Exercise 4.3

Q.1 Express each of the following surd in the simplest form:

(i) $\sqrt{180}$

Solution: $\sqrt{180}$

$$= (180)^{\frac{1}{2}}$$
$$= (2\times 2\times 3\times 3\times 5)^{\frac{1}{2}}$$
$$= (2^2\times 3^2\times 5)^{\frac{1}{2}}$$
$$= 2^{2\times\frac{1}{2}}\times 3^{2\times\frac{1}{2}}\times 5^{\frac{1}{2}}$$
$$= 2\times 3\times\sqrt{5}$$
$$= 6\sqrt{5} \quad \textbf{Ans}$$

(ii) $3\sqrt{162}$

Solution: $3\sqrt{162}$

$$3\left(\sqrt{81\times 2}\right)$$
$$= 3\left(\sqrt{9^2}\times\sqrt{2}\right)$$
$$= 3\times 9\left(\sqrt{2}\right)$$
$$= 27\sqrt{2} \quad \textbf{Ans}$$

(iii) $\frac{3}{4}\sqrt[3]{128}$

Solution: $\frac{3}{4}\sqrt[3]{128}$

$$= \frac{3}{4}\left(\sqrt[3]{64\times 2}\right)$$
$$= \frac{3}{4}\left(\sqrt[3]{4^3\times 2}\right)$$
$$= \frac{3}{4}\left[\sqrt[3]{4^3}\times\sqrt[3]{2}\right]$$
$$= \frac{3}{4}\times 4\times\sqrt[3]{2}$$
$$= 3\times\sqrt[3]{2}$$
$$= 3\sqrt[3]{2} \quad \textbf{Ans}$$

(iv) $\sqrt[5]{96x^6y^7z^8}$

Solution: $\sqrt[5]{96x^6y^7z^8}$

$$= \sqrt{32\times 3\times x^5y^5z^5\times x^1y^2z^3}$$
$$= \sqrt[5]{2^5\times 3\times x^5y^5z^5\times xy^2z^3}$$
$$= \sqrt[5]{2^5x^5y^5z^5}\times\sqrt[5]{3xy^2z^3}$$
$$= \sqrt[5]{2^5}\times\sqrt[5]{x^5}\times\sqrt[5]{y^5}\times\sqrt[5]{z^5}\times\sqrt[5]{3xy^2z^3}$$
$$= 2xyz\sqrt[5]{3xy^2z^3} \quad \textbf{Ans}$$

Q.2 Simplify

(i) $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

Solution: $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$$= \frac{\sqrt{9\times 2}}{\sqrt{3}\times\sqrt{2}}$$
$$= \frac{\sqrt{3^2}\times\cancel{\sqrt{2}}}{\sqrt{3}\times\cancel{\sqrt{2}}}$$
$$= \frac{3}{\sqrt{3}}$$
$$= \frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{3\sqrt{3}}{\left(\sqrt{3}\right)^{\cancel{2}}}$$
$$= \frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$
$$= \sqrt{3} \quad \textbf{Ans}$$

(ii) $\dfrac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

Solution: $\dfrac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

$= \dfrac{\sqrt{21}\sqrt{3^2}}{\sqrt{9\times 7}}$

$= \dfrac{\sqrt{21}\times 3}{\sqrt{3^2}\times\sqrt{7}}$

$= \dfrac{\sqrt{21}\times 3}{3\sqrt{7}}$

$= \dfrac{\cancel{3}\sqrt{21}}{\cancel{3}\sqrt{7}}$

$= \dfrac{\sqrt{21}}{\sqrt{7}}$

$= \dfrac{\sqrt{7\times 3}}{\sqrt{7}}$

$= \dfrac{\cancel{\sqrt{7}}\times\sqrt{3}}{\cancel{\sqrt{7}}}$

$= \sqrt{3}$ **Ans**

(iii) $= \sqrt[5]{243x^5y^{10}z^{15}}$

Solution: $= \sqrt[5]{243x^5y^{10}z^{15}}$

$= \sqrt[5]{3^5x^5\left(y^2\right)^5\left(z^3\right)^5}$

$= \sqrt[5]{3^5}\times\sqrt[5]{x^5}\times\sqrt[5]{\left(y^2\right)^5}\times\sqrt[5]{\left(z^3\right)^5}$

$= 3\times x\times y^2\times z^3$

$= 3xy^2z^3$ **Ans**

(iv) $\dfrac{4}{5}\sqrt[3]{125}$

Solution: $\dfrac{4}{5}\sqrt[3]{125}$

$= \dfrac{4}{5}\sqrt[3]{5\times 5\times 5}$

$= \dfrac{4}{5}\sqrt[3]{5^3}$

$= \dfrac{4}{\cancel{5}}\times\cancel{5}$

$= 4$ **Ans**

(v) $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

Solution: $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

$= \sqrt{7\times 3}\times\sqrt{7}\times\sqrt{3}$

$= \sqrt{7\times 3\times 7\times 3}$

$= \sqrt{7\times 7\times 3\times 3}$

$= \sqrt{7^2}\times\sqrt{3^2}$

$= 7\times 3$

$= 21$ **Ans**

Q.3 Simplify by combining similar terms.

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

Solution: $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

$= \sqrt{9\times 5}-3\sqrt{5\times 4}+4\sqrt{5}$

$= \sqrt{3^2}\times\sqrt{5}-3\sqrt{2^2}\times\sqrt{5}+4\sqrt{5}$

$= 3\sqrt{5}-3\times 2\sqrt{5}+4\sqrt{5}$

$= 3\sqrt{5}-6\sqrt{5}+4\sqrt{5}$

$= \sqrt{5}\left(3-6+4\right)$

$= \sqrt{5}\left(3-2\right)$

$= \sqrt{5}\left(1\right)$

$= \sqrt{5}$ **Ans**

(ii) $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

Solution: $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

$= 4\sqrt{4\times 3}+5\sqrt{9\times 3}-3\sqrt{25\times 3}+\sqrt{100\times 3}$

$= 4\times 2\sqrt{3}+5\times 3\sqrt{3}-3\times 5\sqrt{3}+10\sqrt{3}$

$= 8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$= 8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$= \sqrt{3}\left(8 + \cancel{15} - \cancel{15} + 10\right)$

$= \sqrt{3}\left(8+10\right)$

$= \sqrt{3}\left(18\right)$

$= 18\sqrt{3}$ **Ans**

(iii) $\sqrt{3}\left(2\sqrt{3}+3\sqrt{3}\right)$

Solution: $\sqrt{3}\left(2\sqrt{3}+3\sqrt{3}\right)$

$= \sqrt{3}\times\sqrt{3}\left(2+3\right)$

$= \left(\sqrt{3}\right)^2 \times (5)$

$= 3(5)$

$= 15$ Ans

(iv) $2\left(6\sqrt{5}-3\sqrt{5}\right)$

Solution: $2\left(6\sqrt{5}-3\sqrt{5}\right)$

$= 2\times\sqrt{5}\left(6-3\right)$

$= 2\times\sqrt{5}\left(3\right)$

$= 6\sqrt{5}$ Ans

Q.4 Simplify

(i) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

Solution: $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

$= (3)^2 - \left(\sqrt{3}\right)^2$

$= 9-3$

$= 6$ **Ans**

(ii) $\left(\sqrt{5}+\sqrt{3}\right)^2$

Solution: $\left(\sqrt{5}+\sqrt{3}\right)^2$

$= \left(\sqrt{5}\right)^2 + 2\left(\sqrt{5}\right)\left(\sqrt{3}\right) + \left(\sqrt{3}\right)^2$

$= 5 + 2\sqrt{5\times 3} + 3$

$= 8 + 2\sqrt{15}$ **Ans**

(iii) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

Solution: $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

$= \left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2$

$= 5-3$

$= 2$ **Ans**

(iv) $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

Solution: $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

$= \left(\sqrt{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= 2 - \frac{(1)^2}{\left(\sqrt{3}\right)^2}$

$= 2 - \frac{1}{3}$

$= \frac{6-1}{3}$

$= \frac{5}{3}$ **Ans**

(v) $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$

Solution:

$\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$

$= \left[\left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2\right](x+y)\left(x^2+y^2\right)$

$= (x-y)(x+y)\left(x^2+y^2\right)$

$= \left[(x)^2 - (y)^2\right]\left(x^2+y^2\right)$

$= \left(x^2-y^2\right)\left(x^2+y^2\right)$

$$= \left[\left(x^2\right)^2 - \left(y^2\right)^2 \right]$$

$$= x^4 - y^4 \quad \textbf{Ans}$$