

Exercise 4.2

Q.1 Solve

(i) If $a + b = 10$ and $a - b = 6$, then find the value of $(a^2 + b^2)$

Solution:

$$2(a^2 + b^2) = (a + b)^2 + (a - b)^2$$

$$2(a^2 + b^2) = (10)^2 + (6)^2$$

$$2(a^2 + b^2) = 100 + 36$$

$$2(a^2 + b^2) = 136$$

$$(a^2 + b^2) = \frac{136}{2} = 68$$

$$(a^2 + b^2) = 68 \text{ Ans}$$

(ii) If $a + b = 5$, $a - b = \sqrt{17}$, then find the value of ab .

Solution:

$$4ab = (a + b)^2 - (a - b)^2$$

$$4ab = (5)^2 - (\sqrt{17})^2$$

$$4ab = 25 - 17$$

$$4ab = 8$$

$$ab = \frac{8}{4}$$

$$ab = 2$$

$$ab = 2 \text{ Ans}$$

Q.2 If $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$, then find the value of $ab + bc + ca$.

Solution: $a^2 + b^2 + c^2 = 45$

$$a + b + c = -1$$

$$ab + bc + ca = ?$$

We know that

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(-1)^2 = 45 + 2(ab + bc + ca)$$

$$1 = 45 + 2(ab + bc + ca)$$

$$1 - 45 = 2(ab + bc + ca)$$

$$-44 = 2(ab + bc + ca)$$

$$\frac{-44}{2} = (ab + bc + ca)$$

$$(ab + bc + ca) = -22 \text{ Ans}$$

Q.3 If $m + n + p = 10$ and $mn + np + mp = 27$, find the value of $m^2 + n^2 + p^2$

Solution: $m + n + p = 10$

$$mn + np + mp = 27,$$

$$m^2 + n^2 + p^2 = ?$$

We know that

$$(m + n + p)^2 = m^2 + n^2 + p^2 + 2mn + 2np + 2mp$$

$$(10)^2 = m^2 + n^2 + p^2 + 2(mn + np + mp)$$

$$100 = m^2 + n^2 + p^2 + 2(27)$$

$$100 = m^2 + n^2 + p^2 + 54$$

$$100 - 54 = m^2 + n^2 + p^2$$

$$m^2 + n^2 + p^2 = 46 \text{ Ans}$$

Q.4 If $x^2 + y^2 + z^2 = 78$ and $xy + yz + zx = 59$, find the value of $x + y + z$.

Solution: $x^2 + y^2 + z^2 = 78$

$$xy + yz + zx = 59,$$

$$x + y + z = ?$$

We know that

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$(x + y + z)^2 = 78 + 2(xy + yz + zx)$$

$$(x + y + z)^2 = 78 + 2(59)$$

$$(x + y + z)^2 = 78 + 118$$

$$(x + y + z)^2 = 196$$

Taking square root at both sides

$$\sqrt{(x+y+z)^2} = \pm\sqrt{196}$$

$$x+y+z = \pm 14 \text{ Ans}$$

Q.5 If $x+y+z=12$ and $x^2+y^2=64$,
find the value of $xy+yz+zx$.

Solution: $x+y+z=12$

$$x^2+y^2=64$$

$$xy+yz+zx=?$$

We know that

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$$

$$(12)^2 = 64+2(xy+yz+zx)$$

$$144-64 = 2(xy+yz+zx)$$

$$80 = 2(xy+yz+zx)$$

$$\frac{80}{2} = (xy+yz+zx)$$

$$40 = xy+yz+zx$$

$$xy+yz+zx = 40 \text{ Ans}$$

Q.6 If $x+y=7$ and $xy=12$, then
find the value of x^3+y^3

Solution: $x+y=7$

$$xy=12$$

$$x^3+y^3=?$$

We know that

$$(x+y)^3 = x^3+y^3+3xy(x+y)$$

$$(7)^3 = x^3+y^3+3(12)(7)$$

$$343 = x^3+y^3+252$$

$$343-252 = x^3+y^3$$

$$91 = x^3+y^3$$

$$x^3+y^3 = 91 \text{ Ans}$$

Q.7 If $3x+4y=11$ and $xy=12$, then
find the value of $27x^3+64y^3$.

Solution: $3x+4y=11$

$$xy=12$$

$$27x^3+64y^3=?$$

$$(x+y)^3 = x^3+y^3+3xy(x+y)$$

$$(3x+4y)^3 = (3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$$

$$(3x+4y)^3 = 27x^3+64y^3+36xy(3x+4y)$$

$$(11)^3 = 27x^3+64y^3+36(12)(11)$$

$$1331 = 27x^3+64y^3+4752$$

$$1331-4752 = 27x^3+64y^3$$

$$-3421 = 27x^3+64y^3$$

$$27x^3+64y^3 = -3421 \text{ Ans}$$

Q.8 If $x-y=4$ and $xy=21$, then
find the value of x^3-y^3

Solution: $x-y=4$

$$xy=21$$

$$x^3-y^3=?$$

We know that

$$(x-y)^3 = x^3-y^3-3xy(x-y)$$

$$(4)^3 = x^3-y^3-3(21)(4)$$

$$64 = x^3-y^3-252$$

$$64+252 = x^3-y^3$$

$$316 = x^3-y^3$$

$$x^3-y^3 = 316 \text{ Ans}$$

Q.9 If $5x-6y=13$ and $xy=6$, then
find the value of $125x^3-216y^3$

Solution: $5x-6y=13$

$$xy=6$$

$$125x^3-216y^3=?$$

We know that

$$(x-y)^3 = x^3-y^3-3xy(x-y)$$

$$(5x-6y)^3 = (5x)^3-(6y)^3-3(5x)(6y)(5x-6y)$$

$$(5x-6y)^3 = 125x^3-216y^3-90xy(5x-6y)$$

$$(13)^3 = 125x^3-216y^3-90(6)(13)$$

$$2197 = 125x^3-216y^3-7020$$

$$2197+7020 = 125x^3-216y^3$$

$$9217 = 125x^3 - 216y^3$$

$$125x^3 - 216y^3 = 9217 \text{ Ans}$$

Q.10 If $x + \frac{1}{x} = 3$ then find the value of

$$x^3 + \frac{1}{x^3}$$

Solution: $x + \frac{1}{x} = 3$

$$x^3 + \frac{1}{x^3} = ?$$

We know that

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$27 - 9 = x^3 + \frac{1}{x^3}$$

$$18 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = 18 \text{ Ans}$$

Q.11 If $x - \frac{1}{x} = 7$, then find the value

$$\text{of } x^3 - \frac{1}{x^3}$$

Solution: $x - \frac{1}{x} = 7$

$$x^3 - \frac{1}{x^3} = ?$$

We know that

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$(7)^3 = x^3 - \frac{1}{x^3} - 3(7)$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$364 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 364 \text{ Ans}$$

Q.12 If $\left[3x + \frac{1}{3x}\right] = 5$, then find the value

$$\text{of } \left[27x^3 + \frac{1}{27x^3}\right]$$

Solution: $\left[3x + \frac{1}{3x}\right] = 5$

$$\left[27x^3 + \frac{1}{27x^3}\right] = ?$$

We know that

$$\left(3x + \frac{1}{3x}\right)^3 = (3x)^3 + \left(\frac{1}{3x}\right)^3 + 3\left(\cancel{3x}\right)\left(\frac{1}{\cancel{3x}}\right)\left(3x + \frac{1}{3x}\right)$$

$$(5)^3 = 27x^3 + \frac{1}{27x^3} + 3\left(3x + \frac{1}{3x}\right)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

$$125 - 15 = 27x^3 + \frac{1}{27x^3}$$

$$110 = 27x^3 + \frac{1}{27x^3}$$

$$27x^3 + \frac{1}{27x^3} = 110 \text{ Ans}$$

Q.13 If $\left(5x - \frac{1}{5x}\right) = 6$, then find the

$$\text{value of } \left(125x^3 - \frac{1}{125x^3}\right)$$

Solution: $\left(5x - \frac{1}{5x}\right) = 6$

$$\left(125x^3 - \frac{1}{125x^3}\right) = ?$$

We know that

$$\left(5x - \frac{1}{5x}\right)^3 = (5x)^3 - \left(\frac{1}{5x}\right)^3 - 3\left(\cancel{5x}\right)\left(\frac{1}{\cancel{5x}}\right)\left(5x - \frac{1}{5x}\right)$$

$$(6)^3 = 125x^3 - \frac{1}{125x^3} - 3(6)$$

$$216 = 125x^3 - \frac{1}{125x^3} - 18$$

$$216 + 18 = 125x^3 - \frac{1}{125x^3}$$

$$234 = 125x^3 - \frac{1}{125x^3}$$

$$125x^3 - \frac{1}{125x^3} = 234 \text{ Ans}$$

Q.14 Factorize

(i) $x^3 - y^3 - x + y$

Solution: $x^3 - y^3 - x + y$

$$= (x)^3 - (y)^3 - 1(x - y)$$

$$= (x - y)(x^2 + xy + y^2) - 1(x - y)$$

$$= (x - y)(x^2 + xy + y^2 - 1) \text{ Ans}$$

(ii) $8x^3 - \frac{1}{27y^3}$

Solution: $8x^3 - \frac{1}{27y^3}$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left[2x - \frac{1}{3y}\right] \left[(2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2 \right]$$

$$= \left(2x - \frac{1}{3y}\right) \left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right) \text{ Ans}$$

Q.15 Find the products, using formula.

(i) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

Solution: $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$$= (x^2 + y^2) \left[(x^2)^2 - (x^2)(y^2) + (y^2)^2 \right]$$

$$\left[(x^2)^3 + (y^2)^3 \right]$$

$$= x^6 + y^6 \text{ Ans}$$

(ii) $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

Solution: $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

$$(x^3 - y^3) \left[(x^3)^2 + (x^3)(y^3) + (y^3)^2 \right]$$

$$= (x^3)^3 - (y^3)^3$$

$$= x^9 - y^9 \text{ Ans}$$

(iii) $(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)$

$$(x^2 + xy + y^2)(x^4 - x^2y^2 + y^4)$$

Solution:

$$(x - y)(x + y)(x^2 + y^2)(x^2 + xy + y^2)$$

$$(x^2 + xy + y^2)(x^4 - x^2y^2 + y^4)$$

$$= \left[(x - y)(x^2 + xy + y^2) \right] \left[(x + y)(x^2 - xy + y^2) \right]$$

$$\left[(x^2 + y^2)(x^4 - x^2y^2 + y^4) \right]$$

$$= \left[(x^3 - y^3)(x^3 + y^3) \right] \left[(x^2)^3 + (y^2)^3 \right]$$

$$= \left[(x^3)^2 - (y^3)^2 \right] \left[(x^6 + y^6) \right]$$

$$= \left[(x^6 - y^6)(x^6 + y^6) \right]$$

$$= \left[(x^6)^2 - (y^6)^2 \right]$$

$$= x^{12} - y^{12} \text{ Ans}$$

(iv) $(2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1)$

Solution:

$$\begin{aligned} & (2x^2 - 1)(2x^2 + 1)(4x^4 + 2x^2 + 1)(4x^4 - 2x^2 + 1) \\ &= [(2x^2 - 1)(4x^4 + 2x^2 + 1)][(2x^2 + 1)(4x^4 - 2x^2 + 1)] \\ &= [(2x^2)^3 - (1)^3][(2x^2)^3 + (1)^3] \\ &= (8x^6 - 1)(8x^6 + 1) \\ &= (8x^6)^2 - (1)^2 \\ &= 64x^{12} - 1 \text{ Ans} \end{aligned}$$

Al-hamd Science academy Notes