y^{-6}z^{12} \\ &= \frac{x^{18}z^{12}}{y^6} \text{ Ans} \end{aligned}$$

$$(iv) \quad \frac{(81)^n \cdot 3^5 - (3)^{4n-1} (243)}{(9^{2n})(3^3)}$$

$$\begin{aligned} \text{Solution:} & \frac{(81)^n \cdot 3^5 - (3)^{4n-1} (243)}{(9^{2n})(3^3)} \\ &= \frac{(3^4)^n \cdot 3^5 - 3^{4n} \cdot 3^{-1} \cdot 3^5}{(3^2)^{2n} \cdot 3^3} \\ &= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^{-1+5}}{3^{4n} \cdot 3^3} \\ &= \frac{3^{4n} \cdot 3^5 - 3^{4n} \cdot 3^4}{3^{4n} \cdot 3^3} \end{aligned}$$

$$\begin{aligned}
 &= \frac{3^{4n} \cdot 3^4 (3-1)}{3^{4n} \cdot 3^3} \\
 &= 3^{4n-4n} \cdot 3^{4-3} \cdot (2) \\
 &= 3^0 \cdot 3^1 \cdot 2 \\
 &= 1 \times 3 \times 2 \\
 &= 6 \text{ Ans}
 \end{aligned}$$

**Q.2 Show that**

$$\left[ \frac{x^a}{x^b} \right]^{a+b} \times \left[ \frac{x^b}{x^c} \right]^{b+c} \times \left[ \frac{x^c}{x^a} \right]^{c+a} = 1$$

**Proof:**

L.H.S

$$\begin{aligned}
 &= \left[ \frac{x^a}{x^b} \right]^{a+b} \times \left[ \frac{x^b}{x^c} \right]^{b+c} \times \left[ \frac{x^c}{x^a} \right]^{c+a} \\
 &= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \\
 &= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} \\
 &= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} \\
 &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\
 &= x^0 \\
 &= 1 \\
 &1 = \text{R.H.S Ans}
 \end{aligned}$$

**Q.3 Simplify**

$$\text{(i)} \quad \frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$\text{Solution:} \quad \frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$\begin{aligned}
 &= \frac{2^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}} \times (2 \times 2 \times 3 \times 5)^{\frac{1}{2}}}{(2 \times 2 \times 3 \times 3 \times 5)^{\frac{1}{2}} \times (2^2)^{\frac{1}{3}} \times (3^2)^{\frac{1}{4}}} \\
 &= \frac{2^{\frac{1}{3}} \times 3 \times (2^2)^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{(2^2)^{\frac{1}{2}} \times (3^2)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} \times 2^{\frac{2}{3}} \times 3^{\frac{1}{2}}} \\
 &= \frac{2^{\frac{1}{3}} \times 3 \times 2 \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2 \times 3 \times 5^{\frac{1}{2}} \times 2^{\frac{2}{3}} \times 3^{\frac{1}{2}}}
 \end{aligned}$$

$$\begin{aligned}
 &= 2^{\frac{1}{3}} \times 2^{+1} \times 2^{-1} \times 2^{\frac{+2}{3}} \times 3^1 \times 3^{\frac{1}{2}} \times 3^{-1} \times 3^{\frac{-1}{2}} \times 5^{\frac{1}{2}} \times 5^{\frac{-1}{2}} \\
 &= 2^{\frac{1}{3} + 1 - 1 + \frac{2}{3}} \times 3^{1 + \frac{1}{2} - 1 - \frac{1}{2}} \times 5^{\frac{1}{2} - \frac{1}{2}} \\
 &= 2^{\frac{1}{3} + \frac{2}{3}} \times 3^0 \times 5^0 \\
 &= 2^{\frac{1+2}{3}} \times 1 \times 1 \\
 &= 2^{\frac{3}{3}} \\
 &= 2 \text{ Ans}
 \end{aligned}$$

$$\text{(ii)} \quad \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$$

$$\text{Solution:} \quad \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{\left(\frac{25 \times 100}{4}\right)^{\frac{1}{2}}}}$$

$$= \frac{6^2 \times 5}{(25)^{\frac{1}{2}}}$$

$$= \frac{6^2 \times 5}{(5^2)^{\frac{1}{2}}}$$

$$= \sqrt{\frac{6^2 \times 5}{5}}$$

$$= \sqrt{6^2 \times 5^{+1} \times 5^{-1}}$$

$$= \sqrt{6^2 \times 5^{+1-1}}$$

$$= \sqrt{6^2 \times 5^0}$$

$$= \sqrt{6^2 \times 1}$$

$$= \sqrt{6^2}$$

$$= 6 \text{ Ans}$$

(iii)  $5^{2^3} \div (5^2)^3$

**Solution:**  $5^{2^3} \div (5^2)^3$

$$= 5^8 \div 5^6$$

$$= 5^{8-6}$$

$$= 5^2$$

$$= 25 \text{ Ans}$$

(iv)  $(x^3)^2 \div x^{3^2}, x \neq 0$

**Solution:**  $(x^3)^2 \div x^{3^2}, x \neq 0$

$$= x^6 \div x^9$$

$$= x^{6-9}$$

$$= x^{-3}$$

$$= \frac{1}{x^3} \text{ Ans}$$

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