

Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).

(i) $3x^2 + \frac{1}{x} - 5$

No (Because of $\frac{1}{x}$) Ans.

(ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$

No (Because \sqrt{x} or $(x)^{\frac{1}{2}}$) Ans.

(iii) $x^2 - 3x + \sqrt{2}$

Yes (Because no variable has power in fraction). Ans

(iv) $\frac{3x}{2x-1} + 8$

No (Because of $\frac{1}{2x-1}$) Ans

Q.2 State whether each of the following expressions is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x} + 5}$

Irrational Ans

(ii) $\frac{x^3 - 2x^3 + \sqrt{3}}{2 + 3x - x^2}$

Rational Ans

(iii) $\frac{x^2 + 6x + 9}{x^2 - 9}$

Rational Ans

(iv) $\frac{2\sqrt{x} + 3}{2\sqrt{x} - 3}$

Irrational Ans

Q.3 Reduce the following expression to the lowest form.

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$

Solution: $\frac{\cancel{120}x^2y^3z^5}{\cancel{30}x^3yz^2}$
 $= \frac{120x^2y^3z^5}{30x^3yz^2}$
 $= 4x^{2-3}y^{3-1}z^{5-2}$
 $= 4x^{-1}y^2z^3$
 $= \frac{4y^2z^3}{x}$ Ans

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

Solution: $\frac{8a(x+1)}{2(x^2-1)}$
 $= \frac{\cancel{8}a(x+1)}{\cancel{2}(x^2-1)}$
 $= \frac{4a(x+1)}{(x-1)(x+1)}$
 $= \frac{4a}{x-1}$ Ans

(iii) $\frac{(x+y)^2 - 4xy}{(x-y)^2}$

Solution: $\frac{(x+y)^2 - 4xy}{(x-y)^2}$
 $\therefore (x+y)^2 = x^2 + y^2 + 2xy$
 $\therefore (x-y)^2 = x^2 + y^2 - 2xy$
 $= \frac{x^2 + y^2 + 2xy - 4xy}{x^2 + y^2 - 2xy}$
 $= \frac{x^2 + y^2 - 2xy}{x^2 + y^2 - 2xy}$

$$= \frac{\cancel{(x-y)^2}}{\cancel{(x-y)^2}}$$

$$= 1 \text{ Ans}$$

$$(iv) \frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x - y)(x^2 + xy + y^2)}$$

Solution:

$$\frac{(x^3 - y^3)(x^2 - 2xy + y^2)}{(x - y)(x^2 + xy + y^2)}$$

$$(a^3 + b^3) = (a - b)(a^2 + ab + b^2)$$

$$= \frac{\cancel{(x^3 - y^3)}(x^2 - 2xy + y^2)}{\cancel{(x^3 - y^3)}} \frac{1}{(x - y)}$$

$$= x^2 - 2xy + y^2$$

$$\therefore (x - y)^2 = x^2 - 2xy + y^2$$

$$= (x - y)^2 \text{ Ans}$$

$$(v) \frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$$

Solution:

$$\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$$

$$= \frac{(x+2)[(x)^2 - (1)^2]}{(x+1)[(x)^2 - (2)^2]}$$

$$= \frac{\cancel{(x+2)}(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)\cancel{(x+2)}}$$

$$= \frac{(x-1)}{(x-2)} \text{ Ans}$$

$$(vi) \frac{x^2 - 4x + 4}{2x^2 - 8}$$

Solution:

$$\frac{x^2 - 4x + 4}{2x^2 - 8}$$

$$\therefore (a - b)^2 = a^2 - 2ab + b^2$$

$$\therefore a^2 - b^2 = (a + b)(a - b)$$

$$= \frac{(x)^2 - 2(x)(2) + (2)^2}{2(x^2 - 4)}$$

$$= \frac{(x - 2)^2}{2[(x)^2 - (2)^2]}$$

$$= \frac{(x - 2)^2}{2(x + 2)(x - 2)}$$

$$= \frac{(x - 2)\cancel{(x - 2)}}{2(x + 2)\cancel{(x - 2)}}$$

$$= \frac{x - 2}{2(x + 2)} \text{ Ans}$$

$$(vii) \frac{64x^5 - 64x}{(8x^2 + 8)(2x + 2)}$$

Solution:

$$\frac{64x^5 - 64x}{(8x^2 + 8)(2x + 2)}$$

$$= \frac{64x(x^4 - 1)}{8(x^2 + 1) \cdot 2(x + 1)}$$

$$= \frac{64[(x^2)^2 - (1)^2]}{16(x^2 + 1)(x + 1)}$$

$$= \frac{\cancel{64}(x^2 - 1)\cancel{(x^2 + 1)}}{\cancel{16}(x^2 + 1)(x + 1)}$$

$$= \frac{4x(x - 1)\cancel{(x + 1)}}{\cancel{(x + 1)}}$$

$$= 4x(x - 1) \text{ Ans}$$

$$(viii) \frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2}$$

Solution:

$$\frac{9x^2 - (x^2 - 4)^2}{4 + 3x - x^2}$$

$$\begin{aligned}
 &= \frac{(3x)^2 - (x^2 - 4)^2}{4 + 3x - x^2} \\
 &= \frac{(3x + x^2 - 4)(3x - x^2 + 4)}{4 + 3x - x^2} \\
 &= \frac{(x^2 + 3x - 4)(-x^2 + 3x + 4)}{(-x^2 + 3x + 4)} \\
 &= x^2 + 3x - 4 \text{ Ans}
 \end{aligned}$$

Q.4 Evaluate

- (a) $\frac{x^3y - 2z}{xz}$ for
- (i) $x = 3, y = -1, z = -2$
- (ii) $x = -1, y = -9, z = 4$

Solution for 1st part

When $x = 3, y = -1, z = -2$

$$\begin{aligned}
 &\frac{x^3y - 2z}{xz} = \\
 &= \frac{(3)^3(-1) - 2(-2)}{(3)(-2)} \\
 &= \frac{27(-1) + 4}{-6} \\
 &= \frac{-27 + 4}{-6} \\
 &= \frac{-23}{-6} \\
 &= \frac{23}{6} \text{ Ans}
 \end{aligned}$$

Solution for 2nd Part.

When $x = -1, y = -9, z = 4$

$$\begin{aligned}
 &\frac{x^3y - 2z}{xz} = \\
 &= \frac{(-1)^3(-9) - 2(4)}{(-1)(4)} \\
 &= \frac{-1(-9) - 8}{-4}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{9 - 8}{-4} \\
 &= \frac{1}{-4} \\
 &= -\frac{1}{4} \text{ Ans}
 \end{aligned}$$

- (b) $\frac{x^2y^2 - 5z^4}{xyz}$ for $x = 4, y = -2$ and $z = -1$

Solution:

$$\begin{aligned}
 &\frac{x^2y^2 - 5z^4}{xyz} \\
 &= \frac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)} \\
 &= \frac{16(-8) - 5(1)}{8} \\
 &= \frac{16(-8) - 5(1)}{8} \\
 &= \frac{-128 - 5}{8} \\
 &= -\frac{133}{8} \\
 &= -16\frac{5}{8} \text{ Ans}
 \end{aligned}$$

Q.5 Perform the indicated operation and simplify.

(i) $\frac{15}{2x - 3y} - \frac{4}{3y - 2x}$

Solution:

$$\begin{aligned}
 &\frac{15}{2x - 3y} - \frac{4}{3y - 2x} \\
 &= \frac{15}{2x - 3y} - \frac{4}{-2x + 3y} \\
 &= \frac{15}{2x - 3y} - \frac{4}{-(2x - 3y)} \\
 &= \frac{15}{2x - 3y} + \frac{4}{2x - 3y} \\
 &= \frac{19}{2x - 3y} \text{ Ans}
 \end{aligned}$$

$$(ii) \quad \frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$$

$$\text{Solution: } \frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$$

$$= \frac{(1+2x)^2 - (1-2x)^2}{(1-2x)(1+2x)}$$

$$= \frac{(1)^2 + (2x)^2 + 2(2x)(1) - [(1)^2 + (2x)^2 - 2(2x)(1)]}{(1)^2 - (2x)^2}$$

$$= \frac{1+4x^2+4x - [1+4x^2-4x]}{1-4x^2}$$

$$= \frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$

$$= \frac{4x+4x}{1-4x^2}$$

$$= \frac{8x}{1-4x^2} \text{ Ans}$$

$$(iii) \quad \frac{x^2-25}{x^2-36} - \frac{x+5}{x+6}$$

$$\text{Solution: } \frac{x^2-25}{x^2-36} - \frac{x+5}{x+6}$$

$$= \frac{(x)^2 - (5)^2}{(x)^2 - (6)^2} - \frac{x+5}{x+6}$$

$$= \frac{(x+5)(x-5)}{(x+6)(x-6)} - \frac{x+5}{x+6}$$

$$= \frac{(x+5)(x-5) - (x-6)(x+5)}{(x+6)(x-6)}$$

$$= \frac{(x+5)[(x-5) - (x-6)]}{x^2 - 6^2}$$

$$= \frac{(x+5)(x-5-x+6)}{x^2-36}$$

$$= \frac{(x+5)(1)}{x^2-36}$$

$$= \frac{x+5}{x^2-36} \text{ Ans}$$

$$(iv) \quad \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2}$$

$$\text{Solution: } \frac{x}{x-y} - \frac{y}{x+y} - \frac{2xy}{x^2-y^2}$$

$$= \frac{x(x+y) - y(x-y) - \frac{2xy}{x^2-y^2}}{(x-y)(x+y)}$$

$$= \frac{x^2 + \cancel{xy} - \cancel{xy} + y^2}{(x)^2 - (y)^2} - \frac{2xy}{x^2-y^2}$$

$$= \frac{x^2+y^2}{x^2-y^2} - \frac{2xy}{x^2-y^2}$$

$$= \frac{x^2+y^2-2xy}{x^2-y^2}$$

$$= \frac{(x-y)^2}{x^2-y^2}$$

$$= \frac{(x-y)(\cancel{x-y})}{(x+y)(\cancel{x-y})}$$

$$= \frac{x-y}{x+y} \text{ Ans}$$

$$(v) \quad \frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18}$$

$$\text{Solution: } \frac{x-2}{x^2+6x+9} - \frac{x+2}{2x^2-18}$$

$$= \frac{x-2}{(x)^2 + 2(3)(x) + 3^2} - \frac{x+2}{2(x^2-9)}$$

$$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2[(x)^2 - (3)^2]}$$

$$= \frac{x-2}{(x+3)^2} - \frac{x+2}{2(x-3)(x+3)}$$

$$= \frac{x-2}{(x+3)(x+3)} - \frac{x+2}{2(x+3)(x-3)}$$

$$= \frac{2(x-3)(x-2) - (x+3)(x+2)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2(x^2-2x-3x+6) - (x^2+2x+3x+6)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2(x^2-5x+6) - (x^2+5x+6)}{2(x+3)(x+3)(x-3)}$$

$$= \frac{2x^2 - 10x + 12 - x^2 - 5x - 6}{2(x+3)^2(x-3)}$$

$$= \frac{x^2 - 15x + 6}{2(x+3)^2(x-3)} \text{ Ans}$$

(vi)

$$\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

Solution:

$$\frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{(x+1) - (x-1)}{(x-1)(x+1)} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{x}+1 - \cancel{x}+1}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2}{x^2-1} - \frac{2}{x^2+1} - \frac{4}{x^4-1}$$

$$= \frac{2(x^2+1) - 2(x^2-1)}{(x^2-1)(x^2+1)} - \frac{4}{x^4-1}$$

$$= \frac{\cancel{2x^2} + 2 - \cancel{2x^2} + 2}{(x^2)^2 - (1)^2} - \frac{4}{x^4-1}$$

$$= \frac{4}{x^4-1} - \frac{4}{x^4-1}$$

$$= \frac{4-4}{x^4-1}$$

$$= \frac{0}{x^4-1}$$

$$= 0 \text{ Ans}$$

Q.6 Perform the indicated operation and simplify.

(i) $(x^2 - 49) \cdot \frac{5x+2}{x+7}$

Solution:

$$(x^2 - 49) \cdot \frac{5x+2}{x+7}$$

$$= [(x)^2 - (7)^2] \cdot \frac{5x+2}{x+7}$$

$$= (x+7)(x-7) \cdot \frac{(5x+2)}{(x+7)}$$

$$= (x-7)(5x+2) \text{ Ans}$$

(ii) $\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+6x+9}$

Solution:

$$\frac{4x-12}{x^2-9} \div \frac{18-2x^2}{x^2+2(x)(3)+(3)^2}$$

$$= \frac{4(x-3)}{(x^2)-(3)^2} \div \frac{2(9-x^2)}{(x+3)^2}$$

$$= \frac{4(\cancel{x-3})}{(\cancel{x-3})(x+3)} \times \frac{(x+3)^2}{2(9-x^2)}$$

$$= \frac{4}{x+3} \times \frac{(x+3)^2}{2(3+x)(3-x)}$$

$$= \frac{\cancel{2} \cancel{2} \cancel{x} (x+3)^2}{\cancel{2} (x+3)^2 (3-x)}$$

$$= \frac{2}{3-x} \text{ Ans}$$

(iii) $\frac{x^6 - y^6}{x^2 - y^2} \div (x^4 + x^2y^2 + y^4)$

Solution:

$$\frac{x^6 - y^6}{x^2 - y^2} \div (x^4 + x^2y^2 + y^4)$$

$$= \frac{(x^2)^3 - (y^2)^3}{x^2 - y^2} \div (x^4 + x^2y^2 + y^4)$$

$$= \frac{(\cancel{x^2 - y^2}) [(x^2)^2 + x^2y^2 + (y^2)^2]}{(\cancel{x^2 - y^2})} \div (x^4 + x^2y^2 + y^4)$$

$$= (x^4 + x^2y^2 + y^4) \times \frac{1}{(x^4 + x^2y^2 + y^4)}$$

$$= 1 \text{ Ans}$$

(iv) $\frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x}$

Solution:

$$\frac{x^2-1}{x^2+2x+1} \cdot \frac{x+5}{1-x}$$

$$= \frac{(x+1)(x-1)}{(x^2+2(x)(1)+(1)^2)} \times \frac{x+5}{-(x-1)}$$

$$= \frac{(x+1)(\cancel{x-1})}{(x+1)^2} \times \frac{(x+5)}{-(\cancel{x-1})}$$

$$= -\frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x+1)}$$

$$= -\frac{(x+5)}{x+1} \text{ Ans}$$

(v) $\frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 + xy}{y(x+y)} \div \frac{x^2 - x}{xy - 2y}$

Solution: $\frac{x^2 + xy}{y(x+y)} \cdot \frac{x^2 + xy}{y(x+y)} \div \frac{x^2 - x}{xy - 2y}$

$$= \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \cdot \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \div \frac{x(x-1)}{y(x-2)}$$

$$= \frac{x\cancel{}}{y\cancel{}} \times \frac{\cancel{y}(x-2)}{\cancel{y}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)} \text{ Ans}$$

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