Exercise 4.4

Q.1 Rationalize the denominator of the following

(i) $\frac{3}{4\sqrt{3}}$

Solution: $\frac{3}{4\sqrt{3}}$

$$=\frac{3}{4\sqrt{3}}$$

$$=\frac{3}{4\sqrt{3}}\times\frac{4\sqrt{3}}{4\sqrt{3}}$$

$$=\frac{3\left(4\sqrt{3}\right)}{\left(4\sqrt{3}\right)^2}$$

$$=\frac{12\sqrt{3}}{16\left(\sqrt{3}\right)^2}$$

$$=\frac{12\sqrt{3}}{16\times3}$$

$$=\frac{\cancel{12}\sqrt{3}}{\cancel{48}}$$

$$=\frac{\sqrt{3}}{4}\ \textbf{Ans}$$

(ii) $\frac{14}{\sqrt{98}}$

Solution: $\frac{14}{\sqrt{98}}$

$$=\frac{14}{\sqrt{98}}$$

$$=\frac{14}{\sqrt{98}}\times\frac{\sqrt{98}}{\sqrt{98}}$$

$$=\frac{14\left(\sqrt{98}\right)}{\left(\sqrt{98}\right)^2}$$

$$=\frac{14\left(\sqrt{7\times7\times2}\right)}{98}$$

$$=\frac{14\times7\times\sqrt{2}}{98}$$

$$=\frac{\cancel{98}\times\sqrt{2}}{\cancel{98}}$$

$$=\sqrt{2}\ \textbf{Ans}$$

(iii) $\frac{6}{\sqrt{8}\sqrt{27}}$

Solution: $\frac{6}{\sqrt{8}\sqrt{27}}$

$$=\frac{6}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6}{\sqrt{8}\sqrt{27}}\times\frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6\left(\sqrt{8}\sqrt{27}\right)}{\left(\sqrt{8}\right)^2\left(\sqrt{27}\right)^2}$$

$$=\frac{6\left(\sqrt{4\times2}\right)\left(\sqrt{9\times3}\right)}{8\times27}$$

$$=\frac{6\times2\sqrt{2}\times3\sqrt{3}}{216}$$

$$=\frac{6\times3\times2\left(\sqrt{2\times3}\right)}{216}$$

$$=\frac{\cancel{36}\sqrt{6}}{\cancel{216}^{\,6}}$$

$$=\frac{\sqrt{6}}{6}\ \textbf{Ans}$$

(iv) $\dfrac{1}{3+2\sqrt{5}}$

Solution: $\dfrac{1}{3+2\sqrt{5}}$

$$=\frac{1}{3+2\sqrt{5}}$$

$$=\frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$=\frac{3-2\sqrt{5}}{(3)^2-\left(2\sqrt{5}^2\right)}$$

$$=\frac{3-2\sqrt{5}}{9-4.5}$$

$$=\frac{3-2\sqrt{5}}{9-20}$$

$$=\frac{3-2\sqrt{5}}{-11}$$ **Ans**

(v) $\dfrac{15}{\sqrt{31}-4}$

Solution: $\dfrac{15}{\sqrt{31}-4}$

$$=\frac{15}{\sqrt{31}-4}$$

$$=\frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$=\frac{\cancel{15}\left(\sqrt{31}+4\right)}{\cancel{15}}$$

$$=\sqrt{31}+4$$ **Ans**

(vi) $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

Solution: $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

$$\frac{2}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\cancel{\sqrt{5}}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}+\sqrt{3}\right)}{\cancel{2}}$$

$$=\sqrt{5}+\sqrt{3}$$ **Ans**

(vii) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution: $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$=\frac{\left(\sqrt{3}\right)^2-2\left(\sqrt{3}\right)(1)+(1)^2}{2}$$

$$=\frac{3-2\sqrt{3}+1}{2}$$

$$=\frac{4-2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{2}}$$

$$= 2-\sqrt{3}$$ **Ans**

(viii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2}{5-3}$$

$$= \frac{5+2\sqrt{15}+3}{2}$$

$$= \frac{8+2\sqrt{15}}{2}$$

$$= \frac{\cancel{2}\left(4+\sqrt{15}\right)}{\cancel{2}}$$

$$= 4+\sqrt{15}$$ **Ans**

Q.2 **find the conjugate of** $x+\sqrt{y}$

(i) $3+\sqrt{7}$

Solution

Conjugate $3-\sqrt{7}$

(ii) $4-\sqrt{5}$

Solution

Conjugate $4+\sqrt{5}$

(iii) $2+\sqrt{3}$

Solution

Conjugate $2-\sqrt{3}$

(iv) $2+\sqrt{5}$

Solution

Conjugate $2-\sqrt{5}$

(v) $5+\sqrt{7}$

Solution

Conjugate $5-\sqrt{7}$

(vi) $4-\sqrt{15}$

Solution

Conjugate $4+\sqrt{15}$

(vii) $7-\sqrt{6}$

Solution

Conjugate $7+\sqrt{6}$

(viii) $9+\sqrt{2}$

Solution

Conjugate $9-\sqrt{2}$

Q.3

(i) **If** $x=2-\sqrt{3}$, **find** $\frac{1}{x}$

Solution: Given that $x=2-\sqrt{3}$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}}$$

$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2+\sqrt{3}}{4-3}$$

$$= \frac{2+\sqrt{3}}{1}$$

$$\frac{1}{x} = 2+\sqrt{3}$$ **Ans**

(ii) **If** $x=4-\sqrt{17}$, **find** $\frac{1}{x}$

Solution: Given that $x=4-\sqrt{17}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}}$$

$$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$= \frac{4+\sqrt{17}}{(4)^2-\left(\sqrt{17}\right)^2}$$

$$= \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+17}{-1}$$

$$= -1\left(4+\sqrt{17}\right)$$

$$\frac{1}{x} = -4-\sqrt{17} \textbf{ Ans}$$

(iii) **If** $x=\sqrt{3}+2,$ **find** $x+\frac{1}{x}$

Solution: Given that $x=\sqrt{3}+2$

$$\frac{1}{x}=\frac{1}{\sqrt{3}+2}$$

$$=\frac{1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$=\frac{\sqrt{3}-2}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$=\frac{\sqrt{3}-2}{3-4}$$

$$=\frac{\sqrt{3}-2}{-1}$$

$$=-\left(\sqrt{3}-2\right)$$

$$=-\sqrt{3}+2$$

$$x+\frac{1}{x}=\left(\sqrt{3}+2\right)+\left(-\sqrt{3}+2\right)$$

$$=\sqrt{3}+2-\sqrt{3}+2$$

$$=2+2$$

$$x+\frac{1}{x}=4 \textbf{ Ans}$$

Q.4 **Simplify**

(i) $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$=\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{\left(\sqrt{5}-\sqrt{3}\right)+\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{5-3}$$

$$+\frac{1\left(\sqrt{5}+\sqrt{3}\right)-\sqrt{2}\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2}+\frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$=\frac{\sqrt{5}}{2}-\frac{\sqrt{3}}{2}+\frac{\sqrt{10}}{2}-\frac{\sqrt{6}}{2}+\frac{\sqrt{5}}{2}+\frac{\sqrt{3}}{2}-\frac{\sqrt{10}}{2}-\frac{\sqrt{6}}{2}$$

$$=\frac{2\sqrt{5}}{2}-\frac{2\sqrt{6}}{2}$$

$$=\sqrt{5}-\sqrt{6} \textbf{ Ans}$$

(ii) $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

Solution: $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$

$$=\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$$

$$=\left(\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)+\left(\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$$

$$+\left(\frac{1}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$

$$= \left(\frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}\right) + \left(\frac{2\times\left(\sqrt{5}+\sqrt{3}\right)}{(\sqrt{5})^2-(\sqrt{3})^2}\right)$$

$$+\left(\frac{2-\sqrt{5}}{(2)^2-(\sqrt{5})^2}\right)$$

$$= \left(\frac{2-\sqrt{3}}{4-3}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}\right)+\left(\frac{2-\sqrt{5}}{4-5}\right)$$

$$= \left(\frac{2-\sqrt{3}}{1}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}\right)+\left(\frac{2-\sqrt{5}}{-1}\right)$$

$$= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$

$$= 2-2-\sqrt{3}+\sqrt{3}+\sqrt{5}+\sqrt{5}$$

$$= \sqrt{5}+\sqrt{5}$$

$= 2\sqrt{5}$ **Ans**

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}-\dfrac{3}{\sqrt{5}+\sqrt{2}}$

Solution: $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}-\dfrac{3}{\sqrt{5}+\sqrt{2}}$

$$= \frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

$$= \left(\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$$

$$-\left(\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{7}}\right)$$

$$= \left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{(\sqrt{5})^2-(\sqrt{3})^2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{(\sqrt{5})^2-(\sqrt{2})^2}\right)$$

$$= \left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}\right)$$

$$= \left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{1}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}\right)$$

$$= \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2}$$

$= 0$ **Ans**

Q.5 **If** $x = 2+\sqrt{3}$, **then find the value of** $x-\dfrac{1}{x}$ **and** $\left(x-\dfrac{1}{x}\right)^2$

(i)

Solution: Given that $x = 2+\sqrt{3}$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3}$$

$$= \frac{2-\sqrt{3}}{1}$$

$$= 2-\sqrt{3}$$

To find the value of $x-\dfrac{1}{x}$

$$x-\frac{1}{x} = \left(2+\sqrt{3}\right)-\left(2-\sqrt{3}\right)$$

$$= 2+\sqrt{3}-2+\sqrt{3}$$

$$= \sqrt{3}+\sqrt{3}$$

$$= 2\sqrt{3}$$

To find the value of $\left(x-\dfrac{1}{x}\right)^2$

We know that

$$x-\frac{1}{x} = 2\sqrt{3}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2 = \left(2\sqrt{3}\right)^2$$

$$= 4\left(\sqrt{3}\right)^2$$

$$= 4(3)$$

$= 12$ **Ans**

(ii) **If** $x=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$**, find the value of** $x+\dfrac{1}{x}$, $x^2+\dfrac{1}{x^2}$ **and** $x^3+\dfrac{1}{x^3}$

Solution: Given that $x=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$$\frac{1}{x}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$x+\frac{1}{x}=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{\left(\sqrt{5}-\sqrt{2}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

$$=\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2-2\sqrt{5}\times\sqrt{2}+\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{5}\times\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{5+2-2\sqrt{10}+5+2+2\sqrt{10}}{5-2}$$

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking square on both sides

$$\left(x+\frac{1}{x}\right)^2=\left(\frac{14}{3}\right)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=\frac{196}{9}$$

$$x^2+\frac{1}{x^2}=\frac{196}{9}-2$$

$$x^2+\frac{1}{x^2}=\frac{196-18}{9}$$

$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

To find $x^3+\dfrac{1}{x^3}$

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking cube on both sides

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$$x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+3\left(\frac{14}{3}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+14=\frac{2744}{24}$$

$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$

$$x^3+\frac{1}{x^3}=\frac{2744-378}{27}$$

$$x^3+\frac{1}{x^3}=\frac{2366}{27}\ \textbf{Ans}$$

Q.6 **Determine the rational numbers** a **and** b **if** $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}+\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$

Solution: Given that

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

$$a+b\sqrt{3}=\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$$=\frac{\left(\sqrt{3}\right)^2+(1)^2-2\sqrt{3}+\left(\sqrt{3}\right)^2+(1)^2+2\sqrt{3}}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2\left(\sqrt{3}\right)^2+2}{\left(\sqrt{3}\right)^2-1}$$

$$=\frac{2\left[\left(\sqrt{3}\right)^2+(1)^2\right]}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2(3+1)}{3-1}$$

$$=\frac{2(4)}{2}$$

$$a + b\sqrt{3} = 4$$

$$a + b\sqrt{3} = 4 + 0\sqrt{3}$$

Comparing both sides

$a = 4$ $\qquad$ $b\sqrt{3} = 0\sqrt{3}$

$$b = \frac{0\sqrt{3}}{\sqrt{3}}$$

$b = 0$ **Ans**