

Review Exercise 4

Q.1 Multiple type questions?

(i) is an algebraic ...

(a) Expression

(c) Equation

(b) Sentence

(d) In-equation

(ii) The degree of polynomial $4x^4 + 3x^2y$ is

(a) 1

(c) 3

(b) 2

(d) 4

(iii) $a^3 + b^3$ is equal to

(a) $(a-b)(a^2 + ab + b^2)$

(c) $(a-b)(a^2 - ab + b^2)$

(b) $(a+b)(a^2 - ab + b^2)$

(d) $(a-b)(a^2 + ab + b^2)$

(iv) $(3 + \sqrt{2})(3 - \sqrt{2})$ is equal to

(a) 7

(c) -1

(b) -7

(d) 1

(v) Conjugate of surd $a + \sqrt{b}$ is;

(a) $-a + \sqrt{b}$

(c) $\sqrt{a} + \sqrt{b}$

(b) $a - \sqrt{b}$

(d) $\sqrt{a} - \sqrt{b}$

(vi) $\frac{1}{a-b} - \frac{1}{a+b}$ is equal to

(a) $\frac{2a}{a^2 - b^2}$

(c) $\frac{-2a}{a^2 - b^2}$

(b) $\frac{2b}{a^2 - b^2}$

(d) $\frac{-2b}{a^2 - b^2}$

(vii) $\frac{a^2 - b^2}{a+b}$ is equal to

(a) $(a-b)^2$

(c) $a+b$

(b) $(a+b)^2$

(d) $a-b$

(viii) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ is equal to

(a) $a^2 + b^2$

(c) $a-b$

(b) $a^2 - b^2$

(d) $a+b$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
a	d	b	a	b	b	d	c

Q.2 Fill in the blanks

(i) The degree of polynomial $x^2y^2 + 3xy + y^3$ is _____(ii) $x^2 - 4$ _____(iii) $x^3 + \frac{1}{x^3} = \left[x + \frac{1}{x} \right] (\text{_____})$ (iv) $2(a^2 + b^2) = (a + b)^2 + (\text{_____})^2$ (v) $\left[x - \frac{1}{x} \right]^2 = \text{_____}$ (vi) Order of surd $\sqrt[3]{x}$ is _____(vii) $\frac{1}{2 - \sqrt{3}} = \text{_____}$

ANSWER KEY

(i) 4

(ii) $(x - 2)(x + 2)$ (iii) $x^2 - 1 + \frac{1}{x^2}$ (iv) $a - b$ (v) $x^2 + \frac{1}{x^2} - 2$

(vi) 3

(vii) $2 + \sqrt{3}$ Q.3 If $x + \frac{1}{x} = 3$, find(i) $x^2 + \frac{1}{x^2}$ Solution: Given that $x + \frac{1}{x} = 3$

$$\therefore (a + b)^2 = a^2 + b^2 + 2ab$$

Putting the values

$$\left[x + \frac{1}{x} \right]^2 = (x)^2 + \left(\frac{1}{x} \right)^2 + 2(x) \left(\frac{1}{x} \right)$$

$$(3)^2 = x^2 + \frac{1}{x^2} + 2$$

$$9 = x^2 + \frac{1}{x^2} + 2$$

$$9 - 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 7 \text{ Ans}$$

(ii) $x^4 + \frac{1}{x^4}$

Solution: Given that $x^2 + \frac{1}{x^2} = 7$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right)$$

$$(7)^2 = x^4 + \frac{1}{x^4} + 2$$

$$49 = x^4 + \frac{1}{x^4} + 2$$

$$x^4 + \frac{1}{x^4} = 49 - 2$$

$$x^4 + \frac{1}{x^4} = 47 \text{ Ans}$$

Q.4 If $x - \frac{1}{x} = 2$ find

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Solution (i)

Given that $x - \frac{1}{x} = 2$

$$\therefore (a+b)^2 = a^2 + b^2 + 2ab$$

Putting the values

$$\left(x - \frac{1}{x}\right)^2 = (x)^2 + \left(\frac{1}{x}\right)^2 - 2(x)\left(\frac{1}{x}\right)$$

$$(2)^2 = x^2 + \frac{1}{x^2} - 2$$

$$4 + 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 6 \text{ Ans}$$

Solution (ii)

Given that $x^2 + \frac{1}{x^2} = 6$

$$\left(x^2 + \frac{1}{x}\right) = x^4 + \frac{1}{x^4} + 2(\cancel{x^2})\left(\frac{1}{\cancel{x^2}}\right)$$

$$(6)^2 = x^4 + \frac{1}{x^4} + 2$$

$$x^4 + \frac{1}{x^4} = 36 - 2$$

$$x^4 + \frac{1}{x^4} = 34 \text{ Ans}$$

Q.5 Find the value of $x^3 + y^3$ and xy if $x + y = 5$ and $x - y = 3$.

Solution: Given that $x + y = 5$

$$x - y = 3$$

As we know that

$$\therefore (x+y)^2 - (x-y)^2 = 4xy$$

Putting the values

$$4xy = (5)^2 - (3)^2$$

$$4xy = 25 - 9$$

$$4xy = 16$$

$$xy = \frac{16}{4}$$

$$xy = 4 \text{ Ans}$$

As we know that

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

Putting the values

$$(5)^3 = x^3 + y^3 + 3 \times 4 \times 5$$

$$125 = x^3 + y^3 + 60$$

$$x^3 + y^3 = 125 - 60$$

$$x^3 + y^3 = 65$$

$$x^3 + y^3 = 65 \text{ Ans}$$

Q.6 If $P = 2 + \sqrt{3}$, find

(i) $P + \frac{1}{P}$

Solution: Given that $P = 2 + \sqrt{3}$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}}$$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{4 - 3}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{1}$$

$$\frac{1}{P} = 2 - \sqrt{3}$$

$$P + \frac{1}{P} = 2 + \sqrt{3} + 2 - \sqrt{3}$$

$$P + \frac{1}{P} = 4 \text{ Ans}$$

(ii) $P - \frac{1}{P}$

As we know that

$$\frac{1}{P} = 2 - \sqrt{3} \text{ and}$$

$$P = 2 + \sqrt{3}$$

$$P - \frac{1}{P} = 2 + \sqrt{3} - (2 - \sqrt{3})$$

$$= \cancel{2} + \sqrt{3} - \cancel{2} + \sqrt{3}$$

$$= 2\sqrt{3} \text{ Ans}$$

(iii) $P^2 + \frac{1}{P^2}$

Solution: Given that $P + \frac{1}{P} = 4$

$$\therefore (a + b)^2 = a^2 + b^2 + 2ab$$

$$\left(P + \frac{1}{P}\right)^2 = (P)^2 + \left(\frac{1}{P}\right)^2 + 2(\cancel{P})\left(\frac{1}{\cancel{P}}\right)$$

$$(4)^2 = P^2 + \frac{1}{P^2} + 2$$

$$16 - 2 = P^2 + \frac{1}{P^2}$$

$$P^2 + \frac{1}{P^2} = 14 \text{ Ans}$$

(iv) $P^2 - \frac{1}{P^2}$

Solution:

$$P^2 - \frac{1}{P^2} = \left(P + \frac{1}{P}\right)\left(P - \frac{1}{P}\right)$$

$$P^2 - \frac{1}{P^2} = (4)(2\sqrt{3})$$

$$= 8\sqrt{3} \text{ Ans}$$

Q.7 If $q = \sqrt{5} + 2$ find.

(i) $q + \frac{1}{q}$

Solution: Given that $q = \sqrt{5} + 2$

$$\frac{1}{q} = \frac{1}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$$

$$= \frac{\sqrt{5} - 2}{(\sqrt{5})^2 - (2)^2}$$

$$= \frac{\sqrt{5} - 2}{5 - 4}$$

$$= \sqrt{5} - 2$$

$$q + \frac{1}{q} = \sqrt{5} + \cancel{2} + \sqrt{5} - \cancel{2}$$

$$q + \frac{1}{q} = 2\sqrt{5} \text{ Ans}$$

$$(ii) \quad q - \frac{1}{q}$$

Solution: Given that $q = \sqrt{5} + 2$

$$\frac{1}{q} = \sqrt{5} - 2$$

$$q - \frac{1}{q} = \sqrt{5} + 2 - (\sqrt{5} - 2)$$

$$= \cancel{\sqrt{5}} + 2 - \cancel{\sqrt{5}} + 2$$

$$q - \frac{1}{q} = 4 \quad \text{Ans}$$

$$(iii) \quad q^2 + \frac{1}{q^2}$$

Solution: Given that $q - \frac{1}{q} = 4$

Squaring both sides

$$\left(q - \frac{1}{q}\right)^2 = (4)^2$$

$$q^2 + \frac{1}{q^2} - 2 = 16$$

$$q^2 + \frac{1}{q^2} = 16 + 2$$

$$q^2 + \frac{1}{q^2} = 18 \quad \text{Ans}$$

$$(iv) \quad q^2 - \frac{1}{q^2}$$

Solution: Given that $q + \frac{1}{q} = 2\sqrt{5}$

$$q - \frac{1}{q} = 4$$

By using formula

$$q^2 - \frac{1}{q^2} = \left(q + \frac{1}{q}\right)\left(q - \frac{1}{q}\right)$$

$$= (2\sqrt{5})(4)$$

$$= 8\sqrt{5} \quad \text{Ans}$$

Q.8 Simplify

$$(i) \quad \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}}$$

Solution:

$$= \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}} \times \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} + \sqrt{a^2-2}}$$

$$= \frac{(\sqrt{a^2+2} + \sqrt{a^2-2})^2}{(\sqrt{a^2+2})^2 - (\sqrt{a^2-2})^2}$$

$$= \frac{(\sqrt{a^2+2})^2 + (\sqrt{a^2-2})^2 + 2(\sqrt{a^2+2})(\sqrt{a^2-2})}{a^2+2 - a^2+2}$$

$$= \frac{a^2 + 2 + a^2 - 2 + 2(\sqrt{a^4 - 2a^2 + 2a^2 - 4})}{4}$$

$$= \frac{2a^2 + 2\sqrt{a^4 - 4}}{4}$$

$$= \frac{2(a^2 + \sqrt{a^4 - 4})}{2}$$

$$= a^2 + \sqrt{a^4 - 4} \quad \text{Ans}$$

$$(ii) \quad \frac{1}{a - \sqrt{a^2 - x^2}} - \frac{1}{a + \sqrt{a^2 - x^2}}$$

$$= \left(\frac{1}{a - \sqrt{a^2 - x^2}} \times \frac{a + \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} \right)$$

$$- \left(\frac{(1)}{(a + \sqrt{a^2 - x^2})} \frac{(a - \sqrt{a^2 - x^2})}{(a - \sqrt{a^2 - x^2})} \right)$$

$$= \left(\frac{a + \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} \right)$$

$$= \left(\frac{a + \sqrt{a^2 - x^2}}{a - (a^2 - x^2)} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{a - (a^2 - x^2)} \right)$$

$$= \left(\frac{a + \sqrt{a^2 - x^2}}{a^2 - a^2 + x^2} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{\cancel{a^2} - \cancel{a^2} + x^2} \right)$$

$$\begin{aligned} &= \left(\frac{a + \sqrt{a^2 - x^2}}{x^2} \right) - \left(\frac{a - \sqrt{a^2 - x^2}}{x^2} \right) \\ &= \frac{a + \sqrt{a^2 - x^2} - a + \sqrt{a^2 - x^2}}{x^2} \\ &= \frac{2\sqrt{a^2 - x^2}}{x^2} \text{ Ans} \end{aligned}$$

Al-hamd Science academy Notes

Unit 4: Algebraic Expressions and Algebraic

Formulas Overview

Algebraic expression:

An algebraic expression is that in which constants or variables or both are combined by basic operations.

Polynomial:

Polynomial means an expression with many terms.

Degree of Polynomial:

Degree of Polynomial means highest power of variable.

Rational expression:

Expression in the form $\frac{p(x)}{q(x)}$, ($q(x) \neq 0$) is called rational expression.

Surd:

An irrational radical with rational radicand is called a surd.

e.g., $\sqrt{3}$, $\sqrt{\frac{2}{5}}$,

Monomial Surd:

A surd which contains a single term is called monomial surd.

Binomial Surd:

A surd which contains sum or difference of two surds is called binomial surd.